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PAPERS

COST OF ENERGY GENERATION

SECOND SYMPOSIUM ON POWER COSTS

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ELEMENTS OF COST

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Synopsis

In recent years the mere mention of the word, "power," especially when uttered by a Government official or used in connection with the word, "water," has been sufficient to send large groups of Americans into hysterics. As a result it has been difficult for any one to approach this subject calmly at a time when a calm approach is needed.

Modern civilization is founded on the utilization of power in a myriad of ways. Of the total energy used in the United States, that produced by water power has remained almost constant at between 3% and 4% for fifty years. The energy used to generate electricity for public utilities in 1936 amounted to less than one-tenth of the total derived that year from mineral fuels and water power. Stated in its true perspective in this manner, the importance of power supply for public utilities appears comparatively small.

There is nothing in the situation with respect to power that cannot be rationalized. When reduced to its elements, the power question resolves itself into simple terms. This paper is concerned with elements of power cost, and the writer submits that these elements, in themselves, are not complicated. Papers will follow which will deal more specifically with individual items of cost.

It should be borne in mind—and many writers on the subject have not consistently done so—that each power development or project is a problem in itself, and that it is unsafe to generalize broadly on the power theme. "Sauce for the goose" is not necessarily "sauce for the gander" in the power field. That method of power generation which is most economical, and which, therefore, should yield the greatest social values in one area, may be the most costly in another. In any locality, changing conditions also may make the method that was most economical yesterday more costly to-morrow. These facts are self-evident to the Engineering Profession, but they have not always been so treated.

The elements of cost of electric power fall naturally into two general groups: (1) Fixed charges which include interest and amortization, depreciation and obsolescence, taxes and insurance; and (2) operating expenses, which include fuel in the case of steam and Diesel generating plants, water, oil, and operating supplies, operators' wages, superintendence, maintenance and repairs, administration and legal expense, and miscellaneous expenses.

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The elements of cost of power are essentially the same whether produced by steam, Diesel, or hydro-electric plants, except the cost of fuel, but the extent to which the various elements enter into the cost varies widely. The purpose of this paper is to outline the elementary parts involved in the cost of producing electrical energy and to indicate how the total cost of energy can be affected.

Fixed Charges

Fixed charges depend primarily upon the investment cost of the various components of the power system, and they represent a very large part of the total cost of water power. In steam plants, however, the fixed charges represent a much smaller proportion of the total cost of power.

The capital cost of hydro-electric developments varies widely and depends on the physical characteristics of the site, such as topography and location, head, stream flow, storage requirements, right of way, etc. The capital cost of steam plants also varies through a considerable but fairly well established range and depends upon the type and capacity of the plant, location, kind of fuel, and fuel economy for which the plant is designed.

The rate of interest paid on the money invested in a power project varies from year to year, depending upon the condition of the money market. At present, both public and private agencies can borrow at from 3.5% to 4% per yr. A public agency usually finances its undertakings by the sale of bonds, and provision must be made for the retirement of the bonds as well as for payment of interest. Private company financing on the other hand is somewhat more involved; funds may be obtained by sale of bonds or by sale of stock, or by a combination of the two. The principal difference between the public and the private agencies in the matter of fixed charges is that the private company endeavors to secure a so-called fair rate of return on its investment, whereas the public agency is generally concerned only with interest and amortization of the capital investment.

Depreciation depends upon the useful life expectancy of the various features of the power development. The usual practice of both public and private power agencies is to provide a depreciation reserve fund for the purpose of replacing the various components of the power system. In the case of a public project financed entirely by sale of bonds which must be retired within, say, 30 or 40 yr, it is doubtful whether depreciation should be included in the features that will have a useful life far beyond the end of the amortization period of the bond. For instance, a large concrete dam, which is a major feature of a hydro-electric development, has a relatively long life and may reasonably be expected to be useful for 50 or 100 yr, or even longer. If depreciation charges are included in the cost of such a feature during the period that its cost is being amortized, it would in effect constitute a double repayment. It is necessary, however, to include sufficient depreciation to provide for replacement of those items which will have to be replaced during the amortization period. Although a dam may have a useful life expectancy in excess of the amortization period, it may be equipped with gates and other mechanical

equipment which will have a much shorter life, and will have to be replaced during the amortization period.

The useful life expectancy of different features of a power system varies considerably. For example, a steam-generating station may have a life expectancy of, say, 30 yr, whereas a hydro-electric plant may have a life expectancy of from 50 to 75 yr. The machinery and equipment in a steam plant operate at relatively high speed and at high temperature; hydro-electric plant equipment operates at slow speed and at much lower temperatures. It is customary, therefore, to assume a higher rate of depreciation for a steam plant than for a hydro-electric plant. Similarly, for other features of a power system, the depreciation in each unit should be based on the best estimate of its probable useful life.

Taxes are an important item in the total fixed charges of a private power company, and they vary considerably in different localities. A public power development on the other hand is usually exempt from taxation. In certain instances, publicly owned power systems have voluntarily included taxes in the rate base, and this seems to be desirable practice. A private power company usually must pay for franchises permitting it to operate, and it must also pay State and Federal income taxes. The public agency is not normally burdened with such charges. In general, the fixed charges are considerably higher for a private than for a public power agency.

Operating Expenses

The cost of operating hydro-electric plants is a relatively small part of the total cost of power generation, and is practically independent of the plant output. The operation costs of steam-electric plants differ markedly from those of hydro-electric plants. The cost of fuel is the major factor in the cost of operating steam plants, and the quantity of fuel consumed varies with the energy output of the plant.

Operating expenses consist principally of operators' wages, oil, operating supplies, maintenance and repairs, and fuel in the case of heat-generated energy. Fuel costs vary materially due to location, distance from source of supply, method of transportation, and competition between fuels; and the cost of a given fuel may fluctuate from year to year. Many modern steam plants are designed to burn two kinds of fuel, such as coal and oil, or natural gas and oil. This arrangement makes it possible to take advantage of the competitive situation and use whichever fuel may be cheaper at the moment. In rare instances, natural gas can be purchased at low prices during periods when surplus gas is available. Some contracts for natural gas have been made on an off-peak basis, with the supplying agency reserving the right to discontinue delivery during periods when the full capacity of its pipe line is required to supply customers other than the power plant. Under such a contract, the steam plant can take advantage of the cheap natural gas when available, but must resort to a higher priced fuel at times when natural gas is not available.

There has been a gradual improvement in the fuel economy of steam plants, due to the use of larger generating units, higher steam pressures, and higher steam temperatures; and it is now possible, with a modern steam plant designed

for high economy, with units of 50 000-kw capacity, or larger, to secure a fuel economy equivalent to 11 000 Btu per net kw-hr.

Station-service requirements generally range from 4% to 10% of the installed capacity. A steam plant will require considerably more power for station service than a hydro-electric plant of comparable size, because of the necessity for operating cooling water pumps, boiler-feed water pumps, coal-handling equipment, etc.

Effect of Load Factor

The energy output of a given power plant will vary directly with the load factor or the plant capacity factor at which the plant is operated. Stated in another way, in order to produce a given quantity of energy the installed generating capacity must be increased as the load factor is reduced.

The effect of plant capacity factor on the cost of power is particularly marked in the case of hydro-electric plants because the annual cost of these plants is fixed and is practically independent of the plant output. The cost of energy generated at a particular hydro-electric plant may be 4 mills per kw-hr if the plant is operated at 80% load factor; but if it is operated at only 40% load factor, it will produce only one-half as much energy and the cost will be increased to 8 mills per kw-hr. In the case of steam-electric plants the load factor has a decided effect on the cost of power although it is not quite so pronounced. The cost of fuel represents a large part of the cost of operating a steam plant and the quantity of fuel consumed varies with the plant output. The cost of production at a particular steam-electric plant may be 4 mills per kw-hr if the plant is operated at a load factor of 80%, whereas if the same plant is operated at only 40% load factor the cost of energy may be increased only to 6 mills per kw-hr.

Effect of Power Factor

The current to be carried by the electrical equipment depends upon the power factor. If the power factor is low, additional carrying capacity must be provided in the generators, buses, transformers, circuit breakers, and transmission lines, thus increasing the cost of this equipment and increasing the cost of power. This is usually not of great importance in the case of hydro-electric developments having long-distance, high-voltage transmission lines because such lines inherently have considerable electrical capacity which tends to correct the low power factor of the load. Under these conditions the hydro-electric plant usually operates at a reasonably high power factor. Steam plants near the load center, on the other hand, do not have the advantage of the corrective effect of long-distance transmission lines, and the electrical equipment in these plants must be capable of carrying the increased current resulting from low power factor, or synchronous condensers must be provided to correct the condition.

Quality of Power Service

Certain classes of power users are much more exacting as to the quality of power service than others; for example, a metropolitan area demands service

as nearly as possible free from power outages. For these areas the speed must be maintained within very close limits and the voltage must be closely regulated. On the other hand, occasional momentary outages and minor fluctuations in speed and voltage may not be of serious consequence in small communities or to isolated industrial users. The extent to which a power system is designed to insure continuity of service, close regulation of speed, and voltage, is reflected in the cost of power. As an example, the City of Los Angeles, Calif., obtains the major part of its power supply from the Boulder Power Plant, and its service requirements are most exacting. The transmission circuits are designed to be lightning proof which necessitated large expense for overhead ground wires, counterpoises, etc. Special characteristics, such as extra fly-wheel effect for low reactance and high short-circuit ratio, are incorporated in the generators to insure system stability under transmission-fault conditions. Exceptionally high-speed oil-circuit breakers, functioning in less than three cycles, are required to isolate a section of the transmission line on which a fault may occur before service to customers is affected. Special devices and equipment to control accurately the frequency and voltage are provided. All these refinements are necessary to provide the high standard of service demanded by a metropolitan area.

Another block of Boulder power is used by the Metropolitan Water District of Southern California for pumping water through its aqueduct, and in this case the service requirements are much less exacting. The District can tolerate occasional brief outages since they involve only a temporary shut-down of the pumps which can be re-started readily as soon as power service is restored. The cost of the District's transmission lines and generating equipment is considerably less than that of the City of Los Angeles because of the difference in quality of service demanded by the different classes of loads.

Spare generating equipment is an essential part of every power system. It is necessary to have sufficient spare capacity so that equipment can be taken out of service for inspection, maintenance, and repair, without interfering with delivery of power. The minimum amount of spare capacity must be adequate to permit the largest single generating unit on the system to be taken out of service. The element of dependability of the various sources of power must be considered carefully in determining the required amount of spare capacity. In general, the dependability of hydro-electric generating units is substantially greater than that of steam units, due to the fact that the former operate at comparatively slow speed and at ordinary temperatures, whereas the steam units operate at very high speed and at high temperatures.

Cost of Transmission

Generating stations must necessarily be located where conditions are suitable. A steam plant must have an adequate supply of cooling water for condensing purposes, and must be situated where transportation facilities are available for delivering fuel economically; and a hydro-electric plant must be located where water and head are available. Transmission facilities are necessary for transmitting the power from the point at which it is generated to the load centers, and the cost of transmission is an important item in the

total cost of power delivered at such load centers. In the case of a hydro-electric development situated remote from the load center, the cost of transmission may be as great as, or greater than, the cost of generation.

The output of the generating station must be delivered to the transmission lines at their operating voltage and this necessitates step-up transformers and high-voltage switching and protective equipment at the generating station. The cost of the transforming and high-voltage switching equipment at the generating end of the transmission line is usually dependent largely upon the voltage of the lines and is independent of the distance of transmission.

Provision must be made for transmission-line outages, either by duplicate circuits or by stand-by generating capacity at the terminal end. If the transmission lines are designed and built in accordance with the best modern practice, such outages are infrequent.

The cost of power transmission depends principally upon the length, voltage, and capacity of the line, and upon the load factor at which the line is operated. The annual costs of a particular transmission system are practically fixed and are independent of the amount of energy transmitted; and, as in the case of hydro-electric plants, the higher the load factor the lower will be the cost of transmission. The over-all efficiency of transmission, including the voltage transformation under generating and receiving ends, should generally be better than 90 per cent.

The distance from a power development to a market is another factor that affects the cost of power. The greater the distance the greater the cost of transmission, and although a development in itself may be exceptionally low in cost per unit of capacity, it may require so large an investment in transmission lines to reach a market that the power cannot be delivered in competition with power from other sources. On the other hand, a relatively large investment per unit of capacity may be warranted in a plant situated close to a load center and requiring small investment in transmission facilities.

Inter-Connections

Power supplies in the United States have developed from the steam engine or water-wheel supplying an individual mill or factory to the central station supplying a community; and then to the power system with a combination of steam and hydro-electric generating stations supplying power to a large territory; and, finally, to the inter-connection of power systems which makes it possible to centralize the generation of steam power at advantageous points where fuel and water for condensing purposes are readily obtainable, and to use large capacity and highly efficient generating units. These large generating plants connected by, and feeding power into, a large transmission system or network, make it possible to take advantage of diversity between the different loads on the system, minimize the stand-by capacity and spare equipment required to insure continuity of service, and secure high plant capacity factor and high fuel economy. It also makes it economically feasible to develop large-capacity hydro-electric projects and to utilize large amounts of low-cost, secondary, or seasonal hydro-electric energy which otherwise would be wasted. Inter-connection of power systems has played an important part in reducing the cost of power.

Many large power systems are supplied by a combination of hydro-electric power and steam. Steam plants are the more economical for supplying peak-load capacity due to their low investment cost per unit of capacity, whereas the hydro-electric plant may be the more economical for supplying the energy requirements of the system, notwithstanding their higher cost per unit of capacity due to their low operating cost. Normally, the hydro-electric plants are operated so as to generate the maximum energy and the steam plants are used to supply the system peak-load requirements with a minimum of fuel consumption. During periods of subnormal water supply, however, normal operating procedure may be reversed and the steam plants may be operated on base load at high-load factor to supply the deficiency in energy output of the hydro-electric plants whereas the latter are used to supply system peak-load requirements. Thus, the two sources of power supplement each other and although the total investment in generating capacity required to supply a given system peak may be greater with a combination of steam and hydro-electric than with steam alone, nevertheless, the combination results in cheaper power, especially where large quantities of low-cost secondary or seasonal hydro-electric energy can be utilized in lieu of steam-generated energy.

Market for Power

The market for power and the length of time required to absorb the power output must be carefully considered in determining the economic feasibility of a power development. This is particularly important in the case of large hydro-electric developments involving large initial capital outlay. The major part of the cost of a hydro-electric development is in the dam, storage reservoir, power-house, etc., which must be constructed before power can be produced. The annual cost of such a project may be in excess of the annual revenues for several years until a large part of the available power can be disposed of, and deficits will accumulate. The deficits accumulated during the early years may make the project economically infeasible.

Large Federal Multiple-Purpose Projects

In recent years, a number of large multiple-purpose projects, involving flood control, improvement of navigation, storage of water for irrigation and domestic purposes, silt storage, prevention of encroachment of salt water into fresh-water supplies, and power development have been undertaken by the Federal Government. These are essentially conservation projects of national scope and the power which they make available, although important from the standpoint of defraying part or all of the cost of these projects, is incidental and subordinate to the other more vital functions. These projects are too large and require too much capital outlay to be economically feasible for development by private enterprise.

The cost of power at these multiple-purpose projects depends largely upon the allocation of the cost to the various functions served. For instance, the primary purposes of the Boulder Canyon Project are to regulate the Colorado River in the interests of navigation; to provide flood protection for the Lower Colorado River Valley, including the rich Imperial Valley in Southern Cali-

fornia, and to conserve the flood waters which otherwise would be wasted into the Gulf of California to make them available for irrigation and domestic uses. It seems appropriate to allocate part of the cost of Boulder Dam to items of major Federal interest, and Congress recognized the primary purposes of the project by providing that \$25 000 000 of the cost of the project should be allocated to flood control to be repaid out of surplus revenues if any accrue.

The Bonneville Project on the Lower Columbia River serves to improve navigation as well as to produce hydro-electric power, and the Federal Power Commission is now (1938) considering the proper allocation of the cost of this project as between navigation and power. The Grand Coulee Project on the Upper Columbia River is primarily a conservation project which will provide water for the irrigation of about 1 200 000 acres of land in the Columbia Basin. The reservoir created by Grand Coulee Dam will afford some measure of flood control, will improve navigation, and will greatly increase the low-flow power output of the present and prospective power developments on the Columbia River below that point. The proper allocation of the cost of Grand Coulee Dam as between the various functions it serves, should be considered at the proper time, and the cost of power should be established on the basis of the allocation finally adopted. The Shasta Dam on the Upper Sacramento River in Northern California is another of the large Federal projects. It involves flood control, navigation improvement, salinity control, conservation of flood waters for irrigation and domestic uses, and, incidentally, the development of a large block of power. Here, again, a proper allocation of the cost of the project to the various functions it serves will be necessary in determining the cost of power.

It is useless to argue that the Federal Government should not build projects of this type because they also generate power. Social considerations demand their construction and also demand their full utilization, as well, so that the potential energy they create must be developed and made available for use.

One hears of social and of human engineers. The terms are of little moment. More important is the need for realization by the profession, of the kinship between engineering activities, as such, and the progress of an industrial, a power, civilization. Too few engineers have included in their activities a broad consideration of the social and economic objectives of the major problems upon which they have been called to work. Pre-eminent in the technical fields, all have not yet seized the opportunity to apply their abilities on the broader front.

Thought on the subject of power ranges from that of the evangelist who believes that power should be made as free as air, to that of the tycoon who views power as the silver spoon meant for the mouths of those born to the economic purple. As a professional man, the engineer has spent much of his time with the formulas of the power industry, but has devoted too little of his thought to the implications, social and otherwise, of what he has wrought. Has not the time come for the engineer to apply more of his effort to the determination of a sound basis for the settlement of the power question?

If the profession accepts, as it must, the idea that such great projects as Bonneville, Boulder, Grand Coulee, Shasta, and other dams must be built,

has not the time come for the Engineering Profession to approach them calmly so as to appraise their benefits soundly both as power projects and as conservation projects?

Public agencies have demonstrated, in many localities, that they can manufacture and distribute power efficiently and successfully with public benefits. Privately owned utilities have served as admirably in many other localities. Each power project is a separate problem; because one method has rendered better service in one instance is not conclusive proof that another will not render better service elsewhere. Remembering these facts, has not the time arrived for the Engineering Profession: (1) To concede that inherently there is nothing bad in either public or private ownership of electrical utilities; (2) to accept the fact that either must earn its right to continued existence in any locality by efficient service in the public interest; (3) to lay aside prejudices; and (4) to apply itself with the precise tools it has available in technical and scientific knowledge to the job of solving the power problem?

The electrical industry is monopolistic by nature. It is self-evident that, in a democracy, a monopoly can be tolerated only as long as the people feel that it serves in the public interest. Is it not time, then, for engineers, who play so prominent a rôle in the field of power, to assist, honestly and wholeheartedly, in seeing to it that the public weal is placed foremost among the objectives of the power industry?

HEAT-GENERATED ENERGY

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SYNOPSIS

This paper is devoted to the cost of producing electric power in fuel-burning plant. It is divided roughly into five parts: The first covers the definition of cost as used herein and certain factors affecting the cost; the second treats cost of steam-electric generation; the third refers briefly to the performance and possibilities of binary vapor cycles, with particular reference to the mercury vapor type; the fourth is devoted to cost of Diesel plant generation; and the fifth contains a very brief reference to future possibilities. Power cost, as used herein, refers to the total of operating, fixed charge, and general overhead expense at the point of generation.

INTRODUCTION

When one speaks or writes of cost it is necessary to be very specific if one's statements are to have more than general significance. Whether reference is made to commodity or service, the expression, "cost," is meaningless unless many characteristics of an actual or assumed transaction are given. Thus, if one refers to the cost of apples, one may have reference to the cost of production in a given orchard in a given year, or to the average cost of production throughout a section or over the entire country. Such a person may have reference to the wholesale or retail price at a given time and place; he may have reference to the export price at a given time and place; and so on through a great many possibilities. It is obvious that the cost under any or all of the mentioned conditions, and others, may properly be spoken of as the cost of apples, and it is equally obvious that all such costs might be different at exactly the same time.

The cost of power is an equally flexible expression. It may refer to production cost at the point of generation with or without allowance for power used in the production process. It may mean either of these limitations, enlarged by any number of other charges, such as supervision by staff members, taxes, an allowance of some kind for physical or some other form of depreciation, interest on the investment in production facilities, etc.; or it may mean cost to the producer in any of these forms at a point remote from the place of generation, as, for example, at the end of a transmission line at transmission-line voltage, or at some lower voltage. On the other hand, it may mean cost to the producer in any one of a great number of forms at some point or points in a distribution system; or it may mean cost to the consumer, including not only the total cost of the actual power to the producer at the point of delivery, but also the cost of meter reading and billing, the cost of any services other

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than pure power supply, a proper share of general costs of several varieties, and profit, if there is any.

In view of such a multitude of possible meanings of the title of this paper, and in view of the resulting danger of fallacious comparisons and inferences, it is deemed necessary to define the sense in which cost of power is used herein. The expression is used to designate the total cost of power to the producer at the point of generation and ready for delivery to the transmission or distribution system which is to convey it elsewhere. By total cost is meant not only the cost of labor and fuel and other supplies, but also all supervision, all allowances for depreciation, taxes, and all other forms of overhead expanse. This may be called the "total power plant cost," or the "total cost at the power plant." The significance of the word, "total," should not be forgotten. Attention is called to the fact that cost as used herein is in terms of net output, not total generation; that is, the unit cost is determined by dividing the total cost, as defined, by that number of kilowatt-hours delivered to the line and available for transmission or distribution as the case may be.

For the benefit of those who are not familiar with the facts and economics of power supply, it is desirable to emphasize the fact that the cost of power as defined herein may represent legitimately either a very large part or a very small part of the cost to the ultimate consumer. For example, if a fuel-burning power plant is considered, which is situated in a manufacturing establishment of small extent, and is supplying all its output to that establishment, transmission costs are not involved, distribution costs are small, meter reading and billing are not necessary, and no kind of service other than power supply is given. Under such circumstances the total power-plant cost, as defined herein, represents practically the entire cost to the consumer. If, however, the power must be transmitted at high voltage to a considerable distance and there sold wholesale to a community, the power-plant cost must obviously be increased by all transmission costs, including losses, operating expenses, and fixed charges, and by the profit, in order to obtain the cost to the ultimate consumer. Neglecting the item of profit, it is quite possible for the legitimate cost of delivery at the wholesaling point to equal one and one-half times, or more, the total cost of the power plant. Furthermore, if the power is to be retailed over a wide area, even near the power plant, it is quite possible for a legitimate cost to the consumer to equal seven or eight times the power-plant cost and to vary widely each side of these values depending upon the density and character of load, the character of the territory, the kind of service rendered, etc. It is obvious that mere power-plant cost tells a very small part of the entire story and that many fallacious conclusions may be drawn with respect to power cost under different conditions if this fact is not fully appreciated.

A presentation of the data indicated by the title of this paper would be simple indeed if it were true that the total power plant cost of fuel-generated power were the same every where, or under all conditions, or even at a given fuel cost. No such simple relations exist; the cost of fuel-generated power is uniquely different for different conditioning factors. Moreover, it is practically impossible to predict how these conditioning factors will combine in any case until a specifically detailed study has been made for that case. If one is to

consider such power costs in any general manner it must be done in terms of cost ranges instead of in terms of specific costs. This method has been adopted in the paper.

As stated previously, the total plant cost of power is made up of a number of cost items, a complete list, divided under three convenient and conventional headings, is as follows:

(*1*) *Operating Costs:* Fuel; operating labor; water; miscellaneous supplies and expenses; maintenance (labor and materials); and superintendence, clerical and testing.

(*2*) *Fixed Charge Expense:* Charge for depreciation; cost of money (interest, dividends, etc.); property taxes; and insurance (fire, casualty, liability, etc.).

(*3*) *General Over-Head Expense of Which Parts Should Be Allocated to Power Production:* Staff superintendence; staff accounting; general offices; stores and warehouses; transportation and communication; general and coal-testing laboratories; shops; working capital; regulatory commission expense; public injuries and damages; employee compensation insurance; social security tax; general taxes (income, surplus, sales, excise, franchise, corporation, etc.); legal and miscellaneous general expense.

Nearly every item in this long list does exist as an item of cost in any fuel-power production undertaking. Certain of them may be masked or may be "lumped in" with others so that their identities are lost, but they exist nevertheless. In small operations, laboratory expense undoubtedly would be absent. In private undertakings, such as industrial power plants as opposed to public utility activities, regulatory commission expense would not exist. With such limited exceptions, however, all the items have their equivalent in any production of power from fuel although their existence may not be appreciated and may not be shown by the method of accounting in use.

A mere survey of this list of components of total cost indicates the impracticability of treating every item in a paper of moderate length. Attention must be concentrated on the more important and larger components. Those that are slighted may conveniently be grouped into a percentage loading on others, or handled in some equally simple fashion; but their existence should not be overlooked. Furthermore, it is convenient to treat the steam and the internal combustion methods of power production separately. If this is not done the treatment becomes undesirably complex and difficult to follow.

Steam Power Costs

The methods used for producing power from steam are numerous. They all have in common the use of fuel-fired steam boilers which supply steam to prime movers of some kind; and that is about all they have in common. In general, when one refers to a steam-operated power plant, one visualizes the highly developed, condensing, steam-turbine, type of plant used in nearly all central stations and in many industrial plants. It must be remembered, however, that such plants represent only one of many classes; for example, the steam locomotives of the United States use a large part of the total fuel consumption and are just as truly steam power plants as the most elaborate

central stations. Many industries use steam engines or steam turbines as devices by means of which to skim a certain amount of power from a steam supply required for process purposes. In such cases the major part of the heat originally in the fuel may legitimately be charged to process steam so that the fuel cost of the power may be reduced to comparatively low values. Similarly, under such conditions it is frequently legitimate to charge a large part of the cost of steam-producing equipment to process steam, thus diminishing the fixed charges against power.

Obviously, no paper of reasonable length could attempt to cover all such possibilities. After considerable thought the writers have decided to limit this section to the costs of fuel-generated power produced in condensing steam-turbine stations of what may be called the central station type. This type represents the nation's best achievement in the production of power as a commodity as cheaply as is compatible with reasonable continuity of supply. In adopting this course it seems wise not to neglect realities and, therefore, not to limit the treatment to the most economical central station that can be built to-day. The fact is that central station supply is now an old industry which of necessity carries with it those of the physical constructions incident to its growth which sound economics or the laws and regulations under which it operates have not yet required or permitted it to retire. An idealist might like to see everything but the latest and best retired over night. A realist takes cognizance of the fact that economic considerations do not permit this.

The fact is that the cost of steam-generated power from central stations is a composite of the cost obtained with plants of varying ages and, therefore, representing different stages in the development of the art. Naturally, each property utilizes its most economical equipment to the greatest practicable extent, but there are many limitations made necessary by physical and other factors that must not be ignored.

The writers have attempted to give a reasonably fair "picture" by considering the performances to be expected of plants operating at different steam pressures and steam temperatures, choosing the values so as to cover the entire field as it is found to-day. This has resulted in the choice of the following steam conditions:

Pressure, in pounds per square inch	Temperature, in degrees Fahrenheit
225	650
400	750
600	825
800	910
1 250	925

The values listed are to be regarded as nominal; that is, indicative of the general range in which actual values will be found. For example, 225 lb may represent any value between, say, 175 and 250 lb, whereas 650° F may represent any value between 550 and possibly 675 degrees.

It is also to be noted that no fixed and definitely predictable thermal performance can be assigned to any single combination of steam pressure and

temperature. For one thing, there has been an ever-continuing development of power-plant equipment and power-plant design, so that the possibilities realizable from the use of a given steam condition have changed almost from year to year. Conditions and circumstances associated with different installations result in designing for different performances from what superficially looks like similar equipment, in order to obtain the best economic balances in different cases. Therefore, in presenting the performances from which costs may be calculated, it is necessary to use bands or ranges rather than exactly fixed or unique values.

The very marked effect of condenser vacuum upon the thermal performance of a steam-turbine plant is now generally well known. The attainable vacuum in any location is determined both by the quantity and the temperature of available circulating water and, to some extent, by its quality. In any general study of the type under discussion it is necessary, therefore, to take account in some way of the effect of average attainable vacuum. This has been done by considering a range covering the generally existing values.

It is not so commonly appreciated that the character of the load has a very marked effect upon attainable thermal and economic performances. This is a fact, however, and one fraught with complexities. Each physical unit concerned has a certain characteristic which may be defined as performance *versus* load. In general, the best thermal performance of the assembly of physical units constituting a power plant is attained near what is known as rated load, although this may be modified through rather wide limits at the time of design. Therefore, operation for long periods of time at lower rates of output must of necessity reduce the thermal performance.

This sounds, and is, simple; but it is by no means the whole story. The manner in which the load varies may be of greater significance than its average value over a given period. For example, two plants of like capacity might operate under such conditions that both of them produced in a given time, say, 24 hr, the same number of units of output. One might be operating under such conditions that it carried a perfectly uniform load throughout the 24-hr period, whereas the other produced all, or substantially all, of its output in, say, 9 hr. The characteristics of steam power plants are such that with other factors reasonably equal, the thermal performance of the steadily loaded plant might be appreciably better than that of the other. These are extreme assumptions with respect to central station operation but might be approached rather closely in industrial plants.

Another significant type of variation concerns the size of the load from minute to minute. If there is great fluctuation, the thermal performance is necessarily poorer than it would be if the same total output were produced under more nearly constant conditions. Variation from day to day is also of importance; for example, a plant may be heavily loaded during four or five days of the week and lightly loaded, or actually idle, during the remaining days. These variations and many others affect the initial design of the plant to the extent to which they can be foreseen; but, irrespective of the provisions that may be made in this direction, they affect both the thermal and the economic performance of the plant as it is operated.

Obviously, a given investment will be used to the best advantage with respect to unit fixed charges when the maximum number of units of product is produced. Moreover, a load characterized by high, short-time peaks must necessarily involve a greater investment in plant than one totaling the same number of units of output produced at more nearly a constant rate. It is evident, therefore, that the character of the load must affect the economic as well as the thermal performance.

Power engineers endeavor to express the significant characteristics of load by the use of a number of ratios known as factors. Thus, they use load factor, power factor, use factor, capacity factor, and others. Each has value and each tells something about the load; however, all combined, they still leave much of importance undisclosed. In any such general study as herein intended, it is not practicable to show completely the effects of all the possible characteristics of load. This is sufficiently difficult even in the study of an isolated case, and makes it almost impossible to compare fairly the performances obtained in as few as two isolated cases.

Under the circumstances the writers have had to content themselves with the use of one factor which may be regarded as the most general in character. By virtue of its generality it is correspondingly lacking in definitiveness. The criterion used is the annual capacity factor, or plant factor, which measures the actual output against what it would have been had the plant operated at full rated output throughout the year. This definition requires explanation. There can be little argument as to the actual annual output in any given case; but there are many different but accepted ways of computing what the output would have been had the plant operated at full throughout the year. As used herein, it is assumed that steam generator capacity, prime mover capacity, and electric generator capacity, together with the capacity of all auxiliary equipment, are so well balanced that nothing prevents the operation of the main generators at full rated capacity indefinitely. The maximum possible output that is used in calculating the annual capacity factor is then based on the theoretical operation of the main generators at full rated capacity throughout the year.

In a continuing sense this maximum possible output is an unattainable performance in any real case, even if the load permitted. Equipment does wear out, maintenance must be performed, and occasional breakdowns of major character do occur. Therefore, the annual capacity factor as used herein is, in a sense, theoretical although it does have the value of being exact and relatively easily determined. In actual cases it always falls well below unity or 100%, depending upon the form in which one chooses to express the ratio.

The range used herein extends from 20 to 80 per cent. Old plants that are used only for peak and stand-by service may show an annual capacity factor well below 20%, or even below 10 per cent. When such low values are reached thermal performance is of little significance and power costs from such plants are too erratic to have much meaning in a paper of this kind, although they may have very great significance as constituting part of the power cost of a larger system. Few fuel-burning plants attain an annual capacity factor, as herein defined, in excess of 80 per cent. Even industrial

plants that operate at practically full load for 24 hr per day seldom achieve such a value for any extended period. If a plant operated at full rated capacity 300 days per yr and was completely idle during the remaining days its annual capacity factor would be only about 82 per cent.

In the foregoing discussion of capacity factor, reference was made to the rated capacity of the plant. It is obvious that this must have a fairly exact value in any given case if the calculated value of capacity factor is to be properly significant. However, there are other considerations connected with capacity which may be treated in a rougher or more approximate fashion. For such purposes one may refer merely to the size of the plant, or to its capacity, meaning by that something of the order of, say, 100 000 kw instead of 25 000 kw or 300 000 kw.

Size, in this sense, has a most complicated effect upon power cost. In general, it may be stated that total power cost tends to decrease as the size of the plant is increased. This follows from many considerations such as the possible use of larger units with better thermal performance and smaller unit investment, from fewer man-hours per unit of output, and other favorable factors; the economic justification of ultra refinements of various kinds; the spreading of clerical and superintendence charges over a greater output, etc. So many apparent contradictions can be found, however, that it is not safe to assume that cost of power from a larger plant in one location will necessarily be lower than that from a smaller plant in another location. For example, a small plant may be constructed under conditions that justify the designer in planning for high economy, in making no provision for future enlargement, and in providing a minimum of spare capacity because of stand-by service from a transmission line of large capacity. If cost of power from such an installation is compared with that from a larger plant, in which costly provision has been made for relatively great expansion at some future time, and in which several relatively small units have been used to fit best to an existing load curve and supply adequate reserve capacity, it may well be that for some time the thermal, the labor, and the economic performances of the smaller plant will be better than those of the larger plant, all other things being equal. Many examples of other types might be cited as evidence of the danger of carrying a generalization too far.

For convenience, the writers have assumed plants of 150 000 to 250 000 kw in preparing the numerical data that follow. This statement means they have assumed a well balanced plant capable of delivering energy at such a rate when all equipment is available. No assumptions have been made with respect to the nominal rating of the plant because this is too complicated a consideration for general treatment. It involves choice of the size of units and many other choices that can be made only for known and reasonably specific conditions. Instead, they have lumped such variations of power costs as would occur by virtue of different causes into the margins provided by the use of ranges or upper and lower limits of costs of different types.

It is convenient to consider the different plants in the order of ascending pressures. The nominal 225-lb installation, therefore, is selected as the point of beginning. Practically all such plants of any magnitude which to-day are

used for central station purposes are twenty years old or more. Therefore, they represent a comparatively early development of the art as it is now known. Just what they are capable of, with respect to thermal performance, depends upon the stage of development represented at the time of their construction and upon the changes that have been made since then, either by the improvement or replacement of existing equipment, or by the addition of new equipment.

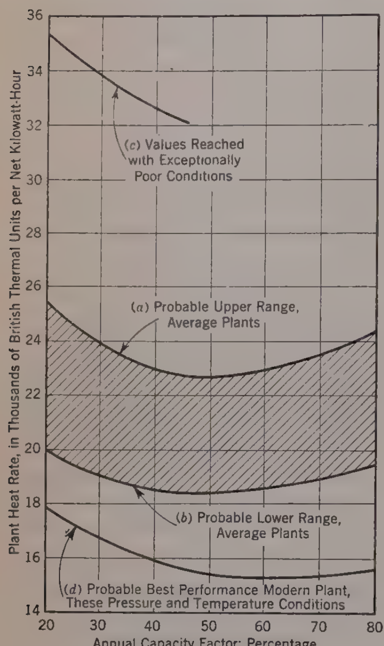

Fig. 1.—Thermal Performance of a Steam Plant; Nominal Pressure, 225 Pounds per Square Inch; 650 Degrees Fahrenheit and a Back Pressure of 1 Inch of Mercury

Several curves showing thermal performances as derived from actual performance data obtained from a number of such plants, are shown in Fig. 1. The hatched area between Curves *a* and *b* indicates the probable range of thermal values for plants representing average conditions, such as typical central station load characteristics, reasonably good grades of eastern or mid-western bituminous coals, units of 20 000 kw, or larger, some modernization since initial installations, etc. Attention is called to the fact that values are all in terms of 1 in., Hg, back pressure. Practically all such plants of which the writers have knowledge are operated at comparatively low capacity factors, some as low as 10% and few as high as 40 per cent. The region beyond 40% is obtained largely by extrapolation and it is questionable in character for two reasons: (1) It is probable that many of the actual plants could not be operated continuously for the long periods of time at the outputs required to give the very high capacity factors, in spite of the fact that they frequently carry full load for shorter periods of time as required to meet the demands of the system; and, (2) the methods of extrapolation may not be very accurate.

Curve *c* in Fig. 1 is introduced to show what may happen under abnormal conditions, such as unusual types of load, very poor or greatly varying fuel characteristics, little or no modernization, etc. The values shown on this curve have not been used in later computations and are included here only to indicate the danger of generalizing to too great an extent in such matters.

Curve *d* represents the probable performance of a plant built for these steam conditions at the present time; that is, a plant designed according to the present status of the art. It is of interest only as indicating the progress which has been made in improving thermal efficiency without reference to the effects of increased steam pressure and temperature. It is not probable that any large plant would now be built with these steam conditions, although exceptionally cheap fuel or other unique conditions might justify such a choice.

Group 1.—Fuel Costs.—The thermal performances as shown in Fig. 1 can be translated easily to fuel costs as soon as the cost of fuel is known. For the purposes of this paper, the writers have chosen to use a fuel cost at the plant of 15 cents per million Btu. This corresponds roughly to eastern bituminous coal of good grade at $4 per ton. It will become evident subsequently that correction for other fuel prices can be made easily. Curves *e* in Fig. 2 (*a*) and 2 (*b*)

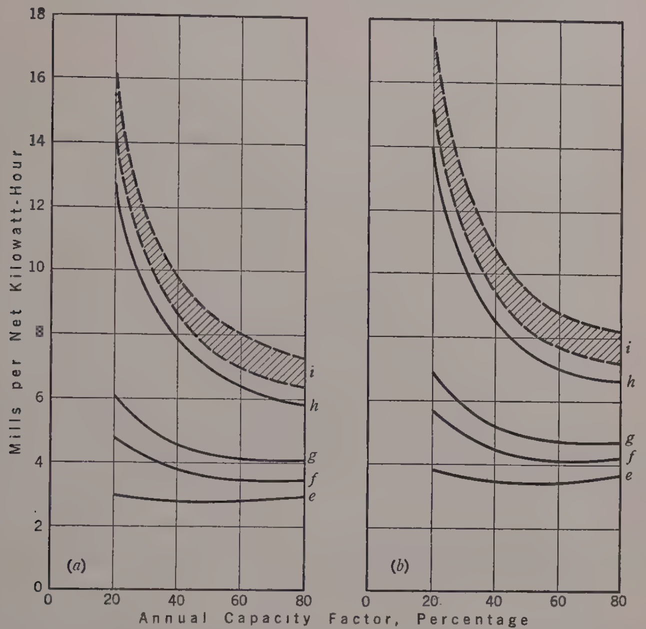

Fig. 2.—Average Lower (*a*) and Upper (*b*) Ranges of Power Cost for a Steam Plant; Nominal Pressure, 225 Pounds per Square Inch; 650 Degrees Fahrenheit; and 1 Inch, Hg, Back Pressure

result from converting Curves *a* and *b* of Fig. 1, into fuel cost in mills per net kilowatt-hour. The method is obvious. It must be remembered that this limits the direct applicability of Curves *e* to 1 in., Hg, back pressure. However, methods of correcting to other pressures are given subsequently herein.

Operating Labor.—The next cost item listed under operating costs is that of operating labor, in which case two major variables must be considered: (1) The output per man-hour as determined by the efficiency of labor, the type of fuel, and the design of the plant; and (2) the average wage paid. This wage varies widely with geographical location, with type of community, and with other factors.

The output per man-hour for the nominal 225-lb plant has been assumed as shown by the full line in Fig. 3. It is applicable to plants burning the better grades of solid fuel and reasonably accurate to a capacity factor of 50 per cent. Below 50%, the performance of most of these plants will fall within ± 30% of the values indicated, as shown by the two dotted lines. Above 50% much will depend upon the character of the plant, if indeed it can be operated to produce

higher capacity factors. As most plants actually are, it is probable that at higher capacity factors the slope of the full line would gradually become less as shown by the dotted extension, indicating increasingly less effective use of men. The same phenomenon would probably occur to an increased extent in plants having the poorer grades of solid fuel. For plants burning liquid and gaseous fuels, the net output per man-hour at any load factor would probably be higher than shown and the slope of the line would tend to remain constant. The data available to the writers are not sufficiently extensive to permit them to be more specific in this regard.

To convert the values shown in Fig. 3 to labor cost, an average hourly wage must be selected. Under the conditions it is necessary to choose some value arbitrarily because it is practically impossible to cover all the local variations in one paper of this type. An average hourly wage of 70 cents has been used herein. Variations of this value are wide in practice.

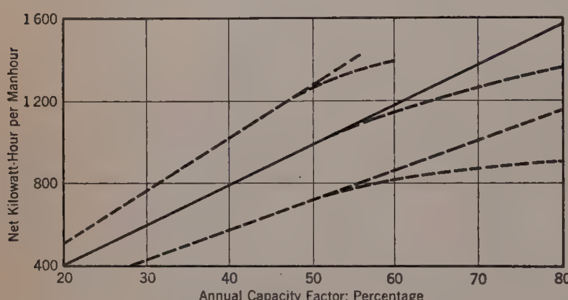

FIG. 3.—NET KILOWATT-HOUR OUTPUT PER MAN-HOUR OF OPERATING LABOR

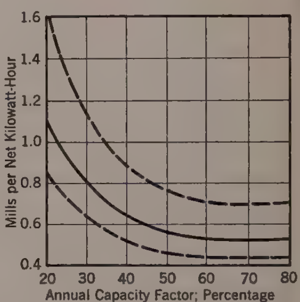

FIG. 4.—MAINTENANCE EXPENSE, LABOR AND MATERIAL

Using 70 cents as the average wage per man-hour for the plants in question, Curves *f* in Fig. 2 are obtained from the values shown by the full line in Fig. 3. These curves are plotted above Curves *e* as a base, so that the ordinates represent the combined costs of fuel and operating labor. The labor costs can be found by difference. As a matter of convenience, the cost of superintendence has been added to the operating labor cost before plotting Curves *f*. The addition is so small as not to make any visible difference in the position of Curves *f*.

Water and Miscellaneous Supplies and Expense.—The next two items of cost, namely, water and miscellaneous supplies and expense, are conveniently grouped together for this kind of generalization. For what may be considered average conditions the grouped cost may be taken at about 0.1 mill per net kw-hr, with a possible variation of ± 50 per cent. However, conditions that are abnormal may cause wider variation as, for example, when water must be purchased from a public supply or when a complicated and costly water treatment is required. The small value adopted as probable cannot be shown clearly in Fig. 2 by drawing an additional curve since this would be only 0.1 mill above Curve *f*. For this reason, the cost of water and miscellaneous supplies has been added to that of maintenance labor and material for plotting purposes.

The task of arriving at reasonable average values for maintenance, labor, and material is difficult. For one thing, standards of maintenance differ greatly from property to property and from region to region. For another, wage scales and costs of materials also vary rather widely. Furthermore, the types of design and construction effect the necessary expenditures; and, by no means least, the type of fuel and certain related factors have a very great influence. Coals with a high sulfur content, ash-fusing at a low temperature and a high water content, all tend to increase maintenance to a significant extent.

For the nominal 225-lb plants, the writers have chosen to use the values given by the full line in Fig. 4. These values are fairly representative of a high grade of maintenance in plants burning reasonably good grades of bituminous coal. The dotted lines indicate probable variations from the values assumed. With very bad fuel or very bad water, the maximum values shown might easily be exceeded. Adding the 0.1 mill for water and miscellaneous supplies and expense to the values shown by the full line in Fig. 4 gives the data for plotting Curves g in Fig. 2. As before, these are plotted above Curves f as a base.

Superintendence, Clerical and Testing.—The last item of operating costs is that designated by superintendence, clerical and testing. Although the service rendered under this item is very valuable, the outlay required to procure it is generally comparatively small. In terms of cost per kilowatt-hour it is generally about 0.1 mill. Obviously, this is too small an increment to plot in Fig. 2, and, for this reason, it was included in the operating labor charge when plotting Curves f.

The total operating cost is then given by the values in Curves g for the two limiting thermal performances assumed. It will be shown subsequently how these performances may be corrected for conditions different from some of those assumed in developing the data thus far presented.

Group 2.—Fixed Charges.—The second large group of cost items is fixed charge expense. A review of the components of this group, as given previously, will show that all are essentially functions of the investment. In fact, each is frequently expressed as a percentage of the investment. It is obvious, therefore, that they can be evaluated immediately if the investment and the various percentages are known.

In connection with investment it should be remembered that this must include, in one way or another, not only the actual plant property, such as land and improvements, buildings, and plant apparatus and equipment, but also those things necessary to convert the plant from a static to an operating entity. In the case of a coal-fired plant, for example, there must be available a sufficient stock of coal to insure power production in the face of an irregular receipt of coal. In the case of certain northern sections of the United States to which such coal is transported *via* the Great Lakes, at least a six-months' supply must be stored when navigation closes. A certain sum of money, therefore, is always tied up in stored coal and represents an investment on which carrying charges must be paid. Again, as an example, a certain amount of working capital must be available in order to pay wages and invoices as they become due. Working capital thus contributes to total investment and, in one way or another, carrying charges must be paid upon it.

The costs resulting from these types of investment are best placed in the third large group ("General Over-Head"), instead of in the second. The carrying charges resulting from the coal storage then go into the general classification of "Stores," and the costs resulting from working capital become part of a larger account of the same character.

Unit investment in power plants, that is, the investment in dollars per kilowatt of capacity, is frequently assumed to be a kind of universally applicable value easily derived. This is very far from the truth. Conditions vary greatly from year to year, from place to place, from method to method adopted or available for construction, and, to a great extent, with the conditions to be met with respect to character of fuel, character of load, provisions for reserve, etc. In years when business is slack, prices and wages tend toward low values and the efficiency of labor tends to increase. A plant built under such conditions should cost appreciably less than one of exactly the same type constructed in the same place under boom conditions. Some districts in which plants are constructed have such combinations of real estate values, building codes, construction practices, and wages that a relatively high cost of plant necessarily results. Some locations require elaborate substructure and expensive canal work, whereas others permit the simplest type of substructure and negligible cost for canals. In some cases, an ample supply of circulating water is available and, in others, expensive provisions for storing and cooling must be made. The result is that one cannot state categorically what a given type of plant should cost or should have cost. It is necessary to study a large number of facts and conditions derived at reasonable figures. How, then, is one to establish a probable investment for the class of plants concerned herein? The unvarnished truth is that one cannot. All one can do is to choose more or less arbitrarily, with full appreciation of the fact that the choice is arbitrary, or consider an entire possible range.

Most of the nominal 225-lb plants were built, or at least partly built, in times of comparatively low prices. Therefore, unless exceptional conditions prevailed in other respects, it would be expected that they should have cost less than they would, had they been built in a period of higher prices. However, in many cases, they have been enlarged during periods of higher prices and, in others, they have been rehabilitated or improved in one way or another, thus increasing the original investment both as to total and as to unit values.

Such data as are available to the writers indicate that the investment in nominal 225-lb plants, as they now exist, will probably fall somewhere between \$85 and \$110 per kw of capacity under what may be termed average conditions, but that there are exceptional conditions which have produced legitimately both lower and higher values. They have chosen to use \$100 for the purposes of calculation, realizing that resultant values for any other condition can be obtained by direct-ratio calculation.

The percentages of this investment, which are to be assumed for each of the items of cost listed under fixed charges, have been the subjects of endless disputes. There are many different theories regarding the manner in which depreciation shall be considered. Political bodies of like character do not agree and different political bodies having jurisdiction, in one way or another, over a

given power undertaking, promulgate different rules and use different theories and practices contemporaneously. Regulatory bodies are inclined to consider a high rate of depreciation proper with respect to the past when establishing a rate base, but they accept a low rate of depreciation when establishing a rate for the future on such a rate base. The United States Department of Internal Revenue treats depreciation on a standardized basis and without reference to the practices of State commissions. No agreement exists with respect to the relative values of obsolescence and physical deterioration as aspects of depreciation.

The cost of money is obviously an item that can be determined more or less accurately in any given case either historically or as of a particular time; but there is no reason why it should be the same in any two cases or under any two conditions. One company may be able to, or may have been able to, obtain the required funds at the equivalent of, say, 5.5%, whereas another, just as legitimately, may have paid 7%, or even more.

Property taxes, obviously, vary with the community and the community's practices. In regions of high property values the real estate tax is likely to be high. Personal property taxes are an item in some States and not in others.

Insurance of the types included under fixed charges generally represents a quite insignificant proportion of the total in the case of well built and well maintained properties. However, it also varies widely with the character of the property and the practices of the management.

In view of all these variations it seems best, in the present case, to lump all fixed charges into one percentage value. Such a value appears to vary between slightly less than 10% of the investment as a lower limit, to possibly as much as 14% as a high limit. The writers have chosen to use 12% for the purposes of computation, realizing again that the results can be corrected easily to any other value if it is found necessary.

With a unit investment of $100 and a fixed charge of 12% annually, the fixed charge per unit of output is obtained by dividing $12 by the kilowatt-hours produced annually per kilowatt of capacity. Values obtained in this manner are plotted as Curves h in Fig. 2, using Curves g as a base.

Group 3.—General Over-Head.—Conditions applying to the third classification of general over-head expense must vary widely from case to case. However, as has been indicated herein, this type of expense always exists even if the items to be included may vary.

It is exceedingly difficult to obtain sufficiently comprehensive data to enable one to determine for each of many properties the value of each item lumped under this heading. On the other hand, it is not nearly so difficult to obtain a close approximation to the total of all such items. Such data as are available to the writers indicate that these totals generally equal from 10% to 25% of the total of Group 1 and Group 2 when all items of general over-head character are taken into account. One might be inclined to believe that there would be a marked difference between industrial plants and public utility plants in this respect. The fact appears to be that this is not normally the case because the items in Group 3, which have no meaning in the case of industrial plants, usually represent a very small part of the total. However, the methods

of accounting used may, and frequently do, tend to disguise the actual total of the costs in this group.

Practically no data are available that can be used to indicate the relation of general over-head to capacity factor. This may seem strange at first, but inspection of the items making up the general over-head will indicate that, in any given organization, most of them would tend to remain constant over fairly long periods of time even if radical changes in plant output occurred, for example, as the result of a depression or as the result of a business boom. Under such conditions the total at any time would not truly reflect what might be expected if the output remained substantially at the then value. Again, when companies are found which, respectively, operate a plant or plants at radically different capacity factors over long periods of time, the characteristics of the companies and other controlling factors vary so greatly between themselves that any variation of general over-head with capacity factor may well be assumed to be due to causes other than the respective values of the capacity factors.

Faced with such uncertainties the writers have chosen to show the effects of general over-head on total power cost as a rather wide band instead of as a line. The bands shown in Fig. 2, marked *i*, represent what the total power costs would approximate if general over-head amounted to any value between 10% and 25% of the total of Groups 1 and 2 at each capacity factor. Their purpose here is to call attention to the fact that such general over-head must be considered as an item of power cost in a going concern and that it may be expected to be of significant magnitude, rather than to state an exact value.

It will be recalled that the groups of curves in Fig. 2 have been constructed by making certain assumptions and that reference has been made to the possibility of correcting to other assumptions. The first of these corrections is concerned with variation of back pressure and its effect upon performance. Curves *e*, Fig. 2, represent fuel cost at 1-in., Hg, back pressure. For a higher back pressure the cost will be more because the thermal performance will be poorer. For a lower back pressure the cost would tend to decrease more or less, depending upon the design of the turbine and upon the average load carried.

In the case of the older types of turbine, such as would be expected in plants of the kind discussed herein, there would normally be an improvement down to 0.5-in., Hg, back pressure except at the higher loads. At these higher loads the improvement would normally cease at approximately 0.7 to 0.8 in., Hg. It follows that, for low capacity factors which would generally but not always be associated with low average turbine loads, the effect of lowering the back pressure below 1 in. might be expected to be greater than at higher capacity factors.

The differences thus involved, however, are small in comparison with many others that cannot be evaluated and it seems unreasonable to attempt to take them into account in such a general treatment as contained in this paper. Therefore, the writers have included just one curve for back-pressure correction to the performance of a nominal 225-lb plant. This is shown in Fig. 5 (*a*). The part below 1 in. is dotted to indicate the uncertain character of the correction in this region. The correction factors indicated as percentages can be

applied directly to increase or decrease the height of Curves *e* in Fig. 2; or, since all curves in Fig. 2 are plotted with the next lower as base, the same percentage of the values on Curves *e* may be added to, or substracted from, the values of Curves *h*.

It seems pertinent to remark that there are few if any cases in the United States in which a normal year's performance would show an average back pressure, weighted for output or otherwise, of less than 1 in., Hg. Therefore, the negative correction is more of academic than of practical interest. This is another justification for not quibbling over the exact shape of the correction factor below 1 in.

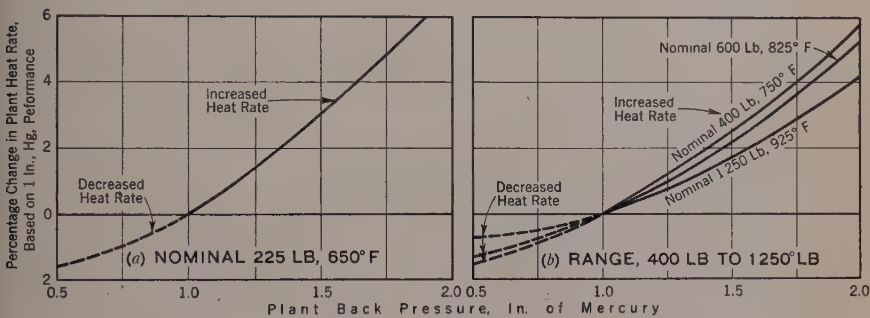

FIG. 5.—AVERAGE CHANGE IN THERMAL PERFORMANCE WITH PLANT BACK PRESSURE BASED AT 1 INCH, HG

Possibly, the next necessary correction is for fuel cost. Curves *e* in Fig. 2 were calculated for a fuel price of 15 cents per million Btu at the boiler, corresponding roughly to about $4 per ton for a good grade of eastern bituminous coal. If any other value such as x cents per million Btu is considered, the values shown by Curves *e* of Fig. 2 must be multiplied by $\frac{x}{15}$. If total power cost is to be shown, it is simpler, of course, to apply the resultant correction to Curves *h* as before.

The next correction factor is for operating labor cost. The values plotted result from assumptions regarding the man-hour output and the average hourly wage. It is obvious that if the man-hour output is assumed at a lower amount, the values indicated by Curves *f* must be increased in inverse ratio, and *vice versa*. It is equally obvious that if a different wage is assumed the values indicated by Curves *f* must be increased in direct proportion and *vice versa*. In making these corrections, as indicated, the presence of 0.1 mill for superintendence and clerical costs in the values shown on Curves *f* is ignored for obvious reasons.

The second group of cost items, fixed charges, is, as stated, based on $100 per kw of capacity and 12 per cent. Alteration in either would produce a corresponding change in the values shown on Curves *h*. The correction would be performed by multiplying the ordinate value between Curves *g* and *h* by the proper ratio.

Reference has been made herein to the effect of oil and gas-firing upon the man-hour output. As a matter of fact, the use of oil or gas instead of solid fuel may have other effects as well, and some of these may result in quite different costs than those obtained for coal-fired equipment. For example, it is obvious that a gas-fired plant receiving its supply through a pipe line owned by others does not have capital tied up in stored coal; it need not have capital tied up in property on which coal is stored; it need not have coal-handling equipment; and it does not have the operating expenses incident to the handling of coal into, and out of, storage and into the plant. These and many other differences may be expected to exist, but no general treatment of the present

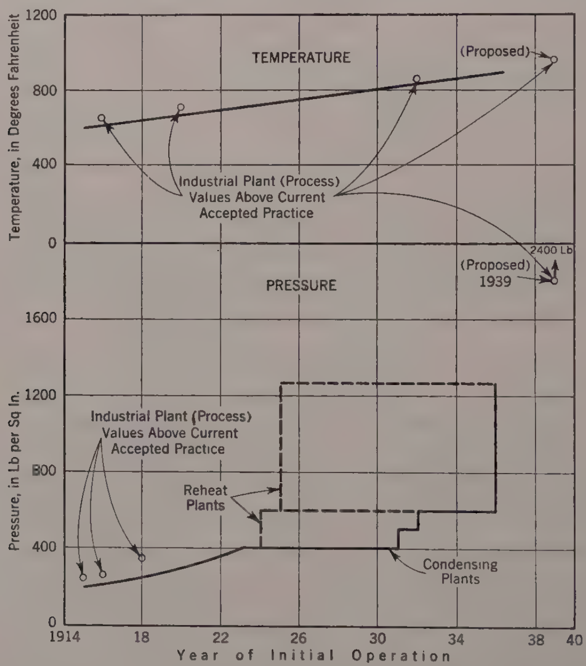

FIG. 6.—INCREASE IN STEAM PLANT TEMPERATURES AND PRESSURES

kind can hope more than to point to that fact. One may state, in general, that, with other conditions equal, the cost of power would be expected to be lower with oil and gas-fired plant than with coal-fired plant; but the significance and importance of the expression, "with other conditions equal," must not be overlooked.

At this point the writers believe it advisable to introduce a caution. It will have been observed that although the economic performance of any plant at any one time must have a very definite value, great latitude of choice exists as soon as one attempts to generalize. The writers caution against attempts to arrive at probable power cost for a given site or for a general condition by assuming all low or all high limits among those that have been indicated. It seems to be true that in this field, as in others, one seldom, if ever, finds all

controlling features most favorable or most unfavorable. Therefore, theoretical power costs, arrived at by making such assumptions, are almost certain to be ridiculous in a practical sense.

Power Costs at Higher Pressures and Temperatures.—Power costs at the higher pressures and temperatures are next considered. These steam conditions have been introduced more or less, successively, as experience, necessity, engineering and design progress, and the development of metallurgy and fabricating processes, respectively, dictated or permitted. The gradual increases of pressure and temperature in the United States are shown in Fig. 6; the year in which each step was taken is indicated. In a general manner, the thermal performance of plants improved during this upward march, is indicated in Fig. 7. Part of the improvement is due to the greater inherent value of higher pressures and temperatures; part to improved plant design, both with respect to the operating cycle chosen and to the arrangement of equipment; and part to improvements in the component equipments.

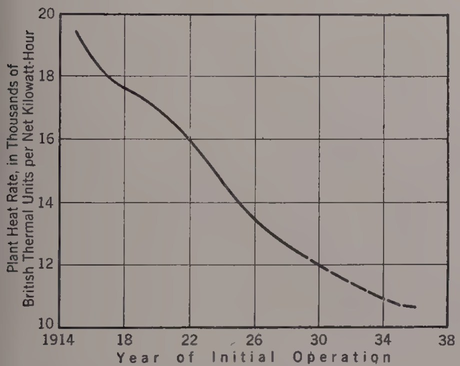

FIG. 7.—IMPROVEMENT IN THERMAL PERFORMANCES (ACTUAL RESULTS FOR AT LEAST ONE YEAR'S PERFORMANCE)

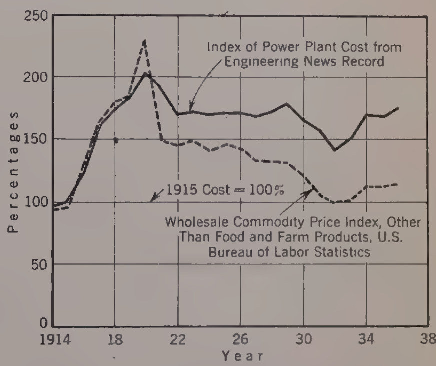

FIG. 8.—INDICES OF STEAM POWER PLANT INVESTMENT AND WHOLESALE COMMODITY COSTS

To one familiar with general price levels, study of Fig. 6 will indicate that the upward climb of pressures and temperatures occurred during a period of relatively high commodity prices, at least up to the end of the last decade. The general effect of changing commodity prices upon the cost of complete power plants is shown very roughly in Fig. 8, for the years 1914 to 1936. Naturally, a combination index such as this is not necessarily accurate with respect to any individual case; it can only reflect conditions in a most generalized or over-all manner. The circumstances in an individual case may legitimately result in wide positive and negative departures from index values.

The phenomena discussed in the immediately preceding paragraphs yield several peculiar results. First, they make it difficult to obtain a true comparison between what may be called basic investment costs of plants built to operate at different pressures and temperatures. Theoretically, one could obtain a perfect index and apply it to the actual known investment costs of different plants built at different times, thus wiping out the effect of varying commodity costs and having the data truly comparable. Practically, this

cannot be done because of the impossibility of obtaining so accurate an index for application to isolated cases; because of the wide differences caused by varying local conditions and varying types of design; and because there is no known method of correcting for the extent to which development costs, engineering errors, modified production methods, etc., have been reflected in the actual finished cost in different cases. In another sense, a comparatively small number of radically different units are involved in this case, and statistical methods are not applicable.

Second, these phenomena make it practically impossible to determine the true economic values of different pressure and temperature conditions as separate from the characteristics of any given case. The thermal performances can be found both by computation and by performance statistics from actual plants; but operating expenses and fixed charges are not so readily obtainable in such form as to permit of accurate generalization.

If one tabulates historical costs of plants as they have been built during this period of shifting indices, shifting pressures and temperatures, and shifting design, there appears to have been a gradual increase of cost with increasing pressure and temperature. As nearly as the writers can estimate from the facts, if the nominal 225-lb plant investment were taken at $100 per unit of capacity as was done before, each step in pressure, as herein considered, would probably add about 5% to 7% to the required investment. On the other hand, if one were to consider what can be done in a given location under given conditions to-day, the writers estimate that, starting with the low-pressure investment as indicated previously herein, there would be a slight decrease to a minimum at about 400-lb to 600-lb pressure and then a slight increase to 1 200 lb. The differences involved appear to be small, possibly not greater than 5% to 10% at most. It is quite conceivable that, as experience is gained, as the designs are developed further, as metals are improved, and as development charges are canceled, even this estimate may be changed so as to show a negligible minimum of investment near the middle of this pressure range, or even a gradually falling investment with increase of pressure and temperature to the end.

In view of the kaleidoscopic nature of the variation of investment with pressure and temperature, the writers have thought it best to assume plant investment at $100 per unit of capacity for all cases that follow. This is to be interpreted merely as a ready means of presenting data that can be corrected easily to any set of assumed conditions, rather than as indicating that all these different plants have cost exactly $100 per kw (see, in this connection, the earlier remarks about the use of the same value in connection with nominal 225-lb plants).

The thermal performances obtained from plants operating at 400, 600, and 1 250 lb, respectively, are shown in Fig. 9. As before, the values for each pressure are given as a range to take cognizance of fact. It will be observed that each range overlaps the adjacent range, illustrating once more how difficult it is to apply theoretical values to actual conditions. The wide range of performances for 400-lb plants reflects partly the result of improvement of design and operation in the comparatively long time during which such plants

have been built and, to some extent, the variations resulting from different local conditions. The upper limits of the 600 and 1 250-lb ranges are, in a sense, incorrect. They give the performances of plants that are really combinations of two pressures, the higher pressure units having been installed in existing plants operating at a lower pressure and tied in in various ways. They are included because these plants are commonly referred to in terms of the higher pressure, ignoring the existence, and the effect, of the lower pressure.

It will be observed that no performance range is shown for the 800-lb plant although the expectable thermal performance of such a plant as of to-day is indicated by a single line in Fig. 9. The values from which these performance

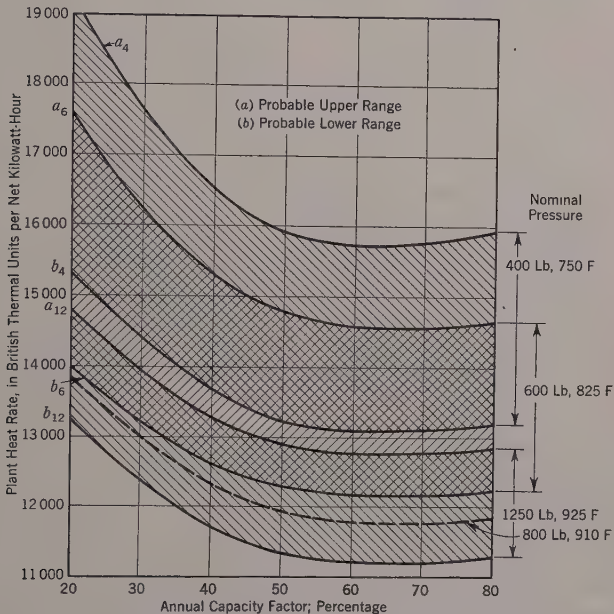

FIG. 9.—THERMAL PERFORMANCE; STEAM PLANT; RANGE 400-POUND TO 1 250-POUND NOMINAL PRESSURE AT 1-INCH HG, BACK PRESSURE

curves for other pressures are developed, are obtained from actual operating plants. It happens that although many 800-lb plants are now (1938) being installed, operating data are not yet available.

The power-cost data for 400-lb plants are assembled in Fig. 10, in which all curves correspond to those of Fig. 2 and are similarly labeled. (These values, however, are not directly comparable with those shown in similar curves in this paper.) Curves *e* are developed from the upper and lower values shown in Fig. 9, and using a fuel price of 15 cents per million Btu as before. Curves *f* are obtained from the line marked "Nominal 400 lb" of Fig. 11, using a wage rate of 70 cents per hr and adding 0.05 mill for superintendence. Attention is called to the fact that some actual plants are giving a measurably greater output per man-hour, as indicated in Fig. 11, and that if such values were used, the positions of Curves *f* would be lowered. Attention is also called to the fact that no

allowance has been made for a probable drooping of the labor performance curve at the upper end, as indicated by dotted lines in Fig. 11.

Curves *g* are obtained from Curve *j*, Fig. 12, with the addition of 0.075 mill for water and miscellaneous supplies. It will be noted that Curve *j* represents

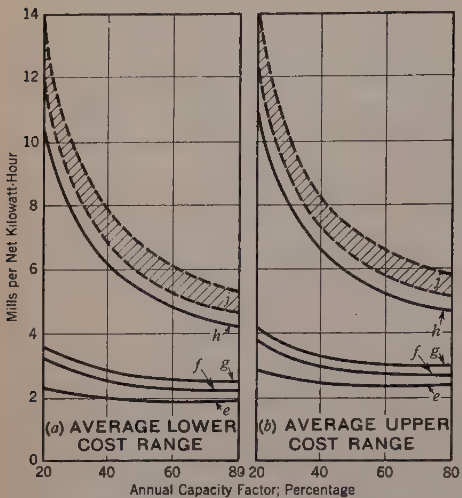

FIG. 10.—AVERAGE UPPER AND LOWER RANGES OF POWER COSTS; NOMINAL 400-LB, 750°F, STEAM PLANT AT 1 IN., HG, BACK PRESSURE

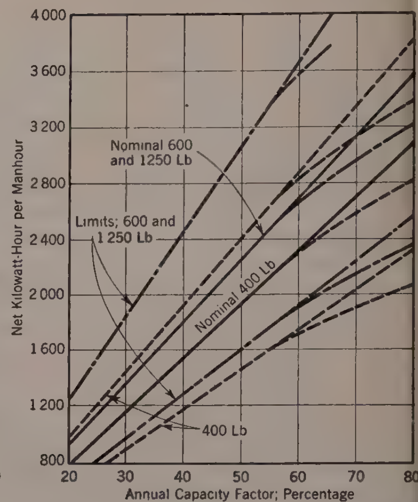

FIG. 11.—NET KILOWATT-HOUR OUTPUT PER MAN-HOUR OF OPERATING LABOR; STEAM PLANT; RANGE, 400 TO 1 250-LB NOMINAL PRESSURE

a value between the observed limits, Curves *l* and *k*. Data available appeared to indicate its position as approximately correct for a weighted average performance.

Curves *h*, as already explained, are based on an arbitrary choice of \$100 per kw of capacity, and 12% fixed charges. The range, *i*, in Fig. 10, is located as before in terms of percentages of the totals through Curves *h*, the lower limit representing 10%, and the upper, 25 per cent.

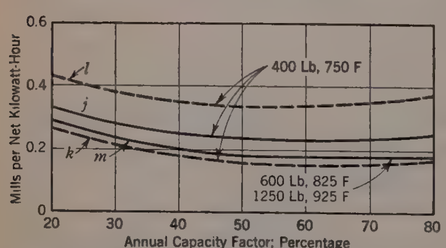

FIG. 12.—MAINTENANCE EXPENSE; LABOR AND MATERIAL; RANGE, 400-LB TO 1 250-LB NOMINAL PRESSURE

Corrections can be made for other conditions and other assumptions as previously noted in connection with the 225-lb plant. The correction to fuel cost for a plant vacuum other than 1-in., Hg, can be made by means of the curve for the 400-lb plant shown in Fig. 5 (*b*). All other corrections are direct ratios as previously indicated.

The power cost data for a 600-lb plant are given in Fig. 13. Curves *e* are developed from the values shown in Fig. 9 and by using a fuel price of 15 cents per million Btu. Curves *f* are obtained from the line marked "Nominal 600 and 1 250 lb," of Fig. 11, using a wage rate of 70 cents per hr and adding 0.05

mill for superintendence. Curves g, Fig. 13, are obtained from Curve m, in Fig. 12. This is the first case in which a range has not been shown. The facts are that plants operating at the two pressures indicated are as yet so few and are so comparatively new that the date available could best be shown by a line instead of by an area. As in the previous case, an addition has been made for water and miscellaneous supplies. For the 600-lb plant, 0.05 mill was added for this purpose.

Curves h, Fig. 13, are based, as before, on the arbitrary choice of an investment of \$100 per kw of capacity and 12% for fixed charges. Range i is located as before in terms of the totals through Curves h using 10% and 25% as lower and upper limits, respectively. Corrections can be made for other conditions and other assumptions as previously indicated. The back-pressure correction is indicated in Fig. 5 (b).

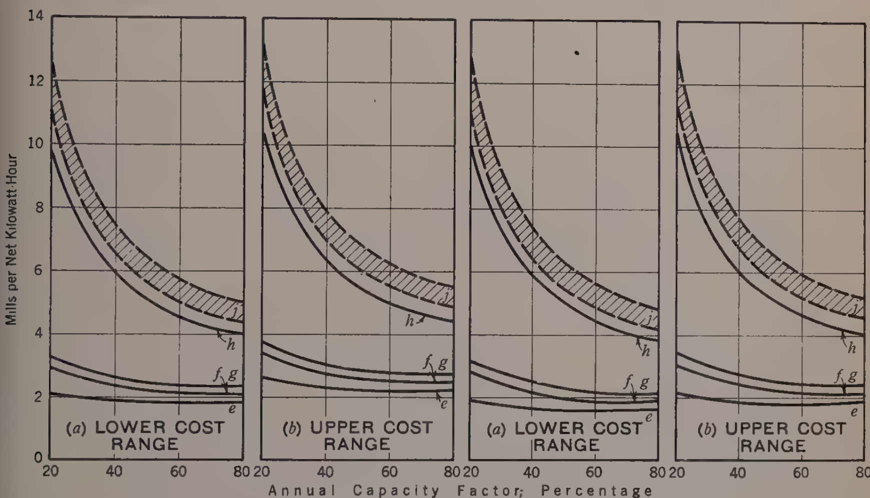

FIG. 13.—AVERAGE RANGES OF POWER COST; NOMINAL 600-LB, 750° FAHRENHEIT, STEAM PLANT AT 1-INCH, HG, BACK PRESSURE

FIG. 14.—AVERAGE RANGES OF POWER COST; NOMINAL 600-LB, 825° FAHRENHEIT, STEAM PLANT AT 1-INCH, HG, BACK PRESSURE

The power costs for a 1 250-lb plant are given in Fig. 14. The fuel costs, Curves e, are developed from the values shown in Fig. 9, using a fuel price of 15 cents per million Btu as before. Attention is called to the fact that the upper range in Fig. 9 is for what are really combination plants although commonly spoken of as 1 250-lb plants. It is probable that complete 1 250-lb plants will give a performance near the lower range under what may be called average conditions. Curves f, g, h, and i, Fig. 14, are obtained as before and by using the same values as for a 600-lb plant.

It is reasonable to assume that a 800-lb plant would give a thermal performance between those of 600-lb and 1 250-lb plants, respectively, and that all other costs would be the same when computed as has been done herein. It does not seem worth while to complicate a presentation of costs actually achieved by injecting costs based on expected performance.

Inspection of Figs. 2, 10, 13, and 14 will show that the predominating items of cost are fuel and fixed charges. The former will vary directly with the price at which thermal units are available. Strategic location in this respect is frequently possible and, under favorable conditions, from 25% to 50% of the fuel cost item can be eliminated. In fact, there are cases in which fuel is available at a price representing a still greater saving. Many who have concentrated upon the reduction of fuel cost have arrived at the suggestion of a mine-mouth plant which would eliminate all, or practically all, the freight on fuel. This subject has been discussed very extensively in recent years. There are comparatively few locations at or near coal mines where an ample condensing water supply is available and where a sizable load is within a distance permitting it to be reached with transmission lines sufficiently short so that transmission costs do not overbalance the fuel savings. Furthermore, the reliability of over-head transmission is not yet generally regarded as sufficiently high to justify depending entirely upon power transmitted in this manner in the case of loads in which a high degree of continuity of supply is required.

Little more can be written about fixed charges than has been presented already. One may hazard a guess as to unit investment in the future, but it can be little better than a guess. It seems probable that as design experience is gained and as the problems of building and installing high-pressure equipment are solved, high-pressure plants will become relatively cheaper. It also seems probable that, in terms of present-day indices, plants for all the pressures considered will be procurable at costs of less than $100 per kw of capacity under favorable conditions.

It is interesting to speculate upon the future development of steam plants. They are already down to an annual performance of approximately 11 000 Btu per net kw-hr. Power engineers know that this can be bettered by increasing the pressure and by the use of a reheat cycle, or by the increase of both pressure and temperature as metal for superheater tubes and pipe lines becomes available at a sufficiently low price to make such higher temperatures economically feasible. They are already technically possible. The designing engineer for a steam power plant can already envisage the possibility of a performance of 10 000 Btu per net kw-hr, or even a little lower than that.

Mercury Vapor-Water Vapor Plant

Several so-called binary vapor combinations have been suggested from time to time as means for reducing the cost of fuel-generated power. Only one of the suggested combinations has thus far shown much promise of economic value. This is the plant using mercury vapor and steam, thermally in series; that is, the heat from the fuel is used primarily to produce mercury vapor and, after the vapor has passed through a turbine, heat remaining in it is used to produce steam for a second turbine.

When the construction of such plant was first undertaken on a semi-commercial scale, its computed thermal performance was so much better than the performance attained with a steam plant that the mercury vapor process held great promise. The very rapid improvement of steam plant, combined with the development of certain practical difficulties in the generation of

mercury vapor, has served to retard the wide adoption of this binary process for power production. On the other hand, there has occurred an ever-increasing use of mercury vapor for process purposes.

Now it appears probable that a binary vapor power plant using mercury vapor and steam could be constructed to give a thermal performance of about 9 000 Btu per net kw-hr. Data with respect to the necessary investment are fragmentary and it is not known whether increased fixed charges would, or would not, offset the results of the improved thermal performance. Presumably, the situation will be clarified with the passage of time.

Internal Combustion Plant

Internal combustion plant is thus far restricted to the use of reciprocating engines and to comparatively small size per prime mover. Large total capacity is obtained by the multiplication of such units. In the United States a few large internal combustion plants have been built which use blast furnace gas for power production in steel plants. With the exception of these rather unique installations, gas-engine and oil-engine plants have been limited to comparatively small total capacity.

Engines using gaseous fuel have not achieved great prominence for the production of electric power, but the Diesel type using liquid fuel has been introduced very rapidly for such purposes in recent years. There are conditions under which it appears to represent the best economic solution of such problems, particularly where comparatively small capacity and high thermal efficiency are desired in combination. Only time can tell whether this is a permanent or a temporary condition. However, there are now indications that point toward the possibility of building comparatively small steam plants of high thermal efficiency, arranged so as to make them competitive with Diesel plants in similar capacities.

The prominence achieved by Diesel engines as prime movers for electricity generation during the past few years justifies consideration of the cost of power obtained from such sources. The writers have arranged cost data for Diesel plant in such ways as to parallel the treatment used in presenting data for steam plant, so that the two shall be, as nearly as possible, directly comparable. Attention, however, is called to the fact that whereas the data for steam were developed for large plants, 150 000 kw, or more, those for Diesel plants are developed for small installations, say, 10 000 kw, and less.

As in the case of steam, all graphs are plotted against annual plant capacity factor. This ratio has been defined herein as the net annual output, in kilowatt-hours, divided by the product of total rated capacity of the plant and the number of hours in the year, namely 8 760. Attention is called to the fact that this factor differs from that commonly used in presenting Diesel plant performance. Direct comparison with the data given herein for steam is permissible; direct comparison with the commonly quoted Diesel plant performance is impossible without preliminary recalculation.

A reasonable fuel cost range is given in Fig. 15. The values indicated by the two curves are based on Diesel engine fuel oil at 5 cents per gal. Corrections can be made by direct ratio for other oil prices. Such prices generally

fall between 4 and 6 cents although there are cases in which oil is purchased at prices much lower than 4 cents, and others in which the higher figure is exceeded. It is to be noted that the rather wide range of thermal performance indicated is influenced more by the type of load carried, the character of operation, and the age of the engine, than by the size of the engine. In this statement, age of the engine refers both to the state of the development of the art when it was constructed and to the extent to which it was worn in use.

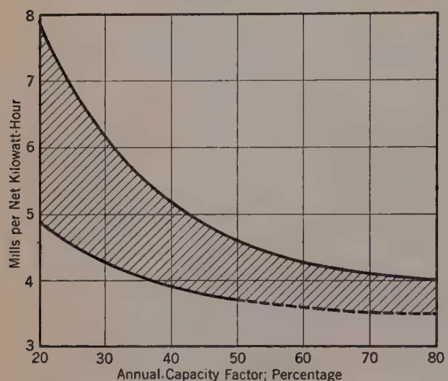

Fig. 15.—Fuel Oil Expense for Diesel Plants (Oil at 5 Cents per Gallon)

The values shown in Fig. 15 have been plotted as Curves *e* in Fig. 16, corresponding to the arrangement used in presenting steam data.

Curves *f* are derived by using the middle graph of Fig. 17 for output per man-hour and an hourly wage of 70 cents, as before. They represent the cost of operating labor and superintendence plotted above Curves *e* as a base. In this case, operating labor is used partly for maintenance purposes, as is common practice in relatively small plants, so that a part of what is indicated herein as operating labor would be more properly chargeable to maintenance. Plant records do not ordinarily permit this allocation. The upper line of Fig. 17 indicates the output per man-hour that may be expected with a well-designed plant containing a few large modern units. The lower line gives values actually obtained under less favorable conditions. Corrections for different man-hour outputs and for different wage rates can be made easily by ratio computations.

Curves *g*, Fig. 16, plotted above Curves *f*, represent the summary of costs of lubricating oil, miscellaneous supplies, and maintenance to the extent that the latter is not included under operating labor as already explained. The values used for the cost of lubricating oil are taken from the middle curve of Fig. 18. The great range of costs found in practice is indicated by the upper and lower curves of that diagram. It should be noted that all these curves are in terms of oil at 55 cents per gal so that different oil prices are not directly responsible for the wide range that is shown. As a matter of fact Diesel plants are using lubricating oils costing from less than 20 cents to more than 60 cents per gal. There is some indication that the total cost of lubrication is greater with the very cheap oils than it is with the more expensive ones, but this should not be regarded as a proved fact, or as generally applicable. The problem is too complicated and is affected by too many other variables to permit of such simple generalization.

The values used for maintenance cost are taken from the middle lines of Fig. 19. It is possible that, in placing this curve as low as they have, the writers have been unduly influenced by the records of more recently installed

plants which may become poorer as these plants age. It seems fairly certain that the line does not belong lower; it may belong as much as 30% higher. The upper range is indicative of what may happen with poorly built, or poorly operated and maintained, equipment.

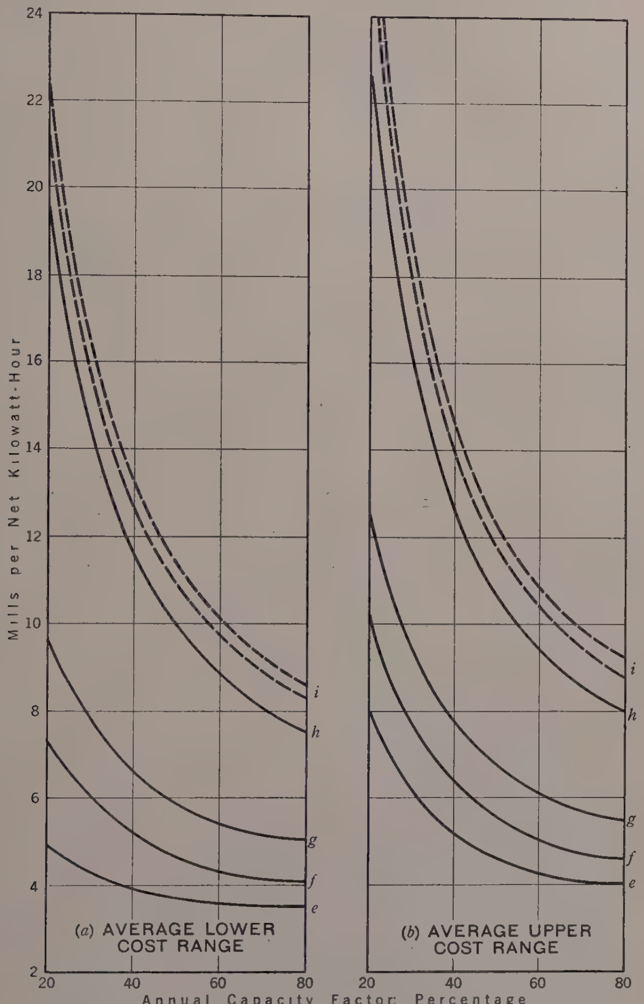

FIG. 16.—AVERAGE RANGES OF POWER COST FOR DIESEL PLANTS

The cost of miscellaneous supplies and expense differs greatly in magnitude, depending upon the local conditions, the grade of "house-keeping," and many other factors. The writers have chosen to use 0.15 mill per net kw-hr at all capacity factors. This appears to be a reasonable average value. Thus Curves *g*, Fig. 16, plotted above Curves *f* as a base, represent the summations of the values given by the middle lines of Figs. 18 and 19 and the constant value, 0.15 mill.

Curves h, Fig. 16, plotted above Curves g as a base, represent fixed charge expense. At any output this is obviously determined by the investment and the percentage represented by fixed charges. For investment the writers have chosen to use \$135 per kw of plant capacity, which they believe represents a fair average value for a complete plant. It is obvious that, with other conditions equal, the unit cost should be greater with a large number of small units as against the same total capacity in a small number of large units. It is also a fact that many local conditions, such as character of soil, availability of water, character of the neighborhood, provision for future enlargement, etc., will all have definite effects upon investment. The choice of \$135, therefore, is not to be considered as equivalent to a statement that any and all Diesel plants should be procurable at such a figure but rather that it represents a fair average of both lower and higher values.

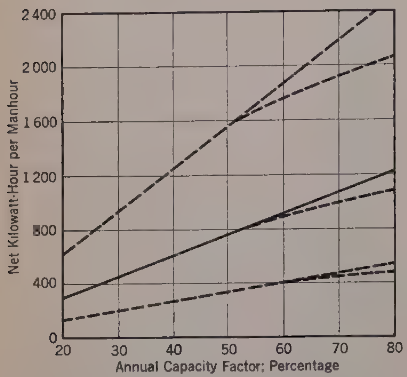

FIG. 17.—NET KILOWATT-HOUR OUTPUT PER MAN-HOUR OF OPERATING COSTS AND SUPERINTENDING LABOR FOR DIESEL PLANTS

The writers have chosen to use 13% for fixed charges as against 12% for steam plant. They believe the higher rate to be justified by the shorter expectable life of Diesel equipment. It is a fact that in some cases such plants have been built recently by municipalities which obtained money at low rates of interest and that in such cases the fixed charges might legitimately be considered as less than 13 per cent. However, it is believed that there are enough operations actually costing more than this percentage to justify its use as a reasonable average value.

Curves h, plotted above Curves g as a base, then, represent fixed charge expense based on \$135 per unit of capacity and 13% as the rate. Obviously, corrections can be made easily for any other value that may be chosen.

The cost of general over-head expense has been shown in Fig. 16 as Range i, just as it was in the case of steam plant. The range is plotted on the assumption that this form of expense will vary between 10% and 15% of the totals shown by Curves h. The choice of an upper limit lower than that used in the case of steam plant is a recognition of the characteristics of the smaller types of properties that would be using Diesel plant. It is not believed that a value less than 10% can be expected when all items of expense are properly accounted.

At this point the writers call attention to the caution given at the end of the section on nominal 225-lb steam plants. Here, as in the case of steam, one should not assume, without warrant, that all controlling factors will be most favorable or most unfavorable for a given case.

Comparison of expectable cost of power from Diesel plant with corresponding values for modern steam plant, as revealed herein, shows plainly that the latter is capable of producing power more cheaply both with and without the

consideration of fixed charges. It should be noted, however, that this result is obtained by comparing values for very large steam plants with those for much smaller plants equipped with Diesel engines. Although not shown by data in this paper, it is true that, in the case of plants of the size represented by existing Diesel installations, the Diesel engine may still give cheaper power than the steam plant. The competitive situation in these sizes is at present in a state of flux because of many current attempts to build cheap, reliable, and thermally efficient steam equipment in the smaller sizes. Progress made thus far appears to the writers to indicate a real challenge to the supremacy of Diesel engines in this respect.

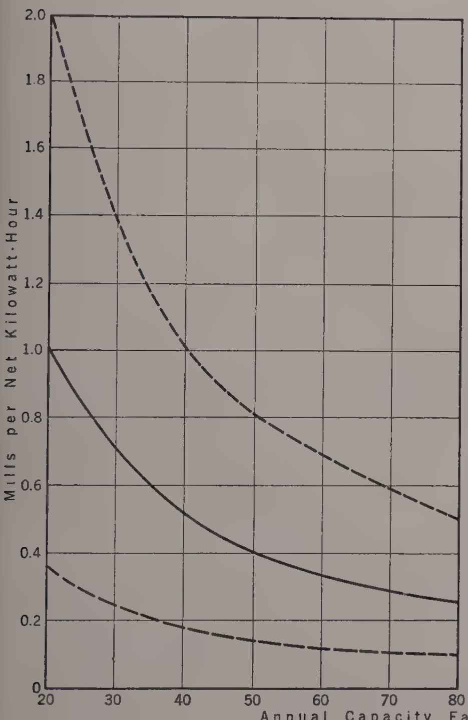

FIG. 18.—LUBRICATING OIL EXPENSE FOR DIESEL PLANTS (OIL AT 55 CENTS PER GALLON)

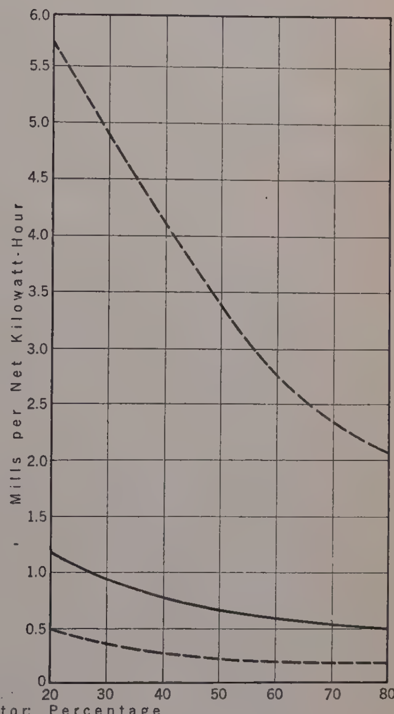

FIG. 19.—MAINTENANCE EXPENSE FOR DIESEL PLANTS

FUTURE POSSIBILITIES

Among other failings, history shows that Man is, in general, a very poor prophet. He has almost always demonstrated inability to imagine the possibility of change because, at the time, he could not envisage the mechanism of it. It is for this reason that the writers hesitate to assume the position of prophet and discuss the future possibilities of fuel-generated power. They recognize too well their own human limitations.

Looking back over the history of fuel-developed power, they recognize the fact that almost every major advance has been made in the face of potential

economic loss; that is, the first cost of equipment was generally so high and the supplementary expense involved in making it work was so great, that it required an optimist to incur the very grave possibility of present loss in the hope of future gains. If one attempts now to look into the future with respect to presently visible means of improving thermal efficiencies, one is immediately confronted with the certainty of over-balancing fixed charges. Time after time this condition has caused studious individuals to state that the then present state of development represented the ultimate; and, time after time, they have been proved wrong by subsequent shrinkage of the predicted first cost and a resultant economic gain.

There appears to be no reason for assuming that the same kind of law will not continue to operate. If it does, it is reasonable to assume that with comparatively slight modifications of present methods along lines which can now be foreseen, thermal efficiencies of the order of about 40% may be realized commercially in comparison with the present achievement of about 30% with steam and 35% with Diesel engines. It is difficult to imagine how present types of plants can be perfected to better performance, but this does not mean that it cannot be done.

History also indicates that as Man approaches the ultimate performance with one type of equipment or method, he develops the ability to produce a new, radically different, and better one. It may be that this is indicated in the present instance by the activity in the field of fuel cells; that is, equipments somewhat similar to electric batteries in which fuel is consumed and electric power produced as a result. This development has gone on almost unnoticed by power-plant men until there is now operating in laboratory size a very promising sample of such a cell. No one can say now whether it can be enlarged sufficiently and commercialized, but careful investigation appears to indicate great probability that it may be. Many difficulties can be foreseen, but none of them appears to be of insurmountable character.

The promise is for thermal efficiencies of the order of double or more than double those presently attainable, and with a unit investment not greater than that now required, possibly even less.

Need one go further with respect to future possibilities in this line? It would seem that the chances for continued reduction of total power cost at the fuel-burning generating plant are indeed very large.

HYDRO-GENERATED ENERGY

BY H. K. BARROWS,[4] M. AM. SOC. C. E.

SYNOPSIS

This paper presents an analysis of the plant characteristics and elements of cost of hydro-generated energy based upon an investigation and study of data regarding fifty-seven hydro-plants scattered throughout the United States. Attention is given particularly to hydro plants in power systems where hydro and steam power are complementary in use, and a method of evaluating such hydro-power costs is suggested, which takes into account both energy and capacity values.

Some of the larger Federal hydro-projects are also discussed and compared in cost with those of private utility companies and a brief survey is made of water-power resources not as yet developed.

ESSENTIAL FEATURES AND TYPES OF DEVELOPMENT

The general requirement of a hydro-electric plant is a flow of water, as uniform as may be, held back usually by a dam so that it may create a head on hydraulic turbines, and by passing through them under this head or pressure, create electric energy by means of the generator, now usually direct-connected with the turbine.[5]

A hydro-development, therefore, requires a dam in most cases. Where the head utilized is only that created by the dam in what is called a "concentrated fall development" the power-house is situated just below the dam, the water reaching the turbines through a flume or penstock and again entering the river after passing through the turbines. In some cases, water from the turbines passes back to the river by a tail-race channel, which often is of considerable length.

Frequently, it is advantageous to direct the water from the dam for some distance down river by a waterway, thus obtaining additional head by what may be termed an "extended fall" development. Depending on the quantity of river flow, as well as the topography near the river and the head developed, the waterway may be an open canal, a closed pipe or penstock, or a combination of both. Wood-stave, steel, and reinforced concrete pipes or penstocks are all in use for such waterways. Occasionally, a tunnel is used for the waterway, where conditions make it economical.

The power-house is commonly rectangular in shape, with vertical turbine-generator units in a line along the length of the building and spaced to permit, in the concrete substructure, the necessary width of approach flumes to the turbines as well as draft-tubes and tail-race channels. The type of flume and

[4] Prof. of Hydr. Eng., Mass. Inst. Tech.; and Cons. Engr., Boston, Mass.
[5] "Water Power Engineering," by H. K. Barrows, 1934.

turbine setting for reaction wheels depends chiefly upon the head developed, as follows:

Head, in feet (approximate)	Type of approach flume
20 to 30, or less	Open pit
20 to 50	Concrete scroll
30 to 350	Plate-steel scroll
350, or more	Cast-steel scroll

The modern hydraulic turbine unit is commonly of the vertical, single-runner type, with a vertical generator set above the turbine and direct-connected with it. Proper control of speed is obtained by the hydraulic governor and pressure regulation (in the case of other than low heads), by a surge tank, usually now of the differential type, placed in the penstock as near to the turbines as practicable.

For higher heads, particularly 1 000 ft, or more (but depending also upon capacity), impulse wheels are used, in which the water passes through a nozzle to the buckets of the wheel. These wheels are usually set on a horizontal shaft and may be single overhung or (for larger units) double overhung, with reference to the generator supports. The general arrangement of such installations is similar to that of reaction turbines, except that no draft-tube is used, the water from the wheel entering a tail-race channel.

The power-house superstructure is usually of brick or reinforced concrete and contains the generators, switching, and other electrical equipment. It is now common practice to install the transformers, lightning arresters, and high-tension electrical equipment in an outdoor yard. In this paper, costs at the switchboard are considered and no transmission costs are included.

Following is an outline of the essential features of the hydro-electric plant as considered in this paper, and common types of construction:

1.—Dam:
- (*a*) Non-Overfall Section:
  - Solid concrete;
  - Hollow concrete;
  - Earth (with or without core);
  - Timber.
- (*b*) Spillway:
  - Solid concrete;
  - Hollow concrete;
  - Tunnel type;
  - Lateral channel type.

2.—Waterway (Including Tail-Race):
- (*a*) Canal or Open Channel:
  - Lined;
  - Unlined.
- (*b*) Penstock or Pipe:
  - Wood stave;
  - Steel;
  - Concrete.

(*c*) Combinations of Canal and Penstock.
(*d*) Flume:
Wood;
Concrete;
Steel.
(*e*) Tunnel.
(*f*) Tail-Race:
Open-channel (protecting wall or embankment).
(*g*) Surge Tank:
Simple;
Differential.

3.—Power House and Equipment:
(*a*) Building:
Substructure (concrete).
Superstructure:
Brick;
Concrete.
(*b*) Equipment:
Hydraulic:
Turbines and governors;
Relief valves.
Electrical:
Generators;
Switching equipment;
Conduits.
Miscellaneous:
Crane;
Station service.

4.—Reservoir Requirements; Changes in:
(*a*) Highways;
(*b*) Railroads;
(*c*) Manufacturing plants;
(*d*) Other structures;
(*e*) Clearing.

5.—Land and Water Rights:
(*a*) Real Estate:
Land;
Buildings.
(*b*) Water Rights:
Flowage;
Power and riparian rights.

Isolated and System Plants

In this paper, attention will be given chiefly to hydro-plants in a power system in which, usually, other hydro-electric as well as steam electric plants are included. This is of importance as affecting requirements for spare units as

well as auxiliary power that may be needed in the case of isolated hydro-plants in which dependence upon continuous service is essential and for which assistance from other plants in a system is not available.

A hydro-plant in a power system also makes possible a better and more economical use of hydro-electric power than can be the case with an isolated hydro-plant.

Plant Capacity with Reference to Flow

Based upon average yearly flow-duration curves constructed with the lower flows toward 100% of the time, hydro-plant capacities, as constructed, generally vary between a flow available from about 60% of the time to one available only a small part of the time. Of course, this reflects, to a large extent, the capacity or use-factor requirements.

TABLE 1.—Installed Wheel Capacities at Hydro-Electric Plants

Plant	Drainage Area, in Square Miles		Head, in Feet		Wheel Capacity			
					Discharge, in Cubic Feet per Second per Square Mile		Percentage of the Time That Discharge Is Available	
	From:	To:	From:	To:	Present†	Ultimate	Present†	Ultimate
Garvins Falls, N. H.	2 340		30		1.41		35	
Vernon, Vt.	6 300		34		2.46		21	
Fifteen-Mile Falls, N. H. and Vt.	1 600		170		7.60		2±	
New England Power Company*	250	500	66	230	3.80		21	
Harriman, Vt.	184		360		9.60		5±	
Sherman Island, N. Y.	2 782		66		2.26		29	
Safe Harbor, Pa.	26 000		55		1.95	4.00	25±	7
Bartlett's Ferry, Ga.	4 200		112		0.97	1.94	53	22
Tallulah Falls, Ga.	190		580		8.4		5±	
Great Falls, Mont.	23 000		52		0.26		75	
Holter, Mont.	20 000		100		0.30		70	
Pit No. 3, Calif.	4 320		280		0.83		17	
Oak Grove, Wash.	266		849		1.58	4.74	100	33

* Plants 2, 3, 4, and 5. † 1938.

Table 1 contains some actual wheel capacities at different plants expressed in both cubic feet per second per square mile of flow and as the percentage of time that flow is available in the average year.

As will be noted from Table 1, system hydro-plants, when not especially designed for day use, generally show a wheel capacity corresponding to the flow available about 20% to 30% of the time. The day or peak-load plants (Fifteen Mile Falls, Harriman, Safe Harbor, and Tallulah Falls) usually have wheel capacities of about 5% or less of the time flow.

Definitions

Load Factor.—The ratio of the average load over some period of time, as a day, week, month, or year, to the maximum or peak load during that time measured during a short time, as a few minutes, is termed the "load factor." Plant load factor applies to any particular hydro-plant under consideration.

System load factor applies to the power system in which the hydro-plant is an integral part.

Capacity or Use Factor.—The ratio of the average load of the hydro-electric plant over some period of time to the plant capacity, is its capacity factor, or use factor. Note that with maximum load at plant capacity, the plant load factor is the same as the plant capacity factor.

Utilization Factor.—The ratio of the average hydro-plant output over some period of time to the power available in the stream with the plant wheel efficiency and head, and as limited by plant capacity, is known as the "utilization factor." Stated more simply, this is the ratio of power output to power available.

The foregoing factors for a hydro-electric plant in a power system vary commonly about as follows (expressed as percentages):

Factors	Comparison
Yearly load	From a small percentage (often less than 10% with peak-load plants) to about 70% or more under very favorable operating conditions
Yearly capacity	From about 30 to 60%
Yearly utilization	From about 40 to 70%

Primary, or Firm Power.—Referring to hydro-power, "primary power," or "firm power" is the dependable power, or that which is continuously available at the plant under the conditions of use. It is defined by the extreme low-water flow, although sometimes it is taken as the flow available 95% of the time where low flows of record are affected by pondage, particularly at week ends.

Primary or Firm Capacity.—Referring to the use of hydro-electric power in system loads, "primary capacity" or "firm capacity" is the part of the total installed capacity that can perform the same function on that part of the load curve to which it is assigned as could be performed by an alternative steam plant. In this case the primary capacity of a hydro-plant would be based on the power available at low-water flow (or sometimes due to back-water conditions), but concentrating the use of this flow at the time of system peak by means of pondage, to replace steam capacity as far as possible.

Elements of Cost of Hydro-Electric Generated Energy

The dominating element of cost is that of the fixed charges upon cost of plant. These charges will ordinarily include:

Items	Percentage
Interest	5 to 7
Depreciation	1 to 2
Taxes and insurance	1 to 2
Total	7 to 11

A fair allowance is 10% under usual conditions. It is to be noted that a hydro-plant often includes a large proportion of essentially permanent construction, such as dams of earth or concrete, tending to lessen the necessary physical depreciation allowance. Moreover, hydro-plants are usually located in the less settled parts of the country, so that the item of taxes is less than that for steam plants in urban regions. Other minor items of cost of hydro-electric energy are attendance and ordinary maintenance items. These costs will not usually total more than 0.1 ct per kw-hr, even with relatively small plants, and for larger plants the total is usually from 0.02 ct to 0.04 ct per kw-hr. Hence, the cost of hydro-electric energy is essentially that of fixed charges.

Evidently, the cost per kilowatt-hour of hydro-electric energy will vary inversely with the plant capacity factor. Thus, a plant of 10 000 kw, costing $150 per kw, or $1 500 000, would have fixed charges of $150 000 per yr. With a capacity factor of 60% the plant would produce 6 000 kw as an annual average load, or about 52 million kw-hr per yr, costing 0.29 ct per kw-hr.

With a capacity factor of 30% there would be a yearly average load of 3 000 kw, or 26 million kw-hr, costing 0.58 ct per kw-hr, which is twice the cost for a 60% capacity factor.

Curves by means of which fixed charges for hydro-plants (in cents per kilowatt-hour) may be obtained for different plant costs per kilowatt and different plant capacity factors may be constructed.[6] From a curve based upon 10% fixed charges, the cost per kilowatt-hour for other percentages of fixed charges, can readily be obtained by proportion. As previously noted, fixed charges are essentially the cost of hydro-electric energy.

Hydro-Generated Energy in a Power System

Two aspects of hydro-electric power as used in a power system are of importance:

(1) At the wetter seasons of the year (particularly in the spring, when water is available up to and beyond plant capacity) the hydro-plant may be used to carry some, or all, of the base load part of the demand, thus saving principally in fuel cost of steam plants; and

(2) In the medium and low-water seasons the hydro-plant may be used to carry peaks, or the upper part of the load curve, thus saving in respect to steam capacity. Aspect (2) is of much importance, as in the upper part of the usual load curve the proportion of energy to that of the daily total is relatively small for a considerable range of required capacity. In other words, the peaks are of relatively short duration, with the result that a hydro-electric plant carrying, say, the upper 10% of daily energy will include approximately 20% to 40% of peak capacity, depending on the system load curve and load factor.

It should also be noted that the extent to which the system peak is already taken by hydro is of importance in the consideration of additional hydro-plants. The first hydro-plant in the system occupies the favored position at the top of the curve, hence a second plant, as it must take a lower and less advantageous position on the load curve, is usually more difficult to justify.

[6] For an example of this style of plotting, see *Electrical World*, August 14, 1937, p. 100 (570).

Existing conditions are thus of importance in determining the economic justification of new hydro-plants.

The relations of the percentage of daily energy carried by water power and the percentage of peak load carried by the hydro-electric plant are shown in Fig. 20 for system load factors of 0.4, 0.5, 0.6, and 0.7. Each of the curves shown therein is the result of a study of groups of load curves through the given range of load factors and will serve for use in preliminary estimates and studies.

Assuming that a hydro-electric plant supplies 10% of the daily energy for a system with an average daily load of 300 000 kw, or 7.2 million kw-hr daily, the hydro-plant carries 720 000 kw-hr daily, or an average load of 30 000 kw, which latter value might represent the primary 24-hr flow at the site. Referring to Fig. 20, the hydro-plant could carry peak loads, as shown in Column (4), Table 2, for different system load factors. The assumed conditions are shown graphically in Fig. 21.

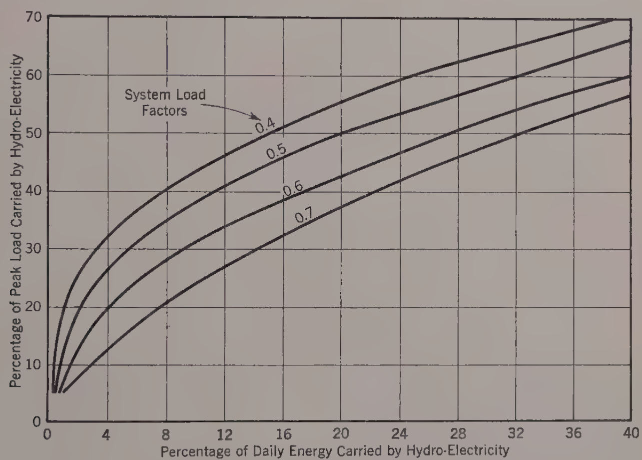

FIG. 20.—SYSTEM PEAK CHARACTERISTICS FOR DIFFERENT SYSTEM LOAD FACTORS

TABLE 2.—DATA SHOWING USE OF FIG. 20

System load factor (percentages)	Peak system load, in kilowatts	CARRIED BY HYDRO-ELECTRIC PLANT		Ratio: Column (4) / 30 000 kw.	$\frac{1}{\text{Column (5)}}$, or hydro-load factors (percentages)
		Percentage of peak (from Fig. 20)	Corresponding kilowatts		
(1)	(2)	(3)	(4)	(5)	(6)
40	750 000	43.5	327 000	10.9	9.2
50	600 000	38.0	228 000	7.6	13.1
60	500 000	31.5	157 000	5.2	19.2
70	430 000	24.	103 000	3.4	29.5

In Column (5), Table 2, is given the ratio of kilowatts carried by the hydro-plant at the time of system peak load to the hydro-plant average 24-hr load. This shows that, depending on the system load factor, if used at the time of

system peak load the plant can supply peak capacity of 3.4 to 10.9 times the amount available on a 24-hr steady use basis. Plant load factors will be the reciprocal of values in Column (5) × 100 and, as will be noted, will vary from approximately 9 to 30 per cent.

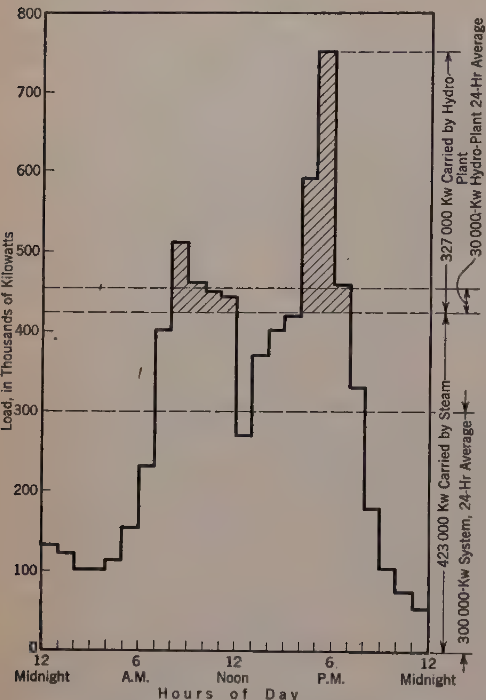

FIG. 21.—USE OF WATER POWER FOR PEAK CAPACITY; SYSTEM LOAD FACTOR, 40 PER CENT

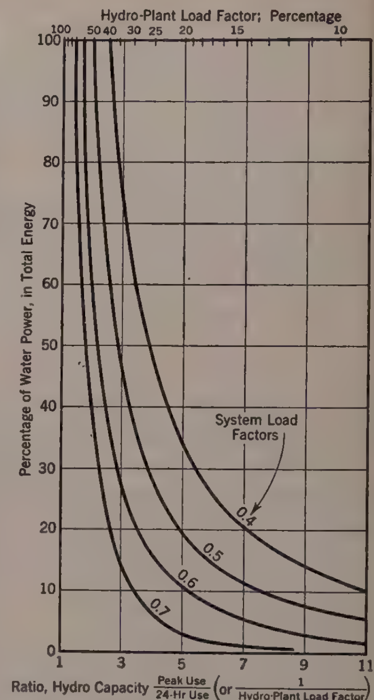

FIG. 22.—USE OF WATER POWER ON SYSTEM PEAKS

Fig. 22 is obtained by computations similar to those for Columns (5) and (6), of Table 2, for various values of percentages of water power in total daily energy (based upon Fig. 20), and shows directly, for different system load factors, the relation of peak hydro-electric capacity (when used at the time of system peak) as compared with its 24-hr use; and by auxiliary scale at the top, the corresponding hydro-load factors.

PONDAGE

Pondage is necessary at the hydro-plant, of course, in order to make possible the use of hydro-electric power at the time of the system peak. The number of hours per day during which water is being used and, by another scale, the hours of collection of water, or pondage, are given approximately by Fig. 23, for different percentages of system peak carried by the hydro-electric plant.

Referring to the previous example and assuming that the 30 000 kw of 24-hr primary hydro-electric power is developed under a head of 200 ft, requiring about 2 250 cu ft per sec with 100% utilization, the data in Table 3 are obtained.

TABLE 3.—DATA FOR THE USE OF FIG. 23

System load factor (percentages)	Percentage of peak carried by hydro-plant (Column (3) in Table 2)	Hours of use (Fig. 23)	Hours of pondage 24 − Column (3)	Pondage Required: Cubic feet per second-hours (Column (4) ×2 250)	Pondage Required: Acre-feet (Column (5) $\times \frac{3\ 600}{43\ 560}$)	Portion of 24-hour flow ponded $\left(\frac{\text{Column (4)}}{24}\right)$
(1)	(2)	(3)	(4)	(5)	(6)	(7)
40	43.5	7.0	17.0	38 000	3 140	0.70
50	38.0	7.5	16.5	37 000	3 060	0.69
60	31.5	8.0	16.0	36 000	2 970	0.67
70	24.0	8.0	16.0	36 000	2 970	0.67

Table 3 demonstrates the use of Fig. 23 and the computation of required pondage for different system load factors, where, as previously noted, the hydro-plant is carrying 10% of the daily energy, under four different system load factors. From 2 970 to 3 140 acre-ft of pondage are required, or from 0.67 to 0.70 of the 24-hr flow.

A further study similar to that of Table 3 for various percentages of the hydro-plant energy to the total daily energy output of the system shows (with only minor variation with system load factor) approximate pondage requirements as follows:

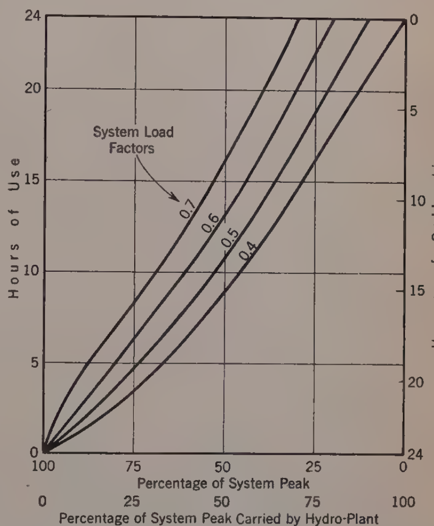

FIG. 23.—PONDAGE; TIME OF USE AND OF COLLECTION FOR DIFFERENT SYSTEM LOAD FACTORS; WATER POWER USED ON SYSTEM PEAK

Hydro-electric energy in percentage of daily energy	Pondage requirements (ratio to 24-hr flow)	Hydro-electric energy in percentage of daily energy	Pondage requirements (ratio to 24-hr flow)
2	0.85	8	0.71
4	0.78	10	0.68
6	0.74	15	0.61

Note that with less than 100% utilization of river flow, less pondage is required; but operation of the hydro-plant for peak loads will usually be with fairly complete utilization.

Although there is some approximation in the foregoing methods the general results are sufficiently accurate to indicate general pondage requirements and show that pondage, to the extent of about 60 to 85% of the 24-hr flow, is required for its peak load hydro-electric use, depending on its percentage of the total daily energy.

Firm Capacity of Hydro-Plant

As previously noted, the usual hydro-plant capacity in a power system corresponds to about the flow available 20% to 30% of the time. Peak or day-load plants will be at about the 5% flow or a somewhat lesser percentage (higher capacity) in some cases. The primary 24-hr flow may be taken at about the 95% flow.

The general form of the average flow-duration curve in regions where water power is in general use is shown approximately by the following two equations:

East and South:

$$\log Q = 2.45 - 0.011\,T \qquad (1)$$

West:

$$\log Q = 2.52 - 0.013\,T \qquad (2)$$

in which Q is the discharge expressed as a percentage of the mean yearly discharge, and T is percentage of time. Table 4 is based upon Equations (1) and (2). It will be noted that ordinary hydro-wheel capacities will be from

TABLE 4.—Flow Relations

Percentage of time that flow is available	Flow as a Percentage of Average Yearly Flow		Ratios to Flow Available 95% of Time	
	East and South	West	East and South	West
5	250	285	10.	15.0
20	170	182	6.8	9.6
30	132	135	5.3	7.1
95	25	19	1.0	1.0

about six to eight times the primary 24-hr power, and peak or day-load hydro-plants ten or fifteen times the primary 24-hr power.

It will be seen that, in many cases, depending upon the extent of hydro-power use and system load factor, the entire hydro-electric capacity may be firm, or largely so, when used on the system peak. Thus, in the example previously given (see Table 2), in which the primary 24-hr power was 30 000 kw, the wheel capacity would be ordinarily about $8 \times 30\,000 = 240\,000$ kw; or, if a peak or day-load plant is considered, $15 \times 30\,000 = 450\,000$ kw. The proportion of plant capacity that would be firm for different system load factors would then be as given in Table 5.

TABLE 5.—Firm Plant Capacities

System load factor (percentage)	Carried by hydro-electric plant (Column (4), Table 2), in kilowatts	Proportion of Plant Capacity That Is Firm	
		Plant capacity, 240 000 kilowatts	Plant capacity, 450 000 kilowatts
40	327 000	All	0.73
50	228 000	0.95	0.51
60	157 000	0.65	0.35
70	103 000	0.43	0.23

Peak Capacity Value of Hydro-Plant

The essential or important result of using water power on the peak of the system load is to save steam capacity. The cost of a steam plant is approximately \$100 per kw.[7] In comparing with water power, the ratio of fixed charges (10% for water power and 12.5% for steam power) would require a factor of 1.25, making the comparative cost of the steam plant, \$125 per kw. This may be regarded as a credit item on the cost of hydro-plant when the latter can be used for system peak loads.

Thus, to illustrate this suggested method of procedure, assuming that a hydro-plant in use for saving in system peak capacity costs \$200 per kw and that three-fourths of its capacity is firm, the credit on account of capacity saving would be, say, $\frac{3}{4} \times \$125 = \$94$ per kw, leaving a net cost of \$106 per kw for the hydro-plant. The yearly capacity factor for such a plant might be 0.5 (if used for base loads in the wetter seasons) and its fixed charges, 0.24 ct per kw-hr. With allowance for operating and maintenance costs, this would be about 0.27 ct per kw-hr as the total cost of hydro-electric energy.

This cost of hydro-electric energy would compete with fuel and attendance costs of steam power. With coal at \$3.00 per net ton and 1.5 lb per kw-hr, fuel would cost 0.225 ct per kw-hr, and attendance about 0.10 ct, or a total of 0.32 ct per kw-hr—showing some margin in favor of the hydro-plant with conditions as assumed.

This method of crediting or allowing for capacity value of a hydro-plant for peak use in a load system and regarding the hydro-electric energy output as affecting fuel saving and attendance charges enables the consideration of the total hydro-electric output and renders unnecessary the segregation of that part commonly called primary. In an actual case the steam capacity saving would be determined by taking into account the primary 24-hr power, pondage, actual wheel capacity, and the actual system load curve. The foregoing method is only approximate and is used merely for illustrative purposes. An illustrative example using actual data for the Conowingo plant is given subsequently herein.

Cost of Hydro-Electric Plants

Supporting Data.—Cost data for hydro-electric plants have been obtained as a basis for study in this paper from two general sources:

(1) From the files and annual reports of the Federal Power Commission[8] at Washington, D. C., plant characteristics and data of cost for thirty-four hydro-plants in different sections of the United States and especially in the West. These factors are given in detail in Table 6 (see project numbers in Column (2)); and,

(2) By correspondence with engineers and other officials of public utility companies, as well as from personal files of the writer, giving similar data of characteristics and cost for twenty-three hydro-plants, fairly well distributed generally. These data are also given in detail in Table 6 and are

[7] "Cost of Generation of Electrical Energy," by Philip Sporn, *Proceedings*, Am. Soc. C. E., December, 1937, pp. 1925–1938.

[8] Annual Repts., Federal Power Comm., 1923–1936, inclusive.

TABLE 6.—CHARACTERISTICS AND COSTS OF HYDRO-ELECTRIC PLANTS

Plant No.	Name (1)	River (2)	State (3)	Date of completion (4)	Drainage area, in square miles (5)	Capacity, in thousands of kilowatts at switchboard (6)	No. of units (7)
		(a) EASTERN PLANTS					
1	Green Island, Project (13) *(a)*	Hudson	N. Y.	1923*	....	6.6	8
2	Wallenpaupack, Project (487)	Wallenpaupack Creek	Pa.	1926*	240	40.0	2
3	Saluda, Project (516)	Saluda	S. C.	1931*	....	130.0	4
4	Bartlett's Ferry, Project (485)	Chattahoochee	Ga.	1926*	4 200	45.0	3
5	Jackson Bluff, Project (682)	Ocklockonee	Fla.	1930*	....	8.8	3
6	Moss Bluff, Project (177)	Ocklawaha	Fla.	1927*	....	0.5	..
7	Wyman	Kennebec	Me.	1931	2 660	72.0*(b)*	3
8	Weston	Kennebec	Me.	1921	....	12.0	4
9	Gulf Island	Androscoggin	Me.	1926	2 860	19.0	3
10	.... *(c)*	....	....	1937	690	3.0	1
11	Mollys Falls	Winooski	Vt.	1926	23	5.0	1
12	Harriman	Deerfield	Vt.	1924	184	43.0	3
13	Sherman *(c)*	Deerfield	Mass.	1926	234	7.0	1
14	Conowingo	Susquehanna	Md.	1928*	27 500	252.0	7
15	Safe Harbor	Susquehanna	Pa.	1932	26 000	178.0	6
		(b) PLANTS IN THE SOUTHERN AND CENTRAL STATES					
16	Waterville, Project (432)	Big Pigeon	N. C.	1930	....	108.0	3
17	Piney, Project (309)	Clarion	Pa.	1925	....	28.8	3
18	Dam No. 7, Project (539) *(a)*	Kentucky	Ky.	1928*	....	2.0	3
19	Ohio Falls, Project (289) *(a)*	Ohio	Ky.	1928*	....	80.3	8
20	Shawano, Project (710)	Wolf	Wis.	1928*	....	0.7	1
21	Mottville, Project (401)	St. Joseph	Mich.	1923*	....	1.7	4
22	Winton, Project (469)	Kawishiwi	Minn.	1924*	....	4.0	2
23	Blanchard, Project (346)	Mississippi	Minn.	1925*	....	12.0	2
24	Twin City, Project (362) *(a)*	Mississippi	Minn.	1924*	....	13.4	4
25	Big Bend, Project (792)	Rapid Creek	S. D.	1930*	....	1.2	3
26	....	....	....	1923	9 827	52.5	3
27	....	....	....	1928	....	100.0	4
28	....	....	....	1926	3 000	99.0	3
29	Allegan	Kalamazoo	Mich.	1937	1 530	2.4*(b)*	3
30	....	....	....	1931	14 000	133.0	6
		(c) WESTERN PLANTS					
31	Lower American Fork, Project (696)	American Fork Creek	Utah	1927*	....	1.0	2
32	Upper American Fork, Project (696)	American Fork Creek	Utah	1927*	....	1.2	2
33	Hyrum City, Project (946)	Blacksmith Fork	Utah	1930*	....	0.4	1
34	Logan, Project (486)	Logan	Utah	1927*	....	2.0	2
35	Santaquin, Project (665)	Santaquin	Utah	1927*	....	0.9	2
36	Stairs, Project (597)	Big Cottonwood Creek	Utah	1927*	....	1.5	2
37	Alpine, Project (671)	Alpine Creek	Utah	1927*	....	1.8	2
38	Battle Creek, Project (675)	Battle Creek	Utah	1927*	....	2.4	1
39	Kern Canyon, Project (178)	Kern	Calif.	1921*	....	8.5	1
40	Borel, Project (382)	Kern	Calif.	1925*	....	10.0	..
41	Exchequer, Project (88)	Merced	Calif.	1926*	....	25.0	2
42	Kerckhoff, Project (96)	San Joaquin	Calif.	1920*	1 488	39.0	3
43	Chelan, Project (637)	Chelan	Wash.	1927*	....	48.0	2
44	Lewiston, Project (621)	Clearwater	Idaho	1927*	....	10.0	2
45	Grangeville, Project (204)	South Fork Clearwater	Idaho	1923*	....	0.6	2
46	Wallowa Falls, Project (308)	East Fork Wallowa, Royal Purple Creek	Ore.	1924*	....	0.8	1
47	Glines Canyon, Project (588)	Elwha	Wash.	1927*	....	12.0	2
48	Ariel, Project (935)	Lewis	Wash.	1932*	....	46.0	2
49	Rock Island	Columbia	Wash.	1933	90 000	60.0	4
50	Mystic Lake	West Rosebud	Mont.	1924	47	10.5	2
51	Gorge	Skogit	Wash.	1924	1 200	55.5	3
52	....	....	....	1922	670	70.0	2
53	....	....	....	1925	4 750	81.0	3
54	....	....	....	1921	580	66.7	3
55	....	....	....	1908	1 970	65.0	6
56	....	....	....	1928	50	50.0	2
57	....	....	....	1931	260	60.0	2

* Began operation. *(a)* Uses Government Dam. *(b)* Ultimate capacity. *(c)* Remote control.

TABLE 6.—*Continued*

Head, in feet	Annual Output		Annual average load, in thousands of kilowatts	Pondage, or storage, in thousands of acre-feet	Ratio: Pondage to wheel capacity	Cost, in Thousands of Dollars					
	Years	Average in kilowatt-hours per million				Dam	Waterway	Powerhouse and equipment	Highways, railways, etc.	Land and water rights	Total
(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)
					(a) Eastern Plants						
14	1933–1936	20.5	2.3	....	....	23	417	1 317	....	....	1 757
370	1932–1936	76	8.7	160	50	1 263	2 197	2 558	124	3 051	9 193
183	1933–1936	264	30.2	1 600	77	11 624		2 954	80	6 110	20 768
114	1936	328	37.4	120	10	4 228	305	1 949	166	1 199	7 847
33	1934	8.5	1.0	34	4.4	....	....	....	....	....	2 920
17	1934	1.5	0.2	....	....	....	....	....	....	....	139
139	....	280	32.0	60	3.8	....	....	....	....	....	14 000
32	1921–1936	47	5.4	....	....	....	....	....	....	....	2 200
58	....	90	10.3	29	2.6	($3 500)				1 200	4 700
41	....	15.5	1.8	6.3	2.9	250	53	223	102	95	723
350	....	5	0.6	9.0	21.0	201	215	190	....	256	862
360	....	136‡	15.5	115.0	33.0	3 720	3 150	1 620	1 210	600	10 300
78	....	25§	2.8	....	....	840	110	590	....	600	2 140
89	....	1 300	148.0	72.0	0.9	10 425	1 745	22 400	8 350	6 900	49 820
55	....	880	100.0	69.0	0.7	7 700	....	18 600	....	800	27 100
					(b) Plants in the Southern and Central States						
860	1933–1936	325	37.0	....	....	....	....	....	....	....	13 800
80	1934–1936	55	6.3	....	....	5 871		4 380	....	389	10 640
18	1936	8.0	0.9	....	....	4	49	420	....	1	474
37	1933–1936	317.0	36.2	....	....	158		7 484	....	14	7 656
14	1933–1936	3.2	0.4	....	....	133		170	....	32	335
13	1934–1936	3.6	0.4	....	....	293	3	165	....	88	549
....	1933–1936	19.5	2.2	....	....	....	....	....	....	....	1 544
46	1933–1936	30.0	3.5	....	....	....	....	....	....	....	3 400
30	1933–1936	43.0	5.0	....	....	13	226	1 231	....	....	1 470
268	1934–1936	2.0	0.2	....	....	10	236	64	....	8	318
63	(13 yr)	266.0	30.3	....	....	($6 669)				3 802	10 471
87	(8 yr)	406.0	46.3	....	....	($9 155)				3 205	12 360
125	(10 yr)	275.0	31.4	1 380	60	($14 824)				3 458	18 282
16	....	9.0	1.0	3 500	1.4	150	....	260	24	177	611
90	....	425.0	48.5	1 200	28	6 800	180	8 000	500	3 000	18 480
					(c) Western Plants						
....	1936	3.9	0.4	....	....	10	99	57	....	2	168
....	1936	5.1	0.6	....	....	10	113	68	....	....	191
92	....	....	....	....	....	16	24	26	....	2	68
220	1936	11.9	1.4	....	....	46	238	136	....	2	422
650	1936	2.5	0.3	....	....	3	68	67	....	2	140
370	1936	5.6	0.6	....	....	135	104	77	....	2	318
1 920	1936	3.9	0.4	....	....	2	130	65	....	1	198
1 790	1936	2.2	0.3	....	....	1	135	104	....	2	242
260	1927–1931, incl., 1936	44.0	5.0	....	....	147	1 262	534	3	41	1 987
270	....	....	....	....	....	51	1 574	364	3	112	2 104
210	1928–1931, incl.	70.0	8.0	285.0	82	3 904	215	933	8	6 400	11 460
350	1927–1931, incl., 1936	193.0	22.0	4.2	1.3	813	3 292	1 426	62	5	5 598
410	1933–1936, incl.	278.0	31.7	....	....	1 771	2 880	2 209	....	2 403	9 263
36	1933–1936, incl.	18.7	2.1	....	....	1 342	545	789	....	682	3 358
64	1933–1936, incl.	2.7	0.3	....	....	164	24	44	1	....	233
1 170	1934–1936, incl.	3.3	0.4	....	....	13	51	25	....	2	91
160	1928–1929	85.6	9.8	24.0	11	690	361	528	....	207	1 786
170	1936	225.0	25.7	225.0	28	5 045	229	2 080	397	922	8 673
32	1936	379.0	43.2	14.0	0.25	8 387	....	4 545	....	1 623	14 555
1 050	....	60.0	6.8	....	....	185	975	520	....	(120)	1 800
270	12 yr	240.0	27.4	....	....	4 424‖	....	3 557	2 385	53	10 419
454†	....	282.0	32.2	....	....	277	2 917	2 891	797	1 638	8 520
315†	....	393.0	44.8	8.4	1.1	2 073	6 268	2 143	772	355	11 611
1 149†	....	462.0	52.7	1 357.0	800	1 185	10 180	2 796	114	123	14 398
465†	....	536.0	61.2	1 466.0	360	894	4 197	2 188	101	3	7 383
2 561†	....	231.0	26.4	108.7	190	1 391	3 175	1 279	436	226	6 507
1 218†	....	291.0	33.2	155.4	108	689	5 470	1 421	168	31	7 779

† Gross. ‡ Includes lower plants. § Includes lower plants; remote control. ‖ Includes waterways.

TABLE 6.—*Continued*

Plant No.	Name	Annual Costs, in Thousands of Dollars: During the years: From:	To:	Operation	Maintenance	Total	Cost, in Dollars per Kilowatt of Capacity: Dam: Cost	Dam: Percentage of total	Waterway: Cost	Waterway: Percentage of total
	(1)	(20)	(21)	(22)	(23)	(24)	(25)	(26)	(27)	(28)
	(a) Eastern Plants									
1	Green Island, Project (13)	1933	1936	34.0		34.0	4	1	63	24
2	Wallenpaupack, Project (487)	1932	1936	34.0	21.0	55.0	32	14	55	24
3	Saluda, Project (516)	1933	1936	45.0	15.0	60.0	89‖	56‖	....	....
4	Bartlett's Ferry, Project (485)	1936		22.0	10.0	32.0	94	54	7	4
5	Jackson Bluff, Project (682)	1934		21.0	5.0	26.0	....	....	....	....
6	Moss Bluff, Project (177)	1934		1.2	0.4	1.6	....	....	....	....
7	Wyman	....	....	....	....	....	....	....	....	....
8	Weston	....	....	....	....	....	....	....	....	....
9	Gulf Island	....	....	....	....	....	184¶	75¶	....	....
10	....	....	....	....	....	....	83	34	18	8
11	Mollys Falls	....	....	....	....	6.0	40	23	43	25
12	Harriman	....	....	....	....	....	86	36	73	30
13	Sherman	....	....	....	....	....	120	39	16	5
14	Conowingo	....	....	105	100	205	41	21	7	3
15	Safe Harbor	....	....	116	45	161	43	28	....	....
	(b) Plants in the Southern and Central States									
16	Waterville, Project (432)	1933	1936	25.0	23.0	48.0	....	....	....	....
17	Piney, Project (309)	1934	1936	35.0		35.0	204‖	55‖	....	....
18	Dam No. 7, Project (539)	1936		1.8	1.3	3.1	2	1	24	10
19	Ohio Falls, Project (289)	1933	1936	53.0	16.0	69.0	2‖	2‖	....	....
20	Shawano, Project (710)	1936		1.8	0.4	2.2	190‖	40‖	....	....
21	Mottville, Project (401)	1936		4.8	0.7	5.5	175	53	2	1
22	Winton, Project (469)	1934	1936	14.0	4.0	18.0	....	....	....	....
23	Blanchard, Project (346)	1934	1936	17.0	5.0	22.0	....	....	....	....
24	Twin City, Project (362)	1933	1936	114.0		114.0	1	1	17	15
25	Big Bend, Project (792)	1936		3.9	7.4	11.3	8	3	196	74
26	....	....	....	....	....	52.0	127¶	64¶	....	....
27	....	....	....	....	....	67.0	92¶	74¶	....	....
28	....	....	....	....	....	73.0	149¶	81¶	....	....
29	Allegan	....	....	....	....	....	62	24	....	....
30	....	....	....	....	....	....	51	37	1	1
	(c) Western Plants									
31	Lower American Fork, Project (696)	1936		6.4	1.0	7.4	11	6	104	59
32	Upper American Fork, Project (696)	1936		3.9	0.7	4.6	8	5	94	59
33	Hyrum City, Project (946)	....	....	....	....	....	40	24	60	35
34	Logan, Project (486)	1936		9.3	2.1	11.4	23	11	119	56
35	Santaquin, Project (665)	1936		3.2	2.3	5.5	3	2	78	49
36	Stairs, Project (597)	1936		5.8	0.3	6.1	90	42	69	33
37	Alpine, Project (671)	1936		2.9	1.8	4.7	1	1	74	65
38	Battle Creek, Project (675)	1936		2.6	0.9	3.5	1	1	56	55
39	Kern Canyon, Project (178)	1931		6.2	0.5	6.7	17	7	149	64
40	Borel, Project (382)	....		....	....	....	5	3	158	75
41	Exchequer, Project (88)	1931		17.0	....	17.0	156	34	9	2
42	Kerckhoff, Project (96)	1931		27.0	11.0	38.0	21	15	84	58
43	Chelan, Project (637)	1936		24.0	6.0	30.0	37	19	60	31
44	Lewiston, Project (621)	1936		13.0	34.0	47.0	134	40	55	16
45	Grangeville, Project (204)	1936		7.4	1.6	9.0	263	70	38	10
46	Wallowa Falls, Project (308)	1934	1936	7.2	0.2	7.4	16	14	64	56
47	Glines Canyon	1928	1929	5.0	2.0	7.0	57	39	30	20
48	Ariel	1936		36.0	7.0	43.0	110	58	5	3
49	Rock Island	1936		34.0	25.0	59.0	140	58	....	....
50	Mystic Lake	....	....	....	....	....	18	11	93	54
51	Gorge	....	....	33.0	26.0	59.0	80‖	42‖	....	....
52	....	....	....	25.0	9.0	34.0	4	3	42	35
53	....	....	....	42.0	7.0	49.0	26	18	77	54
54	....	....	....	38.0	19.0	57.0	18	8	152	70
55	....	....	....	42.0	20.0	62.0	14	12	64	56
56	....	....	....	21.0	7.0	28.0	28	22	63	48
57	....	....	....	35.0	25.0	60.0	11	9	91	70

¶ Total, except land and water rights.

TABLE 6.—*Continued*

Cost, in Dollars per Kilowatt of Capacity								Annual Cost of Power, in Thousands of Dollars				Cost, in cents per kilowatt-hour of output	Annual capacity factor
Power-House and Equipment		High-ways, railways, etc.		Land and water rights		Total		Fixed charges	Operation and maintenance	Water rents	Total		
Cost	Percentage of total	Cost	Percentage of total	Cost	Percentage of total	Cost	Percentage of total						
(29)	(30)	(31)	(32)	(33)	(34)	(35)	(36)	(37)	(38)	(39)	(40)	(41)	(42)
(a) Eastern Plants													
201	75	....	....	....	....	268	100	176	34	5	215	1.05	0.37
64	28	3	1	76	33	230	100	919	55	....	974	1.28	0.22
23	14	1	1	47	29	160	100	2 077	60	....	2 137	0.81	0.23
43	25	4	2	27	15	175	100	785	32	....	817	0.25	0.83
....	....	....	....	....	....	332	100	292	26	....	318	3.74	0.11
....	....	....	....	....	....	283	100	14	2	....	16	1.03	0.37
....	....	....	....	....	....	195	100	1 400	(60)	....	1 460	0.52	0.45
....	....	....	....	....	....	183	100	220	(20)	....	240	0.51	0.45
....	....	....	....	63	25	247	100	470	(30)	....	500	0.56	0.54
74	31	34	14	32	13	241	100	72	(3)	....	75	0.48	0.59
38	22	....	....	51	30	172	100	86	6	....	92	1.84	0.11
38	16	28	12	14	6	239	100	1 030	(40)	....	1 070	0.79	0.36
84	28	....	....	86	28	306	100	214	(5)	....	219	0.87	0.41
89	45	33	17	27	14	197	100	4 982	205	133	5 320	0.41	0.59
104	69	....	....	5	3	152	100	2 710	161	....	2 871	0.33	0.56
(b) Plants in the Southern and Central States													
....	....	....	....	....	....	128	100	1 380	48	....	1 428	0.44	0.34
152	41	....	....	14	4	370	100	1 064	35	....	1 099	1.99	0.22
205	89	....	....	0	0	231	100	47	3	5	55	0.69	0.44
93	98	....	....	0	0	95	100	766	69	105	940	0.30	0.45
243	51	....	....	46	9	479	100	34	2	....	36	1.13	0.52
98	30	....	....	52	16	327	100	55	5	....	60	1.68	0.24
....	....	....	....	....	....	386	100	154	18	....	172	0.88	0.55
....	....	....	....	....	....	283	100	340	22	....	362	1.20	0.29
92	84	....	....	....	....	110	100	147	114	95	356	0.82	0.37
53	20	....	....	7	3	264	100	32	11	....	43	2.15	0.19
....	....	....	....	72	36	199	100	1 047	52	....	1 099	0.41	0.58
....	....	....	....	32	26	124	100	1 236	67	....	1 303	0.32	0.46
....	....	....	....	35	19	184	100	1 828	73	....	1 901	0.69	0.32
108	43	10	4	74	29	254	100	61	(9)	....	70	0.78	0.43
60	43	4	3	23	16	139	100	1 850	(150)	....	2 000	0.47	0.36
(c) Western Plants													
60	34	....	....	2	1	177	100	17	7	....	24	0.62	0.47
57	36	....	....	....	....	159	100	19	5	....	24	0.47	0.48
65	38	....	....	5	3	170	100	....	....	....	....	....	....
68	32	....	....	1	1	211	100	42	11	1	54	0.45	0.68
76	48	....	....	2	1	159	100	14	6	....	20	0.80	0.32
52	25	....	....	1	0	212	100	32	6	....	38	0.68	0.43
37	33	....	....	1	1	113	100	20	5	....	25	0.64	0.25
43	43	....	....	1	1	101	100	24	4	....	28	1.27	0.10
63	27	0	0	5	2	234	100	199	7	1	207	0.47	0.59
36	17	0	0	11	5	210	100	....	....	....	....	....	....
37	8	0	0	256	56	458	100	1 146	17	2	1 165	1.66	0.32
37	26	2	1	0	0	144	100	560	38	66	664	0.34	0.56
46	24	....	....	50	26	193	100	926	30	....	956	0.34	0.66
79	24	....	....	68	20	336	100	336	47	....	383	2.05	0.21
70	19	2	1	....	....	373	100	23	9	....	32	1.20	0.50
31	27	....	....	3	3	114	100	9	7	....	16	0.48	0.48
44	30	....	....	17	11	148	100	179	7	....	186	0.22	0.81
45	24	9	5	20	10	189	100	867	43	30	940	0.42	0.56
76	31	....	....	27	11	243	100	1 455	59	....	1 514	0.40	0.72
50	29	....	....	(11)	6	172	100	180	20	....	200	0.34	0.65
64	34	43	23	1	1	188	100	1 042	59	....	1 101	0.46	0.49
41	34	11	9	23	19	121	100	852	34	....	886	0.31	0.46
26	18	10	7	4	3	143	100	1 161	49	....	1 210	0.31	0.55
42	20	2	1	2	1	216	100	1 440	57	130	1 627	0.35	0.79
34	30	2	2	0	0	114	100	738	62	72	872	0.16	0.94
26	20	9	7	4	3	130	100	651	28	105	784	0.34	0.53
24	18	3	2	1	1	130	100	778	60	260	1 098	0.38	0.55

distinguishable from Source (1) in that they do not have project numbers (see Column (2)).

Most of the items in Table 6 need no explanation. As applied to Federal projects, head, in feet (Column (8)), is the gross or static head, as the actual working heads were not available. For non-Federal projects, the head is given as the working, or net, head, except as otherwise stated. Annual output (Table 6, Column (10)) is given as an average value for such years as are available. However, where load is increasing the latest year of record is used. The values in Column (13), the ratio of pondage to wheel capacity, are obtained by dividing the pondage capacity, in second-feet-days, by the flow at wheel capacity, in second-feet.

Costs (Columns (14) to (19), Table 6) are as recorded, except as condensed into the break-down used in Table 6. Undistributed over-head costs, where they occur, have been pro-rated. Under the heading, "Cost, in Dollars per Kilowatt of Capacity" (Columns (25) to (36), Table 6) certain projects include only a small item for the cost of the dam where power is developed at a navigation dam.

Under "Annual Cost of Power" (Column (37)), note that fixed charges are taken at 10 per cent. In certain projects an item of production rental is included to allow for fees paid yearly to the Federal Power Commission. The cost per kilowatt-hour (Column (41)) is the cost of total power output, including both primary and secondary power.

Range of Conditions.—That the information in Table 6 is representative is indicated by the range of conditions that characterize the 57 plants included. Geographically, 15 are in the East, 15 are in the Southern and Central States, and the remainder (27) are in the West. In point of age, the dates of construction, or of beginning operations, are as follows: 1908 (1); 1920 (1); 1921 (3); 1922 (1); 1923 (4); 1924 (6); 1925 (4); 1926 (7); 1927 (11); 1928 (6); 1930 (4); 1931 (4); 1932 (2); 1933 (1); and 1937 (2).

Other characteristics are:

Description	Range
Capacity, in kilowatts	400 to 252 000
Head, in feet	13 to 2 561
Yearly output, in million kilowatt-hours	1.5 to 1 300
Cost of plant, in thousands of dollars	91 to 50 450
Cost, in dollars per kilowatt of capacity[9]	95 to 479
Cost, in cents per kilowatt-hour[10]	0.16 to 3.74
Yearly capacity factor	0.10 to 0.94

The fifty-seven plants have a total capacity of about 2 190 000 kw; they cost about $384 000 000, or an average of about $175 per kw. Their total yearly output is about 9.5 billion kw-hr (or about one-fourth the total public utility water power output of the United States for 1936), at an average cost of 0.44 ct per kw-hr.

[9] Uses Government dam.
[10] Plant built in 1908.

Effect of Head and Capacity upon Cost.—The cost per kilowatt of capacity is plotted against head in Fig. 24. The size of the plant is approximated by circles of varying diameter, the numbers referring to the plant number in Table 6. In general, the unit cost decreases as the head increases and the larger plants generally show lower unit costs. By grouping plants according to head, as in Table 7, general tendencies are revealed more clearly, as shown by the dotted line drawn through the group points in Fig. 24.

TABLE 7.—ELEMENTS OF PLANT COST WITH REFERENCE TO HEAD (Fifty Plants)

Item	Average Head and Range, in Feet, and Number of Plants							
	37 (13–64) 14 Plants		209 (78–370) 24 Plants		568 (410–860) 5 Plants		1 550 (1 050–2 561) 7 Plants	
	Cost, in dollars per kilowatt	Percentage of total cost	Cost, in dollars per kilowatt	Percentage of total cost	Cost, in dollars per kilowatt	Percentage of total cost	Cost, in dollars per kilowatt	Percentage of total cost
Dam	123	44	63	30	14	10	13	9
Waterway	13	5	53	25	59	41	85	61
	136		116		73		98	
Power-house and equipment	97	34	54	26	49	34	36	26
Highways, etc.	5	2	7	3	3	2	2	2
Land and water rights	43	15	34	16	18	13	3	2
Totals	281	100	211	100	143	100	139	100
Range of cost per kilowatt	152–479	....	124–458	....	114–193	....	101–216	....

Four plants in Table 6 are omitted from Table 7, because use is made of a Government dam, and three because the head is not known. Table 7 also shows group costs per horse-power and the percentage of the following five items of cost breakdown:

(1) The cost of the dam varies from \$123 per kw for the low head groups to \$13 for very high heads and is from about 40% to 10% of the total cost;

(2) The cost of the waterway varies from \$13 per kw for low heads to \$85 per kw for very high heads and is from about 5% to 60% of the total cost (the cost of the dam plus the waterway remains more nearly constant, varying from \$136 per kw for low heads to \$98 per kw for very high heads, and is from about 50% to 70% of the total cost);

(3) The cost of the power-house and equipment varies from about \$100 to \$35 per kw, and is about 30% of the total cost;

(4) In general, highways, railroads, etc., are relatively unimportant; and,

(5) Land and water rights vary from about \$40 to \$5 per kw, and are about 15% or 20% for the two lower groups, lessening to about 5% at very high heads. In the case of some of the plants in Table 7, the item of water rights is (in part, anyway) covered by yearly rental charges.

It must be kept in mind that the foregoing items are average costs based upon groups of plants as noted. Individual plants will vary greatly from the

values given. The averages, however, show the trend of unit cost of different parts of the hydro-plant as the head varies.

Cost of Water-Power Output

Effect of Head, Capacity, and Capacity Factor upon Cost.—In Fig. 25(*a*) hydro cost per kilowatt-hour of output is plotted against head, these data being taken from Table 6. Capacity is shown approximately by the size of circles and capacity factor by radial lines in the circles as noted. The numerals refer to plant numbers in Table 6.

Although, in general, the cost per kilowatt-hour tends to lessen as head increases and to lessen with the size of the plant, the effect of capacity factor is more predominant. Note, for illustration, the group of points between 300-ft and 400-ft head, in Fig. 25(*a*). The upper points have low capacity factors and the lower points, high capacity factors. If all are reduced to, say, a 0.50 capacity factor, the points will be relatively near together. This adjustment has been made for all the points of Fig. 25(*a*) and replotted in Fig. 25(*b*) on the basis of a 0.50 capacity factor for all plants and, as will be noted, the major variations in the plot of Fig. 25(*a*) are thus largely eliminated.

The effect of capacity factor upon cost of output is also shown in Fig. 26, where cost per kilowatt-hour of output is plotted against capacity factor, irrespective of head. This also reveals more clearly the smaller cost of output from the larger plants.

Referring again to Fig. 25(*a*), the range in actual costs per kilowatt-hour is seen to vary as Table 8(*a*). From Fig. 25(*b*) the range, adjusted to a constant capacity factor of 0.50, is as shown in Table 8(*b*). The latter data

TABLE 8.—Costs of Water-Power Output, in Cents per Kilowatt-Hour

Head, in feet	(*a*) Range of Actual Costs		(*b*) Range of Costs, Adjusted to a Constant Capacity Factor of 0.50	
	From:	To:	From:	To:
Less than 100..........	0.3	3.7	0.2	1.2
100 to 500............	0.16	2.2	0.3	1.1
More than 500.........	0.35	1.3	0.25	0.55

reveal more clearly the range in costs due to variation in what may be called "site cost factor," or the natural conditions at sites tending toward a greater or less construction cost. The capacity, or use factor, refers to the method of plant operation and, obviously, the differences in cost in Table 8(*a*) and Table 8(*b*) are due solely to different assumed methods of operation, and consequent difference in capacity factors.

Primary Cost of Water Power

Frequently, plant cost per kilowatt, as well as cost per kilowatt-hour of hydro-electric energy, is based upon the power corresponding to the 24-hr primary flow. This is logical in the case of an isolated plant, but misleading

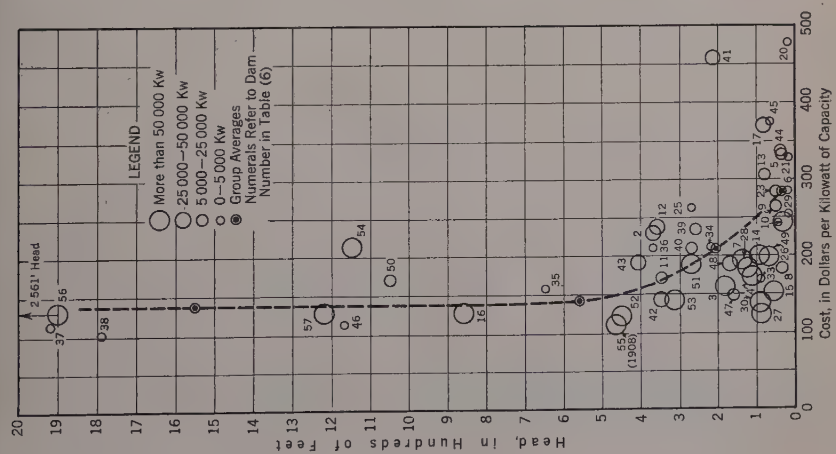

FIG. 24.—PLANT COST OF WATER POWER AS AFFECTED BY HEAD AND CAPACITY

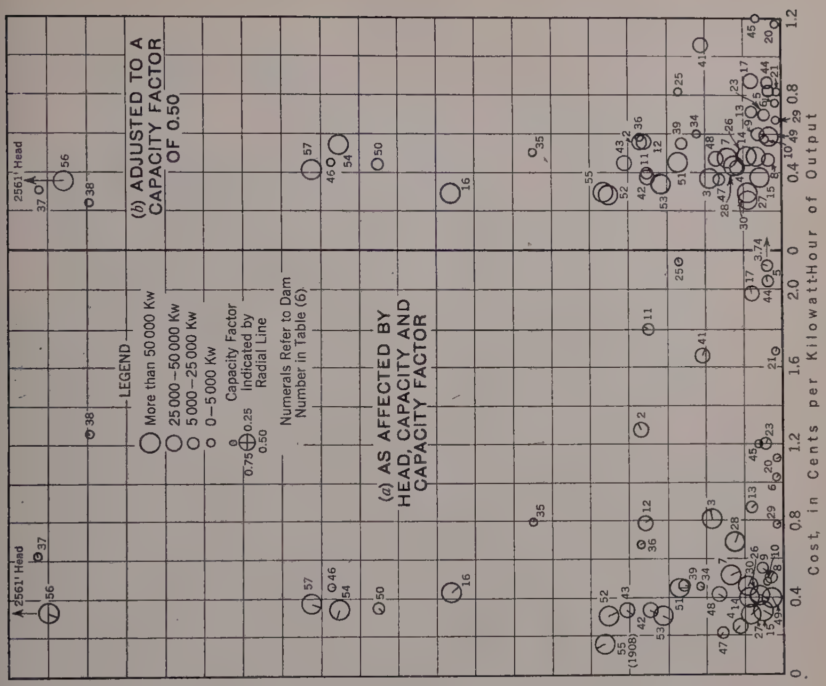

FIG. 25.—COST OF HYDRO-ELECTRIC ENERGY

where the plant is tied in with a load system. In the latter case it is better to credit the hydro-plant with its system peak capacity saving, as has been suggested, and compute the net cost of hydro-electric energy for comparison with steam power fuel saving and attendance.

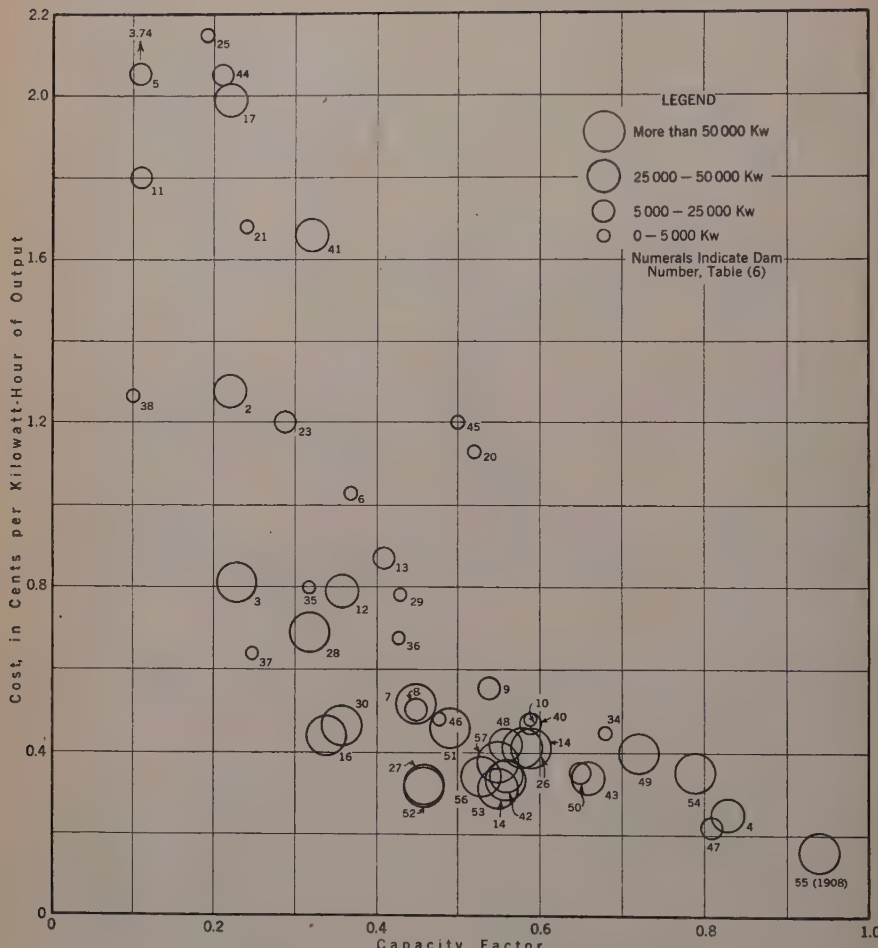

FIG. 26.—COST OF POWER AS AFFECTED BY CAPACITY AND CAPACITY FACTOR

Thus, considering the Conowingo Plant (No. 14, Table 6), in Maryland, on the Susquehanna River (The Philadelphia Electric Company System, September, 1936), to which the following additional data apply:

Average daily low-water, 24-hr power output, in kilowatts	11 000
Peak capacity carried by hydro-electric plant, in kilowatts	210 000
Ratio, $\dfrac{\text{Peak capacity}}{\text{Plant capacity}}$	0.83

By the method suggested, the energy cost per kilowatt-hour of water power would be determined as follows:

Cost of plant		$49 820 000
Credit for saving in steam capacity, 210 000 kw @ $100		21 000 000
Yearly fixed charges on plant @ 10%	$4 982 000	
Credit for saving in fixed charges on steam capacity, $21 000 000 @ 12.5%	2 625 000	
Net fixed charges on plant	2 357 000	
Hydro-plant operation and maintenance	338 000	
Total yearly net cost		$2 695 000

or $\frac{\$2\ 695\ 000}{1\ 300\ 000\ 000 \text{ kw-hr}} = 0.21$ ct per kw-hr, which compares with the cost of fuel plus attendance of equivalent steam power.

In the foregoing analysis no allowance has been made for the higher degree of dependability of hydro-power as compared to steam such that the hydro really replaces somewhat more than an equivalent kilowatt of dependable steam capacity. This is difficult to evaluate exactly but is perhaps of the order of 10 per cent.

Assuming an added 10% saving in steam capacity due to the greater reliability of hydro-power would, in the foregoing example, result in a steam capacity saving of 231 000 kw, and a total saving of $23 000 000, and reduce the energy cost of the water power to about 0.19 ct per kw-hr instead of 0.21 ct per kw-hr.

Advantages of Water-Power in a Power System

Water power used in a power system has three advantages:

(1) Hydro-plant units, large or small, can be placed in operation quickly (in a few minutes), which is important and valuable under sudden large load increases, and particularly in emergencies when some large power unit suddenly goes off the line due to accident or break-down. This quick placing in operation is not possible with steam plants.

(2) Additional, or increment, capacity for the hydro-plant with reference to its system peak use is relatively low in cost, as commonly this is limited chiefly to equipment, whereas the increment cost of a steam plant must include the entire plant. Ordinarily, increment costs for water power will be from $40 to $70 per kw of capacity, whereas for steam this will be $80 to $100 per kw.

(3) Hydro-plant units are generally more reliable than steam units, records showing, for modern hydro-plant units, that the non-scheduled outage "when in demand" is so small as to be negligible.

PUMPED STORAGE, PEAK LOAD, HYDRO-PLANTS

This special use of the hydro-plant is fairly common in Europe, but thus far in the United States only two such plants have been constructed: The Rocky River Plant, in Connecticut, and the Lake Keuka Plant, in New York.[11] There are further opportunities for the use of this method of handling peak loads and the development of a pump-turbine unit with two-speed motor-generator unit, as noted by Mr. Frank H. Rogers,[12] adds materially to their practicability and economic feasibility.

OPERATING AND MAINTENANCE COSTS, HYDRO-ELECTRIC PLANTS

The yearly cost of operation and ordinary maintenance based upon data from forty-five plants in Table 6 is shown on Fig. 27. As will be noted, the

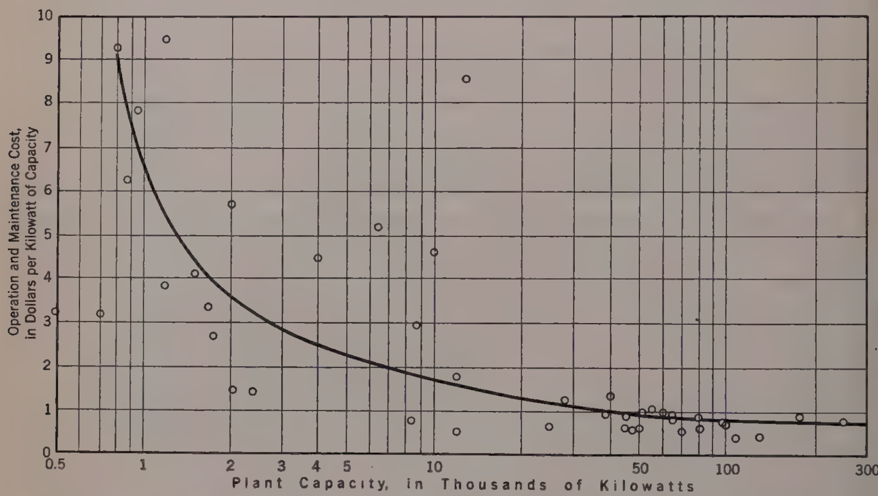

FIG. 27.—ANNUAL HYDRO-PLANT OPERATION, AND MAINTENANCE COSTS, IN DOLLARS PER KILOWATT OF CAPACITY

median curve shows, for a small plant (1 000 kw), costs varying between about \$4.00 and \$9.50 per kw yearly and averaging about \$7.00 per kw. For large plants (more than 100 000 kw), the cost varies between \$0.50 to \$0.90 and averages about \$0.75. Intermediate costs may be noted by reference to Fig. 27. Variations from the median curve for individual plants are considerable.

Cost of operation and maintenance is also shown on Fig. 28, per kilowatt-hour, plotted against total yearly output. It varies between about 0.20 ct per kw-hr for small plants to about 0.02 ct per kw-hr for large plants, with considerable variation from the median curve. Harmonizing the data from Figs. 27 and 28, data of approximate average operating and maintenance costs have been determined in Table 9. These data are given both as total yearly

[11] "Improvements in the Utilization of Energy," by Joel D. Justin, M. Am. Soc. C. E., *Proceedings*, Am. Soc. C. E., December, 1937, pp. 1912–1924.

[12] "Hydro-Generation of Energy," by Frank H. Rogers, *Proceedings*, Am Soc. C. E., December, 1937, pp. 1893–1911.

TABLE 9.—Approximate Average Operation and Maintenance Costs of Hydro-Electric Plants

Plant capacity, in kilowatts	Cost of Operation and Maintenance: Total yearly cost, in dollars	Cost of Operation and Maintenance: Cost, in cents per kilowatt-hour (0.5 capacity factor)	Plant capacity, in kilowatts	Cost of Operation and Maintenance: Total yearly cost, in dollars	Cost of Operation and Maintenance: Cost, in cents per kilowatt-hour (0.5 capacity factor)
1 000	6 000	0.140	50 000	45 000	0.020
5 000	12 000	0.055	100 000	80 000	0.018
10 000	18 000	0.041	150 000	120 000	0.018
20 000	25 000	0.029	200 000	160 000	0.018
30 000	32 000	0.024	300 000	220 000	0.017
40 000	40 000	0.023	....	....	....

costs and, as costs based upon a 0.5 capacity factor, the cost per kilowatt-hour of output. Except for small plants, as has been previously noted, operation and maintenance costs are a relatively small proportion of water power costs.

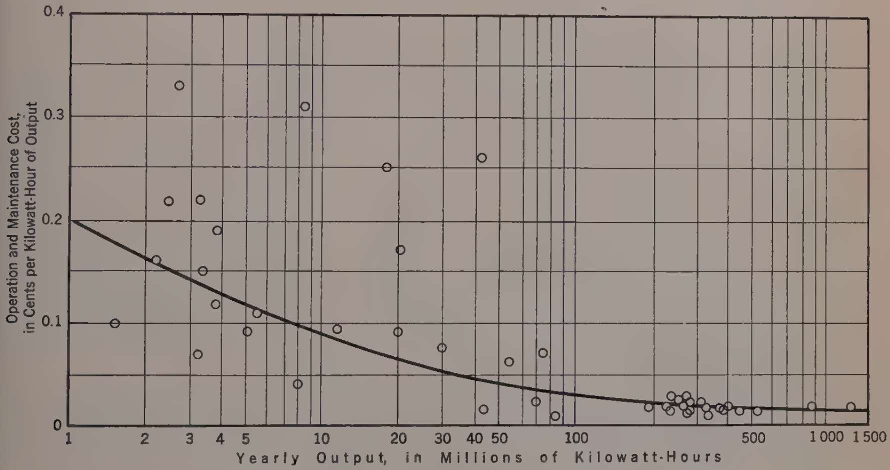

Fig. 28.—Operation and Maintenance Costs, in Cents per Kilowatt-Hour of Output

Federal Water Power Projects

Several large Federal projects for the development of water power have been under way during the past few years. A brief review of these projects in respect to cost appears pertinent in this paper. They include chiefly Boulder Dam, Bonneville, and Grand Coulee in the West, and the projects of the Tennessee Valley Authority in the South.

Boulder Dam (Colorado River).[13]—This is a combined irrigation, power, water supply, and flood-control project, costing about $79 000 000 for the dam and reservoir. The ultimate project contemplates about 1 300 000 kw of

[13] "Government Hydro versus Private Steam Power," H. R. Doc. No. 52, 75th Cong., 2d Session, 1937; see also, "Undertakings Without Precedent," *Engineering-News Record*, November 29, 1934, pp. 675–702.

capacity under a head of about 530 ft, at a cost of about $43 000 000 for the plant. The total reservoir capacity is about 30 500 000 acre-ft, of which 9 500 000 acre-ft, or about 31%, are reserved for flood control. If the remaining 69% of the cost of dam and reservoir (or about $54 000 000), is allocated to power use, this makes the total cost for power purposes about $97 000 000, or $75 per kw of capacity. The yearly output of firm power is estimated at about 4 billion kw-hr. Fixed charges at 6.5% would be $6 300 000 per yr, operation about $700 000, or $7 000 000 total yearly cost, at about 0.17 ct per kw-hr (without transmission).

Bonneville (*Lower Columbia River*).[13, 14]—This is essentially a water power project, but includes provision for irrigation. The ultimate project contemplates 432 000 kw of capacity under a head of about 54 ft, at a total cost of approximately $65 000 000, or $150 per kw of capacity. The fixed charges (at 6.5%) will be about $4 200 000, the operation and the maintenance cost about $300 000, or a total yearly cost of about $4 500 000, which is about 0.22 ct per kw-hr for an output of about 2 billion kw-hr per yr (without transmission).

Grand Coulee (*Upper Columbia River*).[13, 15]—This is a water-power project, although there is a supplemental irrigation project. The complete project is estimated to cost about $196 000 000, with about 1 900 000 kw of capacity, or slightly more than $103 per kw under a head of about 300 ft. The ultimate yearly output will be approximately 9 billion kw-hr; fixed charges (at 6.5%) $12 700 000; operation and maintenance, $1 000 000; or a total yearly sum of nearly $14 000 000, which is about 0.15 ct per kw-hr (without transmission).

Tennessee Valley Authority (*Tennessee River*).[16]—These projects include a comprehensive development of the Tennessee River by relatively high dams for the combined objective of creating power, flood control, and navigation.

Eleven major projects are planned which, in general order, going up stream are: Gilbertsville, Pickwick, Wilson, Wheeler, Guntersville, Chickamauga, Watts Bar, Coulter Shoals, Norris, Fontana, and Fowler Bend. Ultimate power installation includes about 1 800 000 kw of capacity and nearly 6 billion kw-hr yearly of primary power.

Power development alone (not including dams, reservoirs, etc.) will cost about $135 000 000, or $75 per kw of capacity. The total cost of the eleven projects, including power development cost, is estimated at about $520 000 000, or if all is charged to power, about $290 per kw of capacity.

For the ultimate development the fixed charges (at 6.5%), operation and maintenance, and total power costs would be as shown in Table 10. The cost of this water power will thus depend upon the proportion allocated to flood and navigation benefits. If dams and reservoirs are entirely allocated to these two items, the water power will cost about 0.20 ct per kw-hr; if they are entirely allocated to power, the cost will be about 0.60 ct per kw-hr.

The cost of water power from the four Federal projects is summarized in Table 11(*a*). In comparing these costs of power with those from the plants

[14] "A Study of Wholesale Cost of Bonneville Power," Oregon State Planning Board, 1934.

[15] "Grand Coulee High Dam," *Engineering-News Record*, December 23, 1937, pp. 1021–1024.

[16] Annual Rept. T.V.A., for year ending June 30, 1936; also, First Deficiency Appropriation Bill, T.V.A., Committee Rept., 74th Cong., 2d Session, 1936, Testimony of David N. Lilienthal and Arthur E. Morgan, M. Am. Soc. C. E., pp. 113–292; Second Deficiency Appropriation Bill, T.V.A., Committee Rept., 75th Cong., 1st Session, 1937, Testimony of Arthur E. Morgan, pp. 339–451.

TABLE 10.—Cost of Projects by Tennessee Valley Authority

Item	Basis: Power development cost alone ($135 000 000)	Basis: Total cost ($520 000 000)
Fixed charges, in dollars	8 800 000	33 800 000
Operation and maintenance cost (including reservoir operation), in dollars	2 200 000	2 200 000
Total yearly	11 000 000	36 000 000
Cost, in cents, per kilowatt-hour (6 billion kilowatt-hours)	0.183	0.60

in Table 6, it must be kept in mind that fixed charges for the Federal water power projects were taken at 6.5% whereas the others are on a 10% basis. In Columns (6) and (7) of Table 11(a) a comparison of costs with these two bases is given.

TABLE 11.—Cost Summary

Item No.	Plant	Head, in feet	Ultimate plant capacity, in thousands of kilowatts	Unit capacity, in thousands of kilowatts	Annual output of ultimate project, in billions of kilowatt-hours	Cost, in Cents per Kilowatt-Hour, for Fixed Charges of: 6.5%	10%	Allocation; cost of dam and reservoir
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	(a) Federal Water-Power Projects							
1	Boulder Dam	530	1 300	80	4.00	0.17	0.26	31% to flood control
2	Bonneville	54	432	43	2.00	0.22	0.34	To power
3	Grand Coulee	300	1 900	105	9.00	0.15	0.23	To power
4	Tennessee Valley Authority	40 to 400	1 800	23 to 45	6.00	0.18 to 0.60	0.26 to 0.90	To navigation and flood control To power
	Totals	....	5 432	....	21.00	....	....	....
	(b) Large Water-Power Plants Given in Table 6							
5	Conowingo	89	252	36	1.30	....	0.41	To power
6	Safe Harbor	55	178	30	0.88	....	0.33	To power
7	No. 52	454	70	35	0.28	....	0.31	To power
8	No. 53	315	81	27	0.39	....	0.31	To power
9	Waterville	860	108	36	0.33	....	0.44	To power
	Total, or average	....	....	....	3.18	....	0.36	....

The Federal plants are also quite large. It is to be noted that their total output would be about 50% of the entire public utility water-power use in the United States for 1936. Compared on the basis of 10% fixed charges, the cost per kilowatt-hour is only little less for these large Federal projects than for some of the large plants listed in Table 6, such as Conowingo, Safe Harbor, Waterville, and Nos. 52 and 53 in the West (see Table 11(b)). In the

case of the Federal projects other than Boulder Dam the use or marketing of the power must be deferred for an indefinite time, so that an added item of accrued carrying charges should be included. This has not been allowed for in computing Table 11(*a*).

Undeveloped Hydro-Electric Power

The total developed water power in the United States as of January 1, 1937, was about 17 million hp, or approximately 12 million kw.

The total potential water power (including developed power) available 90% of the time was about 43 million hp, or about 30 million kw. The ten leading States in this respect are:

State	Horse-power, potential water power (available 90% of time)
Washington	8 800 000
California	4 600 000
Oregon	4 400 000
New York	4 300 000
Arizona	3 700 000
Idaho	2 700 000
Utah	1 500 000
Montana	1 300 000
Tennessee	1 300 000
Alabama	900 000
Total, ten States	33 500 000

Thus, about three-fourths of the potential water power is found in these ten States (all but three of which are in the West).

It must be kept in mind, however, that the plant capacity would greatly exceed the minimum, or 90%, power, and probably 80 million hp or more of capacity would be required to utilize all the available water power in the United States. Of the total output of public utility plants in the United States in 1936, of about 114 billion kw-hr, about 41 billion kw-hr, or 36%, was hydro-electric power.[17] By general districts this was distributed as follows:

District	Billion kilowatt-hours	Percentage of total
Atlantic Coast States	17.4	43
Central States	7.8	19
Mountain States	3.3	8
Pacific States	12.5	30
Totals	41.0	100

Although many of the better sites for the development of water power have been utilized, there are still many sites that may be commercially utilized as the market for power is extended. Even in the Atlantic Coast District

[17] "Production of Electric Power for 1936," Rept. of Federal Power Comm.

where the present use of water power is greatest, there are still potential opportunities for comprehensive river developments for power, including storage, as well as for redevelopment in some cases.

Acknowledgments

This paper has been made possible through the generous contribution of cost and other supporting data for many water power plants.

Acknowledgments are made to the Federal Power Commission, through Thomas R. Tate, Chief of the Division of Power Resources and Requirements, U. S. Department of the Interior, for access to, and use of, its project files and for its kind assistance. Many engineers and executives of the public utility companies have generously responded to the request for data relating to hydro-electric projects. Mr. Arnold H. Engborg, of the writer's Office Staff, has also assisted in the preparation of this paper.

COMBINED ENERGY GENERATION

By Ezra B. Whitman,[18] M. Am. Soc. C. E.

No attempt will be made in this paper to consider anything but the cost of combined steam and hydro-electric generation although the same principles must be followed in determining the cost of any other combination. The cost of combined energy generation will vary with every combination, and it is not possible to determine such cost for one combination and apply it to another, although the method of determining such cost may be the same in each case. This paper, therefore, will discuss principally the methods that must be followed in determining the cost of combined energy generation and the variable factors that affect such cost.

The Development of Combined Energy Generation

The first plants for the generation of electricity by water power were constructed about 1890 and the concept of the hydro-electric plant in energy generation has undergone many changes since that time. These first plants were usually the sole source of power. This being the case, they were constructed with a capacity of generating equipment for utilizing only the minimum flow of the stream. This was the firm power that could be depended upon throughout the year. They were built in those sections of the United States where fuel costs were high and where water power was available. A number of them were built in California before the discovery of oil and gas in this section.

In order to increase the amount of firm power, storage was resorted to and large storage reservoirs were built in conjunction with the hydro-plants so that the water could be drawn upon at times of minimum flow.

At other plants, where the cost of storage was prohibitive, steam plants were built in conjunction with the hydro-plants. As the larger systems, covering vast sections of the country were developed, the hydro-plants became generating units in these systems in which both steam and water were used as sources of power.

Types of Plants with Combined Energy

At present (1938) there are some plants which are mostly hydro-electric, with steam plants to be used during times of minimum flow. Such plants are the simplest ones for which to determine the cost of the combined energy generation. Many of them began to furnish power entirely from water, but as the power requirements grew, they could not supply the demand at the time of low flow, and steam plants were installed to meet these demands at such times.

[18] Cons. Engr. (Whitman, Requardt & Smith), Baltimore, Md.

The opposite combination is that in which steam plants were first built to furnish the required power. The cost of furnishing peak power by steam is quite high, due to the fact that the boilers must be kept hot, and steam-generating units must be prepared to pick up the peak load during the day as the load increases; and when the load diminishes toward nightfall, the steam units are put out of service, but kept in condition so that they can be placed in operation again as the load increases. If there is a water-power site available that can be developed within a reasonable cost, it is much more economical to carry the peak loads by hydro-electric generation than it is with steam generation, and, therefore, a number of plants have been developed in which water is combined with steam for the purposes of carrying the peak loads.

The third type of combination is found in those plants in which large hydro-electric installations can be made and operated in connection with the steam plants so that the water-power plants, at times of high flow, carry the base load and the steam plants carry the peaks; and, during periods of low flow, the water-power plants are used to carry the peak while the steam generation carries the base load.

Firm Capacity of Water-Power Plants

In the early days of hydro-electric development, the firm capacity of a water-power plant was considered to be only the power that could be generated by the minimum flow of the stream. If the generating capacity of the plant was greater than could be operated at times of minimum flow, this capacity was looked upon as secondary power. Its sale was quite limited, and it could be used only by such industries as could afford to close down during the times of low flow.

With the combination of steam and hydro-electric power that has developed in recent years, the firm capacity of a water-power plant is looked upon in an entirely different sense. It is possible now to use practically all the power that can be developed in a water-power plant in a large system, so that this distinction between firm and secondary power no longer exists. At present, what might be termed the firm capacity of a hydro-plant can be defined as that part of the total installed capacity that is capable of doing the same work on that part of the load curve of the system that a steam plant could perform. This firm capacity varies with the shape of the load curve of the system that is being supplied by both the steam and the water. It does not necessarily come at the time of minimum flow of the stream, which might come in August or September when the load on the system is not the peak. Such a peak usually occurs during December, and in order to determine this firm capacity, the minimum stream flow for that month should be used as a basis for determining the firm capacity of the plant.

Factors to Be Considered in Determining the Cost of Combined Energy Generation

The cost of steam generation and the cost of hydro-electric generation have been covered by two other papers in this Symposium but it will be necessary to review to some extent the principles of determining such cost in

order to show how the cost of combined energy generation must be determined. The first factor to be determined is the construction or capital cost of both the steam plants and the hydro-plants used in the combination.

There is a wide variation in the cost of modern steam plants. In the 1937 report of the Power Authority of the State of New York, the costs of modern steam-generating stations are shown to vary from \$62 per kw to \$151. On the other hand the cost of hydro-plants will vary from about \$125 per kw of installed capacity to \$350, and more. As this first cost of both the steam and the hydro-plants is a large element in determining the cost of the combined energy generation, it can readily be seen that it is impossible to ascertain the cost of combined energy in one system and use that cost as a measure of the cost of the combined energy in another system.

These wide variations of first costs in both steam and hydro-plants may be thoroughly justified by the conditions under which the different plants are built. Steam plants built along the Ohio River, such as those serving Cincinnati, Ohio, and adjacent areas, using the river water for condensing purposes, will be much more expensive than a plant constructed in Baltimore, Md., which uses the tidal waters for condensing purposes. At times of flood the Ohio River rises to a height of 70 ft above the normal river stage, and the foundation expense and the cost of getting the condensing water for such a plant is many times what it would be in Baltimore where the tidal range is ordinarily about 13 in., with a maximum of approximately 5 ft.

With hydro-plants, although the cost of the power-house machinery is comparatively constant, the cost of the dam construction and lands will depend upon the topography of the particular site. Then, there is the question of rebuilding roads that are flooded and, in a number of cases, relocating railroads, telegraph and telephone lines, and the flooding of villages that must be purchased.

It is necessary to determine the construction cost of both the steam plants and the hydro-plants used in the production of combined energy, in order that the fixed charges (which are a large part of the cost of energy generation in either type of plant) may be determined. These fixed charges are: (1) The cost of money; (2) taxes and insurance; and (3) depreciation and obsolescence.

The cost of money with a private company depends very largely upon the financial condition of that company. Large companies that have been soundly administered can secure their money at a much cheaper cost than is possible for the smaller companies and particularly for a new untried project. Although it is true that the large private companies can sell their bonds at a much lower rate of interest at present than was possible before the depression, on the average, the private companies have raised only about one-half their money by the issuance of bonds. An investigation of twenty-seven large utility companies shows that 51.3% of the total capitalization of these companies was the funded debt, and the other 48.7% of the capitalization was raised by the sale of either preferred or common stock. Before the depression, it was not at all unusual for companies on the average to pay between 7% and 8% for the money used for capital purposes. At present (1938), it is doubtful whether

private companies could raise both senior and junior securities at a cost of much less than 7 per cent.

Taxes and insurance vary in different localities, but on the average for steam plants about 2% is usual. Hydro-plants are usually built in the country districts and have lower taxes, which will vary from 0.5% to 1.5%, with an average of about 1 per cent. In some States special taxes are placed on private utilities developing water power and, therefore, the taxes are much higher.

Depreciation and obsolescence are much heavier on steam plants than on hydro-plants. The latter have reached a point at which the machinery is already capable of operating at an efficiency of more than 90 per cent. With steam plants the development of the art has been very rapid since the beginning of the Twentieth Century. The thermal efficiency of steam engines has increased from about 18% to about 36 per cent. Taking into account both physical depreciation and obsolescence of steam plants, it is usual to allow approximately 2.5% to 4% for this item, with an average of about 3.5 per cent. With regard to the hydro-plants, there is little or no depreciation on lands and heavy structures such as dams, the greatest depreciation occurring in connection with the hydraulic and generating equipment. For hydro-plants this item is usually estimated to vary between 0.5% to 1.5%, with an average of approximately 1 per cent.

It is usual, therefore, to compute fixed charges at the present time for steam plants as approximately 12.5%, although these charges have been computed as low as 11% and as high as 15 per cent. With regard to the hydro-plants, at present the average fixed charges would be about 9% although they have been as low as 7% and as high as 11 per cent.

Operating and Maintenance Costs

In order to determine the cost of combined energy generation, not only must the construction cost be secured, but the operating and maintenance costs of both the steam and the hydro-plants must be determined. Again, these costs are widely variable. With steam plants, the largest single item is the cost of coal, which varies from approximately $1.75 at the mouth-of-mine plants to $5.00 or more in regions situated at long distances from the coal fields. The writer will not attempt to describe the many elements of the cost that enter into the operating and maintenance costs of steam plants. One of the controlling elements is the load factor and the shape of the load curve. Load factors on different plants vary from 35% to 70% which causes a wide variation in operating costs even if the two plants are identical, and both of them are able to secure coal at the same price.

Transmission Costs

Another element that enters into the cost of power delivered at the load center is the cost of transmission. As a rule, the steam plants can be built in the load centers, although this is not always the case. With hydro-plants, the rule is the opposite, most of them being built at a distance from the load center; and these transmission costs, where they occur, must be computed

against either the steam or the hydro-plant in determining the cost of combined energy generation at the load center.

Increment Costs of Hydro-Electric Plants

It is not unusual in building hydro-plants to make provision for the future installation of additional units. As the load on the system grows, it becomes possible to utilize more of the flow of the river advantageously than might be possible at the time that the plant was originally constructed. The cost per kilowatt of installing such units is only a fraction of the cost of the original installation, taking into account the cost of lands, dam, and other elements over and above the actual power-house construction, with the water-wheels, water turbines, and generating and transforming equipment.

The Problem of Determining the Cost of Combined Energy Generation

The cost of steam generation is the yardstick for measuring the value of hydro-electric generation. The studies that are made to determine the cost of combined energy generation are practically the same as would be made by a company that was faced with the problem of determining whether it would be better to develop a hydro-project or add additional steam capacity to its generating equipment.

The question that must be answered is whether the proposed plant will reduce the cost of power? If this can be answered in the affirmative, the plant will be built; otherwise, additional steam equipment will be added. In many modern power systems, in which both steam and water power are used as sources of power generation, the two sources are not competitive but are really complementary sources of power supply. It has frequently been found that, with the combined sources of energy generation, the power can be delivered to the customer more cheaply than would be the case had the company depended upon either steam alone or water power alone.

This may be true, where the cost of the water power is considerably more than the cost of that part of the power which is generated by steam. There have been occasions in which it was very difficult to convince public service commissions that it was in the interests of the consumers to use hydro-generated power at a considerably higher cost per kilowatt-hour than the steam power being generated in the same system. Of course, the economic use of water power, even if it costs more than the steam, is justified by the fact that it will replace that part of the power in the load curve which comes on the peaks and which it would be more costly to generate by steam. Many large steam systems have modern base-load plants that produce electricity at a low cost. The older plants, which are more expensive to operate and cost more for the production of a kilowatt-hour of power, are used only for peak and emergency purposes. The actual power produced in such old steam plants is small and may be only a small percentage (or none) of the total annual production; yet their capacity in the system is required for peak and emergency service and if the company did not have such plants, it would be necessary to build them, or new plants of equal capacity. Under these circumstances, it is much more

economical to hold these old plants ready for service than to build complete new equipment for this use.

In some of the large systems, such as the Philadelphia Electric Company in 1925, there were eight steam plants and in 1923 the smallest and least economical of the plants was called upon to produce only 39 000 kw-hr out of a total of nearly 2 000 000 000 kw-hr. In a conversation with the president of one of the large Eastern power companies, for which there had been installed in about 1918 or 1919 several 20 000-kw steam-generating units, the writer was told that the company had not operated these units for several years and hoped that it never would. Nevertheless the sources of supply were such that had there been an interruption to the one of the larger sources, it would have been necessary to have started these units, which will be held in the system as a reserve for many years in the future.

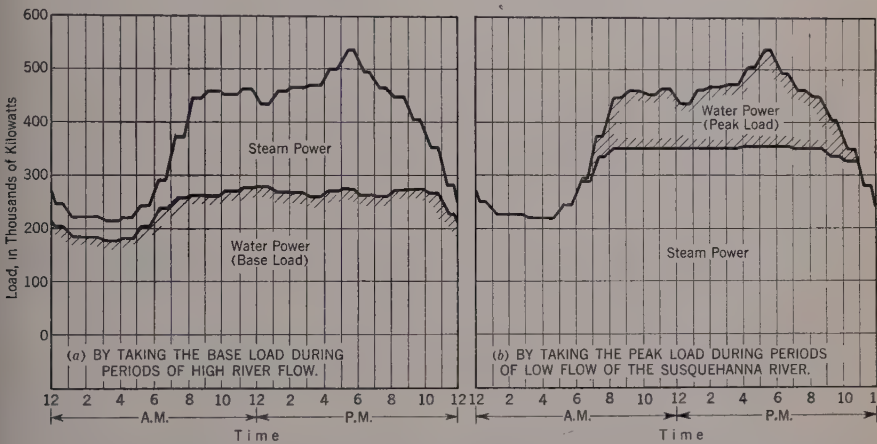

Fig. 29.—Combined 24-Hour Load Curve of Baltimore, Md., and Washington, D. C., Showing How Systems Are Served by Water Power from Holtwood and Safe Harbor Plants of Pennsylvania Water and Power Company.

Fig. 29 illustrates how the modern water-power and steam-power plants are used in a complementary manner. Fig. 29(*a*) shows how the water power would be used in a period of high flow of the river to carry the base load and Fig. 29(*b*) shows how the water power is used at a period of low flow to carry the peak loads. This "hydro" is supplied from the Holtwood and Safe Harbor Plants, owned by the Pennsylvania Water and Power Company, and the Safe Harbor Water Power Corporation, respectively. The Holtwood plant was built before 1913 and the Safe Harbor plant about 1932. When the second plant was completed, Baltimore was not inter-connected with Washington, D. C. Within the past few years, however, this inter-connection has been made so that these two plants serve both Baltimore and Washington. With this increased load, the firm capacity of the combined hydro-electric plants has been considerably increased and thereby its value to the two systems is greater than it was when Baltimore alone was served.

In order to demonstrate the methods of arriving at the cost of combined energy generation, the writer presents very briefly, herewith, the method used

in determining the value of the Conowingo Plant to the Philadelphia (Pa.) Electric System at the time when this project was being considered by the Maryland and the Pennsylvania Public Service Commissions.

The Economic Value of the Conowingo Water Power in the Philadelphia Electric System

In 1925, application was made to the Maryland Public Service Commission and the Pennsylvania Public Service Commission for approval of the building of the Conowingo Power Plant on the Susquehanna River. The plant proposed contemplated the installation of six 50 000-hp turbines and six 40 000 000-kw generating units. This site had first been considered as a source of power more than a generation ago with Baltimore as the ultimate market for such power. Although Baltimore could have absorbed the power generated by the low flows of the river, it was not possible for Baltimore to absorb the power that could be generated economically at this site during the four months of the year when the stream flow would make it possible to operate the contemplated plant at full capacity. Philadelphia, however, with a much larger market for the power, offered the opportunity for the economic development of this site. Even with that city, it was estimated that it would not be until 1930 that the full economic value of this plant could be realized with the installation of six 40 000-kw turbines.

The study included the possibility of installing four more 40 000-kw units. The stream flow would provide for full operation of these ten units for approximately 20% of the time.

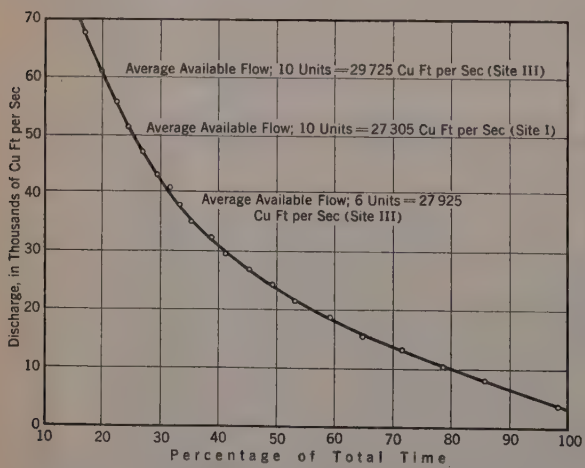

Fig. 30.—Conowingo Development, Duration of Flow Curve, Susquehanna River at Site III, 1891–1922

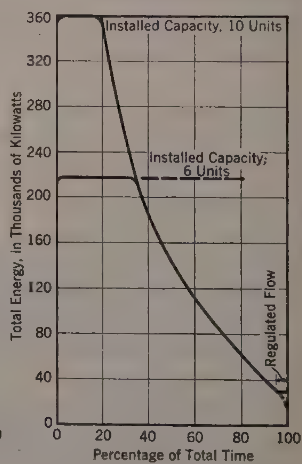

Fig. 31.—Power Duration Curve, Proposed Hydro-Electric Development, Susquehanna River, Near Conowingo, Md.

Stream-flow records were available from 1891 to 1923 and the stream duration curve shown in Fig. 30 was prepared from the flow records. A power duration curve from the data computed on an 85% over-all efficiency of

the water-power plant to and including the high tension bus-bar, was also prepared (see Fig. 31).

The total rated capacity of existing steam plants in Philadelphia at the time of the hearing was 476 480 kw and the load factor on the Philadelphia Electric System in 1923 was 43.8 per cent. In 1924, it was 44.8 per cent. It was estimated that by 1930, the peak, maximum, co-incident load for the steam and water power plants would be 600 000 kw and that the steam plants could carry 405 000 kw and the water-power plants would carry 195 000 kw of this peak.

In 1930, it was also estimated that the average or typical flow-year would generate, at the hydro-electric plant, 1 278 000 000 kw-hr and that 90% of this generated power could be delivered to the system in Philadelphia, or 1 150 000 000 kw-hr. For the same year it was estimated that the steam plants would generate 1 080 000 000 kw-hr. In other words, the hydro-electric plants would produce 51.5% of the total power required and the steam plants would produce 48.5 per cent.

At that time eight steam plants were being used in Philadelphia in the generation of electricity and the maximum demand on each of these stations and the kilowatt-hours generated were shown for 1923 and 1924. Typical, daily, load curves were chosen to show the approximate variations during the year, arriving at a typical curve for each month. Stream-flow tabulations were made on a monthly basis of the average flow within the plant capacities for six and ten units, and the power available from these flows was fitted to the load curves so as to obtain the most economical adjustments between the hydro-electric and the steam plants.

Computations were then made assuming that new steam-generating equipment was built instead of the Conowingo water power. These costs were divided as follows:

Fuel	$ 9 722 000
Labor	1 394 100
Supplies	139 880
Maintenance	1 410 520
Total	$12 666 500

This cost, of course, does not include any fixed charges and amounted to 5.68 mills per kw-hr. It was estimated also that if Conowingo were not built, it would be necessary to build 250 000-kw capacity of steam-generating plant. With Conowingo constructed, it was estimated that it would only be necessary to build 100 000 kw of generating capacity. In other words, it was estimated that Conowingo would take the place of 150 000 kw of steam generation, which would cost $135 per kw to construct, or $20 250 000. Fixed charges saved on a steam plant were estimated to amount to 11.5% of this amount, which is $2 330 000. The annual charges on fuel, supplies, and equipment which were saved were estimated at $82 000.

These 150 000 kw of steam capacity, which the water-power plant could replace, were determined from Fig. 32, showing the estimated average week-day

load curve for December and that part of the load which could be served by hydro-electric plants with the minimum flow of the river. This diagram also shows that with a minimum flow, the hydro-plant could be utilized to take care of 153 000 kw of the demand on the system. It was assumed, therefore, that the firm capacity of the Conowingo Plant in the Philadelphia Electric Company System would be 150 000 kw. The estimated average week-day load curve for the wet month of March, 1927, is shown in Fig. 32(*b*). As a result of these investigations, the following tabulation was prepared to show the value of the hydro-plant in the Philadelphia Electric Company's System, compared to what the cost would have been had that Company constructed new steam generation instead of the Conowingo Hydro-Plant:

Cost of producing all power requirements from steam (1930)		$12 666 500
Cost of steam power after hydro-power comes in (1930)		7 163 000
Operating saving from hydro-electric energy		$ 5 503 500
Fixed charges on $20 000 000 representing 150 000 kw saved in steam-plant investment		2 330 000
Charges on saving in coal and equipment supplies from use of hydro-electricity		82 000
Total representing what the Company could afford to pay for the hydro-plant		$ 7 915 500
Cost of Hydro-Power:		
Rentals	$4 580 000	
Operating expenses, taxes, and depreciation on hydro-plant and lines	1 430 000	
Operating expenses, taxes, and depreciation on investment in Company territory in relation to hydro-electric generation	1 126 000	$ 7 136 000
Remainder representing the saving from use of hydro-plant		$ 779 500

As the hydro-plant was estimated to furnish 1 150 000 000 kw-hr per yr, the saving on this part of the current generated at the Conowingo Hydro-Plant would amount to 0.7 mill per kw-hr or, on the total power generated of 2 230 000 000, the saving would be approximately 0.36 mill. The total cost of the water-power current delivered in Philadelphia was estimated at 6.2 mills per kw-hr. The total cost of the steam generation of all the current was

estimated to cost 5.68 mills per kw-hr; but by utilizing the hydro-power for peak service during the dry flow and base-load service during the wet months, the cost of the combined energy generation was reduced nearly $800 000 per yr, although the hydro-power by itself cost 6.2 mills as compared with the 5.68

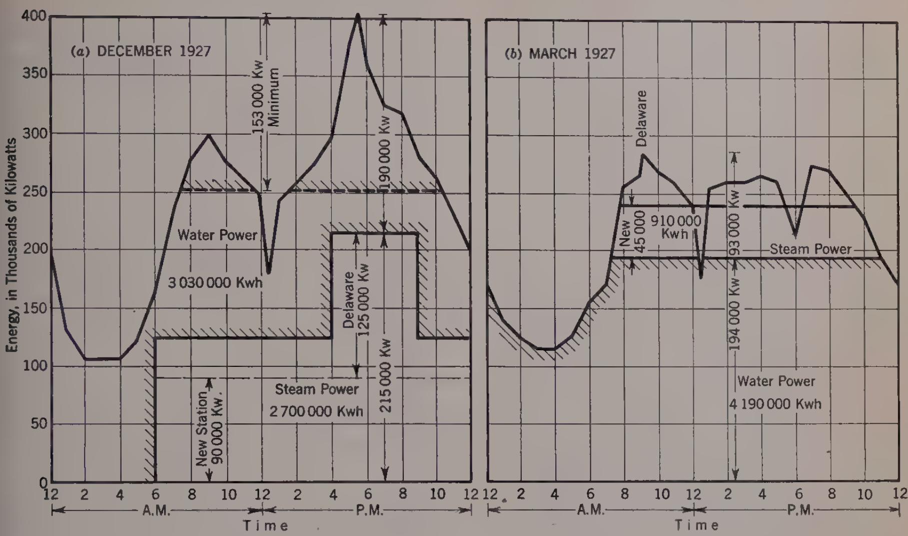

Fig. 32.—Estimated Average Week-Day Load

mills if all the power had been generated by steam alone. This is a typical illustration of the economic advantage of combining steam and hydro-electric generation.

Federal Hydro-Projects

Federal water-power projects were started in the area covered by the Tennessee Valley Authority (TVA) with a view to providing "yardsticks" to determine whether the rates charged by private companies were reasonable. In the TVA case before the Alabama Public Service Commission, the only plant under consideration was the plant at Wilson Dam, Muscle Shoals, Ala. A steam plant costing about 12.25 million dollars was built during the World War and the hydro-plant was started; but until 1920 only approximately 3 million dollars had been spent. The project was completed in 1926 at a total cost of about 47 million dollars so that the hydro and steam plant combined cost about 60 million dollars.

It is true that the steam plant was built under war conditions, but the hydro-plant was not more than well started before 1920, and its construction was completed during the same period that a great number of private plants were built. It was testified that the Government would pay only 3% for its money, therefore, the rate of return required would be only this rate; whereas, private companies had been compelled to pay about 7% for their money. Although many of the private companies had spent considerable sums of money to provide locks and canals for navigation purposes, they had always been forced to absorb

this expenditure in their capital investment for developing electricity. At Wilson Dam, one-third the cost of the entire project was charged to navigation, another one-third was charged to national defense, leaving less than 20 million dollars as the rate base on which the Government would pay 3 per cent.

With regard to the national defense, it is unquestionably true that the private power plants throughout the country engaged in furnishing power for a great number of defense projects have a value for the purposes of national defense in the same sense as the plant at Muscle Shoals; yet no credit was given to the private plants for national defense. In a number of Federal projects, there will also be allocations of the cost of the project, not only to navigation and national defense, but to flood protection, soil erosion, irrigation, and possibly for other uses.

In the aforementioned TVA case, it was also testified that the Government would pay the taxes just as a private company did, but the taxes that the Government proposed to pay were only 5% of the annual income from the plant, whereas it was shown that the private companies were paying taxes nearly three times this 5 per cent.

In practically all the operations in connection with the production and distribution of the electricity, the Government had estimated the governmental costs much below what these corresponding costs were for private companies. No private company would ever have built the great hydro-plants that have been built or are being built by the Government in the valley of the Tennessee River without first making studies as to how this power could be absorbed by the available market, in the same way that the studies were made in connection with the Conowingo Power Plant. If any such studies have ever been made, the writer has certainly never seen nor heard of them.

As the fixed charges on the hydro-plant are by far the greater part of all the charges against the cost of hydro-generation of electricity, the fact that the Government will begin with only 3% as the cost of the money, as against twice this rate for private companies, will give the Government water power tremendous advantage over any project that could be built by private capital. The Government is also in a position such that it can wait for the development of a market to absorb power which it generates and any deficits can be made up by taxation. Private companies are not in a fortunate enough position to be able to spend tremendous sums of money on projects which it may take years to develop to the point where they will pay a sufficient return on the capital invested to attract private capital.

The writer is convinced that it is useless to bicker about the comparative costs of hydro-power and steam power and of privately operated plants and Government-owned and operated plants if the greatest good of all the people of the country is to be served. After all, the cost of generating power is the smallest part of the cost of delivering the power to the customers and particularly the domestic customers. At the most, what is being argued about is a few mills per kilowatt-hour in the cost of the current as finally delivered to the customer. If the Government and the private companies would only sit down together and develop a plan for the most economical utilization of their combined power plants, it would be possible to serve the customers who may

be within the territory that can be served by the Government hydro-plants to the advantage, not only of the customers, but also in such a manner that the investments by the private companies will not be greatly impaired or destroyed. Should the Government go into the TVA area and take from the private companies such cities as Knoxville, Tenn., Chattanooga, Tenn., Nashville, Tenn., Memphis, Tenn., and Birmingham, Ala., this would render useless much of the property of the private plants now serving these communities and would undoubtedly destroy the value of the investments by hundreds of thousands of individuals who own securities in these private companies.

The writer hopes sincerely, that some method may be found whereby the people will get the benefit of the Government plants and that the property of the private companies may be saved for their owners.

DEPRECIATION AND OBSOLESCENCE

By Maurice R. Scharff,[19] M. Am. Soc. C. E.

Synopsis

Depreciation should be used as a generic term for the effects of physical deterioration, obsoleteness, inadequacy and change in use or in demand, and requirement of public authority. The writer treats the engineering aspects of each of these causes of depreciation, and suggests that engineering study offers the greatest hope of progress toward their better understanding.

The determination of annual depreciation expense is then discussed. It is pointed out that such determinations heretofore have customarily been based on one or the other of two erroneous assumptions—either that all the causes of depreciation became effective in proportion to the passage of time, in accordance with a straight line or sinking-fund formula; or, that none of these causes became effective with the passage of time, or prior to the retirement of the property. The writer suggests that a rational engineering approach would recognize that some causes of depreciation do progress with the passage of time, and that others do not.

A procedure for the application of this concept to the determination of annual depreciation expense is summarized briefly, and its relation to the cost of power in competitive industry, public utilities, and governmental projects is discussed.

An opportunity is afforded the Engineering Profession to clarify the subject of depreciation by discussion of its engineering aspects. The writer proposes that civil engineers invite the co-operation of engineers in all branches of the profession in carrying out such a program.

Introduction

About a quarter of a century ago a Special Committee of the Society undertook a study of the problem of valuation of public utilities, the final report[20] on which has achieved recognition as an important contribution to the literature of this controversial subject. Not that this report ended the controversy. On the contrary, it has raged unabated to the present time, and has lost none of its vigor—as has been demonstrated by the renewal late in 1937 of the old debate over the relative merits of "prudent investment" and "fair value" as a basis for public utility rate-making. One chapter of the report was devoted to the subject of depreciation; and although twenty years have elapsed since its publication, one can still agree with the opening sentence of

[19] Cons. Engr., New York, N. Y.

[20] *Transactions*, Am. Soc. C. E., Vol. LXXXI (1917), pp. 1311–1581.

that chapter, in which the Committee states:[21] "Perhaps there is no single subject in connection with valuation that has caused more trouble than depreciation."

On re-reading this report, in preparation for this paper of the cost of depreciation and obsolescence in energy generation, the writer has had the impression that if depreciation had been more clearly recognized at that time as the result of the actual incidence of specific causes, the Committee would have devoted more attention to consideration of the engineering facts regarding these causes and their effects, and would have placed less emphasis upon the relative merits of mathematical formulas for relating depreciation solely to the passage of time. Much of the discussion of the subject in the past, particularly that by accountants and economists, has proceeded from the assumption that depreciation was a simple function of cost, time, and the interest rate. More attention has often been paid to the shape of the curves and the forms of the equations for these functions than to the facts which they purported to explain.

It is the writer's proposal that depreciation be used as a generic term for the effects of several categories of causes. He proposes to discuss this suggestion in somewhat general terms, with reference to cost of energy generation only for purposes of illustration, conceiving this to be the most helpful contribution he can make to this, the Second, Symposium on Power Costs.

The categories of causes of depreciation usually listed in discussions of the subject include deterioration, wear and tear, obsoleteness, inadequacy, change in demand, change in use, and requirement of public authority. The mere enumeration of these causes, however, suggests that their effects cannot be expressed as any simple function of the passage of time, because it is common knowledge that an older unit of property or equipment may be, and often is, less deteriorated, more efficient, safer or more reliable, and less affected by change in demand, change in use, or the requirement of public authority than a similar unit of property or equipment of lesser age.

It is true that the incidence of these causes are events that occur in the universal framework of time, and that the effects of some of them are continuous, and sufficiently regular to permit formulation as simple functions of time. In most cases, however, the incidence of these causes is discontinuous, irregular, and even fortuitous with respect to time; and the engineer recognizes that their effects can be better expressed in terms of functions of physical and chemical change, abrasion, rupture, change of form or dimensions, loadings, stresses and moments, capacities, demands and uses, efficiencies, lost time accidents, service interruptions, competition of substitute services, etc., than in terms of the complicated, high-order functions that would be required to express their relation to the passage of time.

It is most fitting, therefore, that the Engineering Profession should devote attention to the development of a better understanding of the causes of depreciation and the measurement of their effects. If progress can be made in this direction, it may yet come about that accounting for costs and profits in industry and finance, and regulatory practices relating to public utilities will be brought into harmony with the facts relating to depreciation. It has too

[21] *Transactions*, Am. Soc. C. E., Vol. LXXXI (1917), Chapter VI, p. 1448.

often been assumed that the meaning of depreciation is dependent upon accounting and regulatory policies and practices.

Part of the confusion of the past has been due to the fact that in accounting and finance, depreciation has been looked upon as synonymous with amortization of investment. It is a sound financial principle that the value of a capital investment is equal to the present worth of the amortization of the investment plus the return upon the unamortized portion of the investment. This principle can be complied with, however, by the consistent application of any method of amortization whatever, regardless of its relation or lack of relation to depreciation.

Confusion has also been caused by the controversies and contradictions that have grown up around the meaning of "fair value" in regulation of public utilities, and the legal measure of depreciation in determining "fair value." The Courts have performed a valuable service in emphasizing the reasonableness of the correspondence of such legal measure with the facts:[22]

"The testimony of competent valuation engineers who examined the property and made estimates of its condition is to be preferred to mere calcuations based on averages and assumed probabilities."

Further evidence is given in the following:[23]

"In the light of the evidence as to the expenditures for current maintenance and the proved condition of the property—in the face of the disparity between the actual extent of depreciation, as ascertained according to the comprehensive standards used by the company's witnesses, and the amount of the depreciation reserve—it cannot be said that the reserve merely represents the consumption of capital in the service rendered."

It is proposed now to discuss briefly each of the principal recognized causes of depreciation that have been enumerated as follows: (1) Physical deterioration; (2) obsoleteness; (3) inadequacy and change in use or in demand; and (4) requirement of public authority.

Physical Deterioration

Where property is used up and disappears progressively with use as, for example, the lead in a pencil or the material in a grindstone, such use is analogous to the consumption of such supplies as fuel taken from a coal pile. Such consumption, of course, can be measured in simple units of length, volume, or weight.

When the use of property can be isolated for study, it may be possible to measure the progress of deterioration as a function of such use. Wood poles, for example, have a single use—that of sustaining over-head conductors by reaction to vertical compression, on the one hand, and of supporting unbalanced span loads and lateral wind loads through resistance to bending, on the other. It is suggested that deterioration through fiber rot in such a case may reasonably be measured in terms of decline in compression and bending strength at standard fiber stresses.

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[22] McArdle *v.* Indianapolis Water Co., 272 U. S. 400 (1926).
[23] Lindheimer *v.* Illinois Bell Telephone Co., 292 U. S. 151 (1934).

The foregoing examples are cases in which simple objective measurements of the effects of deterioration appear reasonably applicable. In many types of cases, however, the relation of deterioration to objective measurements is less obvious. Thus, when a steam boiler is new it can operate under a definite pressure limitation as prescribed by the A.S.M.E. standards and boiler insurance codes. Furthermore, the tubes, shells, and headers have definite thicknesses of sound metal designed to support the anticipated stresses with reasonable factors of safety. As deterioration occurs, the thickness of sound metal is reduced. De-rating of boilers by insurance inspectors supplies an additional objective measure of the extent of deterioration.

Similarly, changes in the dielectric strength of insulation and in other physical and chemical characteristics occur in the windings of generator coils as a result of use and of changes in temperature and dielectric stress. Extensive measurements of some of these characteristics have been made by manufacturers and by engineering committees in an effort to describe them.

In these and in similar cases, however, it would be impossible, in the present state of knowledge, to secure agreement upon any standard series of objective measurements that would be generally accepted as measuring the extent of deterioration. Engineering estimates of conditions reflecting the extent of deterioration in such cases, therefore, are customarily stated as representing the judgment of the engineer as to the point where the deteriorated condition of the property in question would fall on a scale between 100%, representing condition new, and 0%, representing a condition requiring removal, if all degrees of deterioration were arranged in order of magnitude.

Such a judgment, of course, is in a measure subjective. It has been the experience of the writer, however, that such judgments of a number of competent observers of similar training and experience with respect to an identical condition tend to be reasonably consistent and to group themselves around a mean without excessive deviations.

It may be that in judging condition or extent of deterioration, engineers are in somewhat the same situation as men were when they attempted to convey to one another their relative sensations on touching similar objects at different temperature levels before it was known that temperature was proportional to molecular velocity or that a convenient scale for the measurement of temperature could be established by measuring the length of a column of mercury in a glass tube.

The time may come when the processes of deterioration will be sufficiently understood so that several objective measurements may be recognized as completely describing all the types of physical deterioration that have occurred in an item of property and may be synthesized into a determination of condition that is related as closely to objective observations as the unused length of the lead in the pencil, the remaining stone in a grindstone, the weight of fuel in a coal pile, or the remaining fiber strength of a wood pole.

The writer believes that progress in this direction depends upon engineering study, and proposes that civil engineers invite the co-operation of engineers in other branches of the profession specializing in the design, construction, and operation of power plants in undertaking to devise more complete objective

standards for the measurement of the effects of deterioration of power plant structures and equipment, and of suggesting improved methods of arriving at sound engineering judgments and valid statistical inferences in the light of such objective standards as are already available.

Obsoleteness

Attention is called to the use of the word, "obsoleteness," instead of "obsolescence" as included in the title of this paper, in deference to the use of the latter term in the designation of the Sub-Committee of the Power Division, of which the writer is Chairman. The use of "obsolescence," derived from the inceptive Latin verb ending in "-sco," is misleading, since such verbs describe verbal actions commencing and progressing continuously with the passage of time. "Obsoleteness," derived from a past participle describing a completed verbal action, seems more suitable for describing a condition existing at a particular time, especially when it is intended to refer to a condition resulting from a cause which is discontinuous in time and which becomes effective upon occurrence of specific events, such as the development of new inventions or other steps in the progress of the arts.

It is suggested that the decline in the economic value of existing equipment, resulting from the development of improved equipment, which can be operated at a saving, may reasonably be used as a measure of obsoleteness. Economic value is defined as the sum of the present worths of the expected returns, where such returns are equal to the total revenues less all operating expenses (exclusive of provision for depreciation or amortization).

Progress in the arts tends to affect competitive enterprises by attracting into the field lower cost producers, with resultant lowering of prices, elimination of marginal producers, and decline of the profits and economic values of remaining enterprises using obsolete facilities.

With a relatively inelastic demand, such decline of economic value may be stated as the total of the cost of the old equipment plus the sum of the present worths of its operating expenses (exclusive of depreciation or amortization) over a reasonable period of time, minus the corresponding total for the most modern available improved equipment. The ratio of decline is this difference divided by the cost of the old equipment.

Algebraically, the percentage effect of such obsoleteness may be stated as follows:

$$D_o = 100 \frac{C - [C_e - \sum_{o}^{n} W_p \, (O - O_e)]}{C} \quad \ldots\ldots\ldots\ldots\ldots (3)$$

in which D_o = percentage depreciation due to obsoleteness; C = cost of existing equipment; O = annual operating expenses associated with existing equipment (exclusive of depreciation); C_e = cost of the most economical modern substitute equipment; O_e = annual operating expenses associated with the most economical modern substitute equipment (exclusive of depreciation); and, $\sum_{o}^{n} W_p$ = sum of the present worths of an annual sum over a reasonable period of n years.

In some instances, improved equipment results in greater safety, greater reliability, or improved quality of product, rather than in realizable savings in operating and maintenance expense, and in such cases the measurement of obsoleteness appears less simple and has the subjective character of engineering judgments to which reference has been made in connection with estimates of some forms of physical deterioration, and the judgment is expressed as a percentage on a scale between 100% and zero. In this case, also, there would seem to be opportunity for the development of objective measurement by engineering study, and for progress toward an objective basis for judgment.

Inadequacy and Change in Use or in Demand

As in the case of obsoleteness, these causes of depreciation become effective as a result of specific events, such as addition of new load, loss of load, change in market, or change in method of operation. Inadequacy is rarely encountered in well managed profitable enterprises, for the reason that in such cases facilities are promptly increased to take advantage of opportunities for profitable employment of capital. Excess capacity due to decline in market or change in methods of operation, however, is not uncommon and there is opportunity for clarification of its meaning and development of improved methods of measuring its effects. The analogy of decline in price, to the extent of savings in cost of production that could be accomplished by a producer, with no more than adequate equipment, appears reasonable as a basis of measurement and on this basis the percentage depreciation due to change in demand or in use is, as before:

$$D_u = 100 \frac{C - [C_a - \sum_{o}^{n} W_p (O - O_a)]}{C} \quad \ldots\ldots\ldots\ldots\ldots (4)$$

in which D_u = percentage depreciation due to change in use; and C_a and O_a are the cost and the annual cost of operation of equipment of no more than adequate capacity.

Requirement of Public Authority

The effects of this cause of depreciation are rarely found in property in service, because compliance with such requirements is ordinarily compulsory, and retirement and replacement take place at the effective date of such requirement. Where property is maintained in contravention of legal requirement by competent public authority, either the property right in such property has in effect been destroyed, and depreciation due to this cause should be considered complete; or a liability is imposed upon the owner of the property, the amount of which may be taken as a measure of depreciation due to this cause.

Annual Depreciation Expense

If the existing depreciation in the property is the sum of the effects of the several causes of depreciation that have been discussed herein, the annual depreciation expense should be the sum needed to make the reserve for depreciation representative of existing depreciation at the beginning and at the

end of each year. If it were practicable to make a depreciation study as of January 1 of each year and determine the existing depreciation, it would be possible to compute the reserve requirements at such dates. Then, the annual depreciation expense would be the difference between the required reserves at the beginning and at the end of the year plus the debits to the reserve during the year. The debits to the reserve during the year are the net retirement losses, which are equal to the difference between the sum of the cost of the property retired plus the costs of dismantling and the salvage recovered.

Of course, a complete new depreciation study as of January 1 of each year cannot be made on account of the magnitude of the task and the time required to make the determination of existing depreciation. However, it is possible to approximate, within narrow limits, the probable change in the amount of existing depreciation during a year, and thus to approximate the amount of annual depreciation expense.

The difficulty of determining the annual depreciation expense arises from the discontinuous relation of some of the causes of depreciation to time. The result of this difficulty has been that depreciation due to all causes has generally been treated either as if all these causes became effective directly and solely in proportion to the passage of time, and as if depreciation were identical with amortization of investment by a straight line or a compound interest formula; or as if none of these causes was effective in proportion to the passage of time, and as if depreciation losses occurred only at the time of retirement of property.

It is suggested that a rational engineering approach to the problem would be to recognize that the effects of some causes of depreciation on some property do proceed in a simple functional relation to time and that others do not; and to attempt to formulate a plan for computing annual depreciation which will more nearly correspond with facts than either of the erroneous treatments to which reference has been made.

The writer does not minimize the difficulties of such an approach. He believes, however, that engineering study offers promise of progress in the direction indicated which is well worth the effort of attempting to overcome the difficulties that lie in the way. With this in mind he proposes to describe briefly a procedure which he has developed and used in studies made under his direction.

Of all the causes of depreciation, physical deterioration comes the nearest, and in a greater proportion of cases, to progressing approximately in proportion to the passage of time, whereas obsoleteness, inadequacy, change in use or in demand, and requirement of public authority occur irregularly, and often at the time of the retirement of a property. Admitting freely the inaccuracy of the assumption and pending the development of more complete knowledge of the relation between incidence of the causes of depreciation with respect to the particular property and the passage of time, it is suggested that a helpful guide to the determination of annual depreciation expense in many cases will be found in a computation based upon the assumption that deterioration does progress directly in proportion to the passage of time, and that the other causes of depreciation become effective at the time of retirement and can be provided for as retirement losses.

To make such a computation it must be recognized that the physical condition determined by an engineer, in the manner previously described, is the result of two offsetting influences: (1) Deterioration, which acts to lower the condition of the property; and (2) the replacement of parts through maintenance, which tends to restore the condition of the property.

The physical condition of the property as found is the net result of the effect of these opposing influences during the history of the property. Every dollar spent on maintenance, however, does not improve the condition of the property. Some maintenance expenditures tend only to prevent deterioration and others affect only the location or arrangement of property. The cost of replacing parts on maintenance, also, because of its piecemeal character, may be substantially greater than the cost of the parts replaced as portions of the cost of property used in operations.

If it is assumed: (1) That a constant proportion of maintenance expenditures has been effective in restoring deterioration; (2) that maintenance expenditures have been spread evenly over the property whose condition has been determined; and (3) that parts replaced on maintenance tend to deteriorate at the same average rate as all the property whose condition has been determined, then it is suggested that the relation of these several factors may be expressed by the equation:

$$K_n = 100 \left[1.00 - X\, Y_n + \frac{b_n}{B_n} (I)\, (1.00 - X\, y_n) \right] \ldots\ldots\ldots (5)$$

in which K_n = percentage condition resulting from physical deterioration; X = average annual rate of deterioration in the past; Y_n = weighted average age in years of dollars representing the property; $\frac{b_n}{B_n}$ = ratio of cumulated maintenance expenditures (b_n) to fixed capital balance, representing the property (B_n); I = ratio of maintenance expenditures effective in restoration of condition; y_n = weighted average age in years of cumulated maintenance expenditures; and n = subscript indicating year, January 1 of which is the basis of the study.

Equation (5) may be solved for X, which is the average rate of deterioration independent of the offsetting effects of maintenance. The reciprocal of X may be considered as an average indicated useful physical life, which the property would live if the constant rate of deterioration continued from 100% to 0% condition, and if it were retired solely because of physical deterioration.

With the same assumptions as before, and with amounts budgeted for additions, retirements, and maintenance expenditures for an ensuing year following the date for which existing depreciation has been determined, the annual depreciation expense, D_a, may be stated as follows:

$$D_a = [B_{n+1}\, (1.00 - K_{n+1}\, M_n) - B_n\, (1.00 - K_n\, M_n)] + R_n \ldots\ldots (6)$$

in which M_n = the net percentage effect of all causes of depreciation other than deterioration; R_n = the estimated net retirement loss in the ensuing year; and K_{n+1} is expressed by Equation (5) for $n = n + 1$ (the subscript,

$n + 1$, indicates merely that the figures to which they are applicable are those of a year later than n).

The writer has found this formulation useful (although it is inchoate), in estimating the costs required to be budgeted for depreciation expense during a year following a determination of existing depreciation. Where depreciation reserves have been established in accounts that are the same proportion of the book cost of depreciable property as the percentage depreciation at the beginning of the year determined by engineering study in the manner described, this procedure will tend to preserve such correspondence between reserves and actual depreciation at the end of the ensuing year.

The procedure, of course, is capable of indefinite refinement, as information and a better understanding of the facts of the particular case become available. If there is ground for anticipating that deterioration of some part of the property will not proceed in proportion to the progress of time, correction may be made on this account. If the causes of obsoleteness, inadequacy, or change in use or in demand are known to be in operation in a manner that will result in additional depreciation during the ensuing year, due to these causes, without justifying retirement, a proportionate allowance may be made for these factors. It may be desirable in any case to increase the estimated requirement for depreciation expense by a percentage for contingencies to allow for the obvious limitations on the accuracy of the procedure. If such a procedure should be followed for a number of years it would probably be desirable, possibly at intervals of five to ten years, to check the correspondence between the resultant reserves and the actual depreciation of the property by a re-examination of its condition and by new engineering studies of obsoleteness and of the effects of change in demand or in use.

It does not seem worth while, at this time, to labor the point, or to attempt to develop, in any more detail, the solution of the practical problems that will be encountered in the application of this procedure. The purpose is to emphasize the opportunity for a contribution by the Engineering Profession to a rational solution of this problem which it is believed will have application to competitive industry, to public utilities, and to the determination of the cost of power developed by public projects, to which it is now proposed to make brief reference.

Competitive Industry

In competitive industry the prices of products tend to be determined by equating utility to the marginal consumer to cost to the marginal producer. Profit is the difference between price, as so determined in the market, and cost of production. Neither price nor profit has any necessary relation to the investment in fixed facilities of the particular enterprise under consideration; and the value of the enterprise is the present worth of its prospective earnings including profit and recoupment of investment at such time and in such manner as the profitable operation of the enterprise in the competitive market may make possible.

The policy of management, however, as to amortization of investment may be based on considerations other than the occurrence of actual depreciation and retirement losses. Thus, if a uniform product is manufactured and sold,

it may be desirable that the entire cost of amortization of investment—or possibly that all costs—shall be spread over the total number of units of the product, regardless of the time when, and the rates at which, such costs or losses are actually incurred. On the other hand, a more limited equalization of the effect of depreciation on cost of production in accordance with some other plan may be desirable because of some other special consideration. It has seemed to the writer that it is for this reason that depreciation and amortization of investment have been so generally considered as identical in accounting practice.

It is suggested, however, that knowledge of the actual time and rate of occurrence of losses due to depreciation, based upon competent engineering study, will be helpful to management even when amortization policies are not controlled by depreciation. The tendency to extend governmental supervision over financial and accounting practices in connection with the issue of securities, moreover (as illustrated by developments under the Security and Exchange Acts), suggests the possibility that even in competitive industry a closer correspondence between such practices and the actual facts of depreciation may be necessary or desirable at some time in the future.

Public Utilities

In the public utility industry the situation seems to be quite different. The principle has been established that public utility rates are to be based upon allowance of gross revenues from sales of service equal to operating expense, plus annual depreciation, plus a reasonable return on the fair value of the property used and useful in the public service. Fair value, as prescribed by the Courts, must be based upon consideration of reproduction cost less depreciation, prudent investment less depreciation, and other pertinent measures of value. The tendency of the Courts, in prescribing these general requirements, has been to insist that depreciation deducted in determining fair value shall be "actual depreciation," and that there must be a reasonable correspondence between such "actual depreciation" and the reserves resulting from the annual depreciation included in operating expense.

The freedom of competitive industry to amortize investment in any manner suggested by the special conditions of the industry is limited by the definite requirement that depreciation charges and reserves shall correspond as closely as possible with the facts as to depreciation losses and the condition of the property from time to time, taking into account all the effective causes of depreciation.

The tendency toward clarification of this requirement has received a marked impetus in the two years, 1936–1937, through adoption by various Federal and State regulatory commissions of new classifications of accounts requiring (to quote the language of the classification of the Federal Power Commission, for example) that "each utility shall record as at the end of each month the estimated amount of depreciation accrued during that month on depreciable electric plant."

The method of approach herein outlined complies with these special requirements which are applicable to the public utility industry. Even the

suggested methods of measuring the effects of obsoleteness and excess capacity due to change in demand or in use, derived from the analogy of the economic effect of such changes on price and economic value in the competitive field, seem to be directly applicable. The regulation of semi-monopolistic public utility enterprises constitutes, in effect, the substitution of an artificial simulation of competitive conditions to make up for the lack of the influence of free competition on prices.

The references previously made to opportunities for refinement of methods of calculation, with increasing knowledge of facts and their relations, are especially applicable in the present case. Some equalization of requirements for annual depreciation expense by averaging over reasonable periods of years may also be desirable. In special cases, as where the property of a public utility consists of a single long-lived unit, like a hydro-electric dam, it may even be desirable, as a matter of public policy, to equalize such costs by a compound interest formula over a long period of years.

However, even such departures of amortization provision from depreciation, if desirable, cannot be made intelligently, and cannot avoid that unreasonable disparity between existing depreciation and reserves which has been condemned by the Courts, unless the facts are fully understood on the basis of proper engineering investigation and analysis. Such engineering investigation and analysis may well provide a more generally accepted basis of determining amortization based upon average depreciation experience for the purpose of making economic comparisons of different types of design and of the relative costs of steam and of hydro-electric power. The importance of such development is illustrated by the widespread discussion that followed the publication in November, 1937, of the report of the Power Authority of the State of New York on "Government Hydro *v.* Private Steam Power." In this report it was shown that the average fixed charges of four modern base-load steam-generating stations, furnishing power to distributing companies under contract, included depreciation varying from 1.90% to 3.25 per cent. The Authority adopted 2.0% as a reasonable comparative allowance for depreciation and renewals on both public and private water power plants, and 3.0%, based on sinking-fund accrual at 7% interest, as a reasonable comparative allowance for private steam generating stations.[24]

Cost of Power Produced by Governmental Projects

As in the case of competitive industry, the amortization of investments in public projects of any character may be controlled by considerations of policy unrelated to the occurrence of depreciation in the property representing such investments. It is not intended to suggest that it is a function of this Society, or of any other engineering organization, as such, to discuss such governmental policies. It is suggested, however, that, as in the case of competitive industry, knowledge of the engineering facts relating to the occurrence of depreciation in such property will be helpful to all those concerned in such policies, including governmental officers, tax-payers, and voters; and it is obvious that if helpful comparisons are to be made of costs of power produced

[24] H. R. Committee on Rivers and Harbors, Doc. No. 52. 75th Cong., 2d Session, pp. 26, 37, 38.

by governmental projects, and of those produced by private agencies, it will be helpful to include costs of depreciation in both on a strictly comparable basis.

Conclusion

The purpose of the writer in presenting this discussion of depreciation has been primarily to emphasize the engineering aspects of the problem, to which he believes sufficient consideration has not been given in the past, and to suggest that an opportunity be afforded the Engineering Profession to clarify this highly controversial subject by discussion of its engineering aspects. He ventures to express the hope that civil engineers will take the lead in this engineering activity, and will invite the co-operation of engineers in all branches of the profession in carrying forward a program which, he is convinced, can be made of great value, both to the profession and to the public.

RECAPITULATION

BY W. F. UHL,[25] M. AM. SOC. C. E.

SYNOPSIS

The final paper of a Symposium presumably is useful if it summarizes the salient facts revealed in each paper and points out divergencies of opinion where they may be found. It can also properly present conclusions and some of the economic aspects of the subject-matter.

It was thought best to confine the Second Symposium on Power Costs to generation only, in order that the issue might not be confused. Later, it is hoped that the more complex problem of cost of transmission and distribution may form the subject of a Third Symposium.

A study of the papers comprising this Symposium indicates that the cost of generating power is indeed a variable quantity. This is a well-known fact which it should be unnecessary to discuss at length, but many recent statements emanating from sources that should be well informed indicate that the belief is abroad that power should be produced at a fixed price or that a "yardstick" of some kind can be set up to measure the cost of power.

It is clear that, considering all the variables, it must be a mere accident if the cost of power generated in any two or more stations is the same. No attempt has been made in this paper to summarize facts and conclusions separately.

ELEMENTS OF POWER COST

Recent papers relating to the cost of power indicate that some of the fundamental principles involved are sometimes overlooked. These principles are well established. An honest difference of opinion can exist regarding their application but they may not be disregarded except as a convenient means of promoting unsound projects. Engineers cannot properly become promoters unless they acknowledge such activities.

The total cost of generating power consists of fixed charges and operating costs. These terms are well understood. Fixed charges may vary between wide limits, depending on cost of money (interest), depreciation and obsolescence experience, taxes, and possibly some other minor items. Operating costs can be more readily determined, but where costs are to be determined for the future, there can be honest disagreement on such items as maintenance, fuel, labor, etc.

With steam power plants the major uncertainty is the variable cost of fuel and the increasing cost of attendance. During recent years the rapid advance in the art of steam power generation makes it difficult to estimate depreciation

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[25] Hydr. Engr., Chas. T. Main, Inc., Boston, Mass.

and obsolescence. With hydro-plants the major uncertainty is the available stream flow. This is highly speculative in most cases and, at best, can only be based on records covering past experience where such records are available. Extreme drought and flood conditions are seldom known.

Quality of power as reflected by outages, voltage, and speed control, should receive consideration. Comparative reliability of operation must also be considered and the importance of reliability under certain conditions stressed.

Engineers with experience in the power field are thoroughly familiar with all items that must be considered in arriving at power costs, but very often there is a marked difference of opinion regarding the total fixed charges to be included in such costs. The principal items usually included in fixed charges are: (1) Cost of money; (2) depreciation and obsolescence; (3) taxes; and (4) insurance.

(1).—Cost of Money, or as More Commonly Referred to, Interest.—This item can vary within wide limits, depending principally upon financial conditions, the credit of the borrower, the popularity of the business, the hazards involved, and other matters. Some municipalities and boards, set up by legislative enactment for special purposes, have borrowed money for power plants at interest rates as low as 3% to 3.5%, whereas the average rate of interest for private concerns has generally varied between 6% and 8%, although a part of their money might be borrowed at rates as low as 4% to 6 per cent.

(2).—Depreciation and Obsolescence.—These items depend largely on the economic life of a plant which, in turn, depends principally on the design, materials, workmanship, and quality, in the construction and equipment, the extent to which the plant is used, its operation and maintenance, and the state of the art. It can be readily seen that there can be a wide difference of honest opinion regarding such matters.

Present experience indicates that the economic life of a steam plant is probably from 20 to 35 yr, whereas a well-designed and constructed hydro-plant may have an economic life of from 40 to 50 yr, or longer. In other words, the average rate of depreciation may be as high as 5% for a steam plant and as low as 2% for a hydro-plant.

Where the investment in plant is amortized by a sinking fund, it is necessary to establish the term of the useful life of the plant and the probable rate of return on the fund set aside, both of which items are speculative.

The prevailing practice of superposing high-pressure steam turbine units on old low-pressure plants which may have already been operated 10 to 15 yr, or more, complicates the depreciation problem in such cases. Overhauling the older units may be justified economically where they act as the low-pressure part of the new plant, since they now have an additional useful life of a more or less indeterminate character.

The future advance in the art of steam-power generation alone can determine whether the new superposed unit will have a useful life fixed by the remaining life of the older units or whether the life of the older units will be fixed by the useful life of the new high-pressure unit. The problem is not unlike that which presents itself when old hydro-plants are redeveloped and many structures are found suitable for continued use.

(*3*).—*Taxes.*—The principal taxes to be considered are property taxes to City, Township, County, and State. A portion of the owner's Corporation Franchise, State, and Federal income taxes may also be properly chargeable to power generation.

Property taxes on hydro-plants are often very much less in the country than the property taxes on steam plants which are generally situated near load centers. Social security and unemployment assessments must also be included. Taxes may vary from 1.5% to 4%, or more.

(*4*).—*Insurance.*—Insurance is a relatively minor item but is becoming more important as liability insurance has been gradually increasing. Fire, electrical, boiler, and fly-wheel insurance, as well as insurance against damage by the elements, are often carried. The amount of this insurance depends largely upon individual judgment. Generally, the insurance carried on hydro-plants and their equipment is materially less than that carried in connection with steam plants. Insurance usually varies from 0.25% to 0.75 per cent.

A summary of the foregoing items, constituting the annual fixed charges on power plants, indicates that the total may be as low as about 7%, or as high as about 18% of the capital cost. Fixed charges on hydro-plants usually vary from 8% to 11%, and on steam plants from 12% to 15 per cent.

The principal items usually included with operating cost are fuel, water, labor, maintenance, oil and supplies, repairs, superintendence, accidents and damages, general and miscellaneous expense, management, legal expense, etc.

Operating costs for hydro-plants are relatively small and do not vary much with the output of the plant. For steam plants, operating costs are relatively high and vary greatly, depending largely on the capacity factor of the plant, and for the most part on the cost of fuel. On the other hand, the total annual fixed costs for hydro-plants generally are higher than for steam plants, due to greater capital cost per unit of dependable hydro-plant capacity.

Where more than one plant carries the load of an inter-connected system, the average cost of generation must be considered, as well as the generating cost of some particular plant which may be operated at a high load and capacity factor. Money tied up in coal storage and working capital may also be considered a charge against the cost of generating power.

Heat-Generated Energy

Published cost records and individual experience show that the cost of steam power plants varies widely. Conditions under which they are built, their location, reserve capacity, load conditions, nature of service to be rendered, and other matters, have a marked effect on the capital cost. In studying the capital cost of steam plants, it is important that item for item be compared so that conclusions will not be misleading.

Data published on the cost of plants do not always state what is included. Besides the usual itemized account, including the plant rating, the number, size, and type of generating and boiler units, steam pressure and temperature, type of service, fuel used, number of outgoing lines, voltage of lines, condensing water conditions, etc., it is important to know something about land, buildings, foundations, water tunnels, coal and ash-handling equipment, or equipment

for servicing other fuel where used, boiler and turbine-room auxiliaries, switching arrangements, outgoing feeder lines, step-up sub-station equipment, etc. The extent of reserve capacity should also be stated.

Published costs show the following: One list of 150 plants built between 1882 and 1936 is stated to show an average cost of $100 per kw.[26] Another list of 80 plants built since 1920 shows an average cost of $108 per kw. Five of these plants cost more than $150, and four less than $70 per kw.[27]

Another list of sixteen plants built between 1924 and 1937 shows a range in cost, including step-up sub-station equipment, from $101 to $180 per kw, the average being $139 per kw.[28] Still another list of sixteen plants built since 1927 shows a range of cost from $82.50 to $145.00 per kw, with an average cost of $114 per kw.[29] These costs do not include any reserve capacity. Adequate reserve capacity may add from 20% to 40% to the cost of dependable generating capacity.

The records of cost of steam plants indicate that the size of the plant has little if any effect on the unit cost and the same is true of steam pressure and temperature. However, in general, it may be stated that the total power cost tends to decrease as the size of plant is increased.

Station service requirements for steam power plants are from 5% to 8% of the installed capacity. This service includes pumping, fuel handling and preparation, fans and other auxiliaries, and lighting. The power requirements for station service may be supplied in different ways, depending on the heat cycle. The necessary power may be supplied by the main unit and delivered through house-service transformers; it may be produced by house-service generators; or, the auxiliary equipment may be steam driven.

When steam power plants form a part of an inter-connected power system, a portion or all of their output may be transmitted over high-tension lines, thus requiring step-up sub-stations. Many of the large new steam power plants are some distance from the larger load centers and in such cases the entire output may require transmission at higher than generator voltages. In making comparisons between cost of plants, these questions must receive consideration.

The cost of the unit output of the steam power plant, assuming that the fixed charges are agreed upon, depends principally on the capacity and load factors and the cost of fuel. It will be found that the unit cost of generating steam power will vary just as widely as the capital cost.

Particular attention should be called to the difference in cost of steam power from plants designed with full auxiliaries and reserve capacity and those designed for less reliable service. Fixed charges per kilowatt-hour of output from modern steam power plants costing about $100 per kw and using 12% fixed charges, may vary from 0.17 to 0.7 cent, or more, depending on the capacity factor, which may vary from 80% to 20%, or less. Fuel costs per kilowatt-hour of net output will vary from about 0.17 cent at 80% capacity

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[26] "Progress in Generation of Energy by Heat Engines" (First Symposium on Power Costs), by Geo. A. Orrok, M. Am. Soc. C. E., *Proceedings*, Am. Soc. C. E., December, 1937, p. 1888.

[27] From records of the Federal Power Comm.

[28] *Electrical World*, October 27, 1928.

[29] *Loc. cit.*, November 23, 1935.

factor to 0.3 cent at 20% capacity factor, with fuel costing 15 cents per million Btu and with a 1-in. condenser back-pressure.

Total costs per kilowatt-hour composed of fixed charges, operation, maintenance, and fuel, will vary from about 0.4 cent at 80% capacity factor to 1.1 cents at 20% capacity factor for plants costing $100 per kw, with fixed charges at 12%, and fuel costing 15 cents per million Btu.

These costs are for a net output on low-tension terminals of large modern plants of 150 000 to 250 000-kw capacity with steam pressures from 400 lb to 1 250 lb, and temperatures from 650° F to 925° F, and without provisions for reserve capacity. General over-head expense will add from 10% to 25% to the foregoing unit costs.

Diesel and gas engine installations for the generation of electric energy are mostly made with relatively small units and for comparatively small plants. The total installation of such plants is also small compared with the water power and steam power installations.

Based on an investment cost of $135 per kw, 13% fixed charges, and 5 cents per gal for fuel oil, the total generating costs per kilowatt-hour net output from Diesel engine plants (includingg eneral over-head expense) will vary from 2 to 2.5 cents for operation at 20% capacity factor, to about 0.8 cent for operation at 80% capacity factor. A suitable reserve capacity will add somewhat to these costs.

Hydro-Generated Energy

Comparisons between capital costs of hydro-plants are even more difficult to make than comparisons between costs of steam plants, since the natural conditions to which hydro-plants must be suited are more variable and generally more difficult than the steam-plant locations which are subject to some choice. The installed capacity of hydro-plants with relation to the available stream flow also varies widely, depending largely on the load to be served and the available pondage and storage.

The cost of economic hydro-electric developments varies from $100 to $300 per kw of installed capacity. The cost of the greatest number for which data are available varies from $150 to $200 per kw. Higher costs than $300 per kw may sometimes be justified under special load conditions. In general, unit cost decreases as head and size of plant increase.

One of the principal difficulties in establishing the true capital cost of hydro-plants from which the full output cannot be used immediately, is the added capital cost due to fixed cost on the unused part of the plant. Generating units can be added from time to time, but the major cost is in the water rights and structures other than the power-house.

A comparison of hydro-plant costs based on the installed capacity is almost meaningless since plants are found which can operate to capacity only 3% to 5% of the time, due to stream flow limitations, whereas some plants can operate to capacity from 75% to 100% of the time on the available stream flow.

The earlier hydro-plants were developed for private industrial use at a time when the art of steam power generation was relatively uneconomical. As the power load of the industries increased and continuity of power supply became

more important with the advance of the industrial age, steam power was usually installed to supplement the water power during the seasons of low stream flow. Many of these industrial hydro-plants have been redeveloped in recent years, and generation and distribution of output have been changed from mechanical to electrical. Some of the old structures built for the original hydro-plants, such as dams, canals, conduits, and tail-races (and, in some cases, even the power-house), are utilized to a greater or less extent in the redevelopment.

The cost of such redevelopment generally varies from $75 to $100 per kw of installed capacity. The cost of operation and maintenance of these industrial plants is very low. Many are operated without continuous attendance. Pumped-storage hydro-plants can sometimes be built to carry the peak load of a system and thus improve the load factor on the remainder of the plants in the system. They may also be relied on for emergency reserve capacity. For some reason this type of plant has not been adopted extensively in the United States, but quite a number of them have been built in Europe. Natural conditions which permit construction of such plants at reasonable cost and where their output does not require excessive transmission costs, are factors that have an important bearing on the economic advantage which such plants might have over other peak capacity.

The cost of hydro-generated energy, assuming agreement on fixed charges, may be readily determined if the usable output is known, since costs other than fixed charges are nominal. Operation and maintenance costs per kilowatt-hour of output depend largely on the size of the plant and somewhat on its design and construction. For plants of from 5 000 kw to 10 000 kw capacity, these costs may vary from 0.1 to 0.05 cent per kw-hr and for large plants they may be as low as 0.02 to 0.01 cent per kw-hr. Total generating costs consisting of fixed charges computed at 10% and operation and maintenance vary from 0.25 cent to 3 cents and more per kw-hr. Although, in general, the cost per kilowatt-hour tends to lessen as the head and the size of the plant increase, the effect of capacity factor is more predominant.

Steam and Water Power Combined

The earlier hydro-plants were usually developed as a main source of power. This is true with respect to both industrial and public utility hydro-plants. As the loads increased it was generally found more economical to install a supplementary steam plant to be operated during low-water seasons rather than a second water-power plant. Such supplementary steam plants were built at low cost without much regard for economy of operation since they were not in use much of the time.

The utilization factor of most hydro-plants operated by themselves is generally very low unless a portion of their output is sold as dump power. In some cases in the West and in many places in Canada, hydro-plants always did, and still do, carry the entire power load of very large systems. It is true that occasionally during extreme droughts the output must be curtailed, but usually arrangements are made to drop some of the low-price load which is taken on with the understanding that this can be done.

Water power and steam power are not often economically competitive, but ordinarily are complementary if used to best advantage. Under modern conditions, water power finds its greatest economic use where it supplements steam power, since it is found that in many cases where sufficient flow and pondage are available, incremental hydro-capacity costs less than incremental steam plant capacity. It also has certain advantages as reserve capacity since hydro-plant units can be started and their output placed on the outgoing lines in a few minutes whereas steam units must be kept hot and revolving for some time before they can be put under load. Hydro-capacity can also be used for power-factor correction at small cost.

Hydro-electric energy output should cost no more than the fuel cost of an equal amount of steam-generated energy if credit is allowed for that portion of the hydro-plant capacity which is capable of carrying peak loads or which acts as reserve capacity. Usually, hydro-plants are not developed unless they show some margin of economy over steam.

Small water-power plants near load centers or near the ends of transmission or distribution systems often have a value which they would not have if they were situated at a distance requiring the transmission of the energy generated to a distant market. If they were near a load which would otherwise be supplied over a transmission line from a distant power plant, there is a saving in transmission losses and there are certain advantages due to possible voltage control and power factor correction. If pondage is available, such plants also have a capacity value. There is also the additional advantage that during transmission difficulties at least a part of the local and near-by load can be supplied from such a plant.

It is often claimed that Niagara Falls furnishes a water power that is sufficient in itself and does not require a supplementary source of power, since the stream flow is such that a practically constant output of water power is available. However, even Niagara power cannot be fully utilized, except as a part of a large system having a substantial amount of steam power, since a load requiring a constant output has not yet been discovered.

Niagara, "tied in" with steam power, can carry a large base load, and thus can be utilized quite fully; whereas, if operated by itself, there would be many periods when much of the available water would be wasted due to lack of demand. Ultimately, every water power can be more fully utilized if operated in conjunction with heat energy plants.

Economic Importance of Cost of Power

Available statistics indicate that in 1936 the total stationary power plant installations in the United States amounted to about 75 000 000 hp, divided as follows:

Steam plants	56 684 000
Hydro-plants	16 075 000
Oil and gas engine plants	2 000 000
Total	74 759 000

An average cost of $100 per hp (about $135 per kw) would make the total investment in stationary power plants about $7 500 000 000 as of 1936. The actual investment is probably much larger, and the annual investment in such plants has become one of the major items of national business.

If the cost of power is compared with the cost of other commodities for which there is a similar universal demand, it would appear that the unit cost of its generation is unimportant, particularly since such cost is only a small part of the cost of delivering it to the ultimate consumer, as in the case of public utility service.

The total power generated by the public utilities alone, during 1937, was about 125 billion kw-hr. A difference of 1 mill per kw-hr represents a sum of $125 000 000. This indicates that power use in the United States has reached such proportions that small savings in generation are extremely important. However, it is much more important that an abundant and reliable supply of power be available than a source of cheap power which may be unreliable.

By far the greatest power capacity in this country is in the automotive field, probably about 1 billion hp. The power produced by this automotive equipment can certainly not be classed as cheap power, but automobile and airplane engines to-day furnish a supply of reliable power, a condition one would scarcely care to change if reducing the cost of this power resulted in less reliability.

Cost comparisons have often been made between power generated in different parts of the country and between steam and water power. Assuming that all other factors affecting cost of power are considered, such comparisons may be very misleading unless a careful analysis is made of the quality of the power supply. Quality embraces such matters as continuity of supply throughout the seasons, outages of short duration, speed and voltage control, etc. In making power-cost comparisons, the quality of the power supply must be given due consideration.

Power cost is a relatively unimportant item to many industries but the continuity and quality of power supply may be extremely important when profitable business is being processed. Reduced production or spoiled materials are more important than cost of power to most industries.

The ratio of power cost to the total cost of production is small in most industries. It is less than 10% of the total cost of manufacturing in all except perhaps ten or a dozen industries. In many industries the cost of power is less than 1% of the cost of manufacturing.

The ratio of the power cost to the total value of products for 100 different industries averages about 2.64 per cent.[30] The following information on the cost of power was reported by one newspaper:[31]

"A shade over 10 per cent of the total cost of manufacturing our paper goes to the Ontario Hydro-electric Commission at Niagara Falls for purchased power, and a shade less than 9 per cent covers the cost of making the paper and its delivery in Chicago. * * * If our mill was fully equipped with modern machinery, the cost of power could be brought down to about 6.6 per cent. * * * The ink factory power cost is 2.1 per cent of the cost of manufacturing. * * *

[30] *Magazine of Wall Street*, 1926, Vol. 38, p. 568.

[31] *Public Service Magazine*, Vol. 53, No. 4, October, 1932, pp. 101–102.

In the Tribune Tower Building all electricity for lights, elevators, etc., etc., amounts to 1.5 per cent of the total revenue received from the building. * * * The cost of electricity for operating the Tribune WGN Radio Station and Studio is 2.5 per cent of the total cost of maintaining the Studio. * * * Electricity for the warehouse and dock amounts to 0.7 per cent of the total cost of operating the property."

Power is one of the few commodities the cost of which has steadily decreased while the cost of other commodities has been doubled and trebled. Even during the 1929 depression when the demand decreased materially, and in spite of ever-increasing taxation, the cost of power to the consumer was steadily reduced. The Consolidated Edison Company of New York, serving both gas and electricity to the great metropolis, reports that its tax bill is now (1938) about $1 000 000 per week, or $52 000 000 per yr. It is the largest single item of expense except labor.

The tax increase in recent years is astonishing—as much as 70% in seven years. In 1930, taxes took 12 cents out of every dollar of income. In 1937, taxes took 20 cents of every dollar paid by the consumer for service.

The rates charged by the Company have been lowered sharply in these years, so much so that the bill to consumers was $44 000 000 less in 1937 than it would have been on 1929 rates; but if the Company paid no taxes and the entire tax saving were passed on to the consumers, their bills could be reduced 17% more.

Power production as an industry was practically unknown until the efforts of Edison, Thompson, and their contemporaries, established electricity on a commercial basis about 1883, and it was not until about twenty years later that records were obtainable for comparisons as to the cost of power, its use, and its influence upon the welfare of the country.

TABLE 12.—COST OF POWER, SHOWING GROWTH OF THE POWER INDUSTRY IN THE UNITED STATES

Description	1912	1922	1936
Generating capacity, in kilowatts	5 165 400	14 313 350	34 466 000
Invested capital, in thousands of dollars	2 062 655	4 817 000	13 300 000
Thousands of kilowatt-hours of energy generated	11 569 100	44 422 000	108 854 000
Operating cost, in thousands of dollars*	151 270	533 068	934 200
Taxes, in thousands of dollars	13 147	73 772	272 258
Number of domestic customers	3 101 000	10 211 230	21 648 300
Kilowatt-hours of energy sold per customer	264	359	720
Average rate, in cents per kilowatt-hour	8.9	7.4	4.74
Number of people living in electrically lighted houses	14 000 000	44 000 000	89 700 000
Percentage of population	16	39	70
Ratio of investment to receipts	6.85	4.50	6.00
Investment, in dollars per dollar of annual taxes	157.0	65.50	49.0

* Does not include fixed charges.

The economic aspects of the cost of power may be reflected by a few comparisons (see Table 12) which show the growth of the power industry in the United States during the last half century, the increasing additions of the invested capital to the national wealth, and the increasing contributions through taxes to the support of the municipal, State, and Federal Governments.

Conclusions

Some of the more important conclusions may be summarized as follows:

(1) The total cost of producing power consists of fixed charges and operating costs. Fixed charges are the principal cost in producing hydro-electric power and are also a major item of cost in producing steam power.

(2) The capital cost of power plants, both steam and water power, is variable, depending upon many factors, but principally on the location and nature of service to be rendered.

(3) The cost of power generation is also variable, depending principally on the cost of fuel and the output of the plant in the case of steam plants, and upon the capital cost and the available stream flow in the case of hydro-plants.

(4) Reliability of power supply is an important factor which may affect both the capital and operating costs of power plants.

(5) Steam and water-power plants are seldom economically competitive but are ordinarily complementary if used to best advantage.

(6) When investments are made in large new power plants, the output can seldom be absorbed immediately and, in the case of some of the large hydro-plants, not for many years. In such cases the yearly fixed charges and maintenance costs on the unused part of the plants, less credit for increased economy of operation, if any, must be added to the capital cost of the plant until it is fully utilized.

(7) Particular attention should be called to the difference in cost of steam power from plants designed with full auxiliaries and reserve capacity, and those designed for less reliable service.

(8) Abundance of reliable power is of much greater importance than cheap power from unreliable sources.

(9) The power industry is of increasing importance to the welfare of the nation and has made tremendous contributions to its wealth and income.

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PAPERS

ECONOMICS OF SEWAGE TREATMENT

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Synopsis

A general analysis of the economics of various basic processes of sewage treatment is presented in this paper which draws attention to a number of influencing and controlling factors that affect the comparison of the various processes as applied to a specific situation. Data are presented on construction and operation costs and on the reductions accomplished by the various processes indicative of the usual conditions and of the range to be expected.

Introductory Statement of the Problem

Sewage treatment processes or combinations of processes are available for accomplishing varying degrees of treatment, including relatively complete removal of pollution, to meet the requirements of practically any condition. The number of processes available presents the problem of determining for a particular situation the method and degree of treatment which most economically and practically satisfies the needs of that definite problem. From a purely economics viewpoint, and for a fixed set of conditions, there is only one solution to the problem. However, a number of factors other than cost, affect the decision in a number of instances. A variety of considerations well known to sanitary engineers, serves to determine the degree of treatment necessary in a particular problem. Furthermore, the method capable of accomplishing the degree of treatment determined upon, is dependent upon a still larger number of influencing practical considerations. These practical factors may be of controlling importance and may make necessary the elimination of certain methods from considerations; or they may be of lesser import and serve merely to affect the judgment of the engineer in the determination of the most desirable and economical form of treatment.

Without question the most important item affecting the economic analysis of the sewage treatment problem is the determination of the degree of treatment necessary. In view of the fact that this question is definitely one which must be answered for every particular problem, no detailed discussion will be

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made herein. The fact that the degree of treatment required may vary from practically nothing to almost complete removal of pollution, necessarily affects the selection of the process of treatment in view of the limitations of the various processes now in use. The further fact that the cost varies with the degree of treatment provided, makes this latter consideration fundamental. The analysis of the type of process which will most economically provide the necessary degree of treatment is generally, from the standpoint of cost to the community, of much less importance than the determination of whether any intermediate, or relatively complete, treatment is necessary.

The aim of this paper is to show, in a general manner, the economic relationship between the various methods of treatment as well as between the different processes involved within the particular method.

Economics Problems Involved

In the selection of the degree and method of treatment best adapted to a particular situation, a variety of economics problems present themselves. Before proceeding with detailed discussion of the problems, it is well to consider them in general terms.

Disposal by Dilution.—The most common and generally the most economical manner of disposing of sewage and waste matter is to discharge it into the available receiving bodies of water—ocean, lake, river, or creek—either without treatment or after varying degrees of pollution removal.

As a natural drainage channel, a watercourse receives the normal surface pollution of the area tributary to it, and by the processes of natural purification converts the organic matter into stable, non-putrefactive matter. The processes underlying this change are physical, chemical, bacteriological, and biological in character, and depend for their effectiveness upon the existence of a proper and sufficient quantity of the elements and organisms which accomplish the changes. When, however, because of the growth of communities along such watercourse and their consequent discharge of sewage and trade wastes into the river or stream, the organic load becomes too great, the oxygen supply in the river is reduced, with a consequent impairment in the activity of the desirable organisms. When the balance between the oxygen resources and the oxygen demand of the organic matter discharged into the stream becomes unfavorable, some form of treatment becomes necessary to supplement the natural processes. The time when treatment becomes necessary and the extent of treatment required are dependent on the unfavorableness of the balance and the use to which the river will be put.

When adequate dilution is available, proportionate to the sewage and wastes discharged into it, the disposal of the sewage by emptying into a near-by lake or watercourse offers an economical solution to the sewage disposal problem. The proportionate degree of pollution that the river can receive is dependent on the characteristics of the receiving body of water, and the use to which it is put. In any event, there is an upper limit to the capacity of the stream or lake to receive polluting material and when that condition is reached, treatment to some degree, to relieve the unsatisfactory condition, becomes necessary.

Preliminary Treatment.—Many problems present themselves in connection with the economics of preliminary treatment. An important consideration is the most economical period of sedimentation when considered in relation to the reduction in suspended solids and bio-chemical oxygen demand (B.O.D.). The comparison between fine screening and sedimentation is worthy of consideration.

Secondary Treatment.—The advantage and economy of one type of secondary treatment as compared with others capable of performing the required degree of treatment present a number of problems. Within the various types themselves a number of considerations are of importance, such as the effect of various rates of application on the cost of trickling filters, the economical period of aeration in the case of the activated sludge process, and the effect of the addition of varying quantities of chemical in the case of chemical treatment. In all cases consideration must be given to the varying reductions accomplished, and to the controlling effect, in special cases, of the limitations on maximum reductions capable of accomplishment by the various processes. The extent to which preliminary treatment is carried before the secondary process is begun, as well as the type of such treatment, is an important economics problem.

Sludge Disposal.—One of the most important factors in arriving at a decision as to the economics of various types of treatment is the method of sludge disposal. Of the various methods available in any project, a study of the economics and applicability of such methods must be made, having in mind the local influencing or controlling factors. What might be a satisfactory and economic method under the conditions of one situation, might be objectionable or extravagant in another, stressing the fact that in sludge disposal, perhaps more than in sewage treatment, every situation must be considered by itself. Generalizations are difficult, and likely to be misleading when applied to sludge disposal. It is for this reason that the writer has refrained from evaluating the various sludge disposal methods in a general manner; for example, it is apparent that the controversial subject of the economics of sludge digestion or no digestion, in connection with disposal by vacuum filtration and incineration, cannot be answered in general terms when it is realized that variations in a large number of influencing factors definitely affect the results of the investigation. As a further example, vacuum filtration and incineration would not be justified in a situation where climatic and soil conditions and favorable plant environmental factors make disposal of digested sludge on sand drying beds particularly advantageous and economical. The variety of methods of sludge disposal available to the engineer, with their consequent questions of fertilizer production, power production with gas engines, and utilization of heat in incineration, considered in relation to characteristics of sewage and sludge, very definitely make the solution of the sludge disposal problem a specific one, with known conditions, rather than one capable of generalized discussion. The estimates shown herein for the various types of treatment are based upon digestion and air drying on open sludge beds. However, the substantiating costs from various plants include a number of different methods of sludge disposal, such as digestion and drying, vacuum

filtration, incineration, fertilizer production, lagooning, etc. Although the writer has similarly analyzed the costs for various methods of sludge disposal, and realizes the importance of this factor in the determination of the economics of sewage treatment, it was felt to be impracticable to include the analysis in this paper because of length limitations.

Miscellaneous.—For reasons of brevity a number of accepted forms of treatment, such as pre-aeration in connection with trickling filters, chlorination, intermittent sand filters, and Imhoff tank treatment, are not discussed, or if mention is made of them, the statements are of a general nature.

Method of Comparison

To compare the merits of the different processes, accomplishing as they do, varying reductions in the strength of the influent sewage, the total annual charges of treatment by the various processes have been reduced to a cost per 1% reduction in suspended solids and B.O.D. Although, in special cases, bacteriological and nitrogen analyses may be important, the suspended solids and B.O.D. determinations are believed to furnish the most reliable general index of the efficiency of varying treatment processes.

It is the fact that the economic relationship between the various processes is tied in with, and depends very closely on, the analytical results obtained, that explains the large quantity of material included herein which is not strictly economics. Costs, analytical results obtained, adaptability, and other influencing or controlling factors must be carried "hand in hand" in the determination of the most favorable and economical treatment for a particular situation.

From a cost standpoint the various processes have been compared on the basis of the total annual charges per million gallons of sewage treated per 1% reduction in either suspended solids or B.O.D. Aside from the varying reductions accomplished by the different processes already discussed, this unit or yardstick does not take into consideration the fact that the higher the degree of treatment the greater the cost, as will be shown subsequently herein. In other words, the cost of removing the last 10% of suspended solids or B.O.D. in a sewage is proportionately much greater than the first 10%, as is generally recognized. This fact must be realized by the reader in comparing the costs on the unit used herein or, otherwise, conclusions will be drawn from the data which are contrary to the facts.

A further variation which the writer's unit does not definitely show is the effect of size of plant on the costs per unit or reduction. For various reasons the costs per million gallons in the larger plants are generally lower than in the smaller ones, and this variation is greater in some types of treatment than in others. This variation must also be considered in studying the data herein.

With these exceptions the method of evaluating the various processes is believed to be proper and correct and when cognizance is taken of the effect of these factors, a true picture of the cost and place for the various processes can be attained.

General Statement on Cost Data

The construction cost estimates presented herein are based on a level which, in general, prevailed from 1923 to 1930. They are believed to be reasonably close to the *Engineering News-Record* cost index of 200. Although, consciously or subconsciously, the writer had the general conditions existent in Minneapolis and St. Paul, Minn., in mind in preparing the estimates, the cost data are believed to be of general applicability in so far as cost level is concerned, except for unusual conditions. Substantiating cost data from actual plants bear out this statement. The previous statement applies, as noted, to the general cost level, but it is not intended to cover the multitude of factors which by their favorable or unfavorable effects might make one form of treatment more economical in a particular situation than another.

The term, total annual charges, as used in the paper, includes both operating and maintenance costs and fixed charges on the investment. Fixed charges include interest and amortization on bonds issued which is the usual method of financing sewage works improvement. A value for fixed charges of 6% of the construction cost annually has been used, realizing, however, that the magnitude depends for a particular situation on the form of the bond issue, the bond period, and the interest rate, which is influenced by a number of economic factors. This percentage represents an approximately average rate and is approximately the rate commonly secured. The effect of higher and lower rates will be shown in a later comparison.

Bond amortization should more properly be considered as a depreciation item which takes into account the comparative lives of the various plants, rather than one of bond retirement which may cover long or short periods, depending on local requirements and public feeling at the time of issuance. Allowance has been made for this fact by considering a relatively long bond period, and providing in the estimates for operation and maintenance cost an item of "Maintenance, Supplies, Repairs, and Renewals" to cover all such costs as might be necessary to maintain the plant in a first-class operating condition.

Characteristics of Sewage Considered

Although an attempt is made to make the comparisons and conclusions of general applicability, the condition and situation in Minneapolis and St. Paul have been uppermost in the writer's mind. This fact is obviously both necessary and beneficial, since the effect of certain features, such as characteristics of the sewage, capacity of plant, unit costs, etc., would make an abstract generalization worthless, if not even delusive. The characteristics of the sewage considered have an important effect on the results of any economic study of sewage treatment.

Except where specifically stated to the contrary herein, sewage having the following characteristics will be assumed:

Suspended solids, in parts per million	185
Five-day B.O.D., in parts per million	200

The foregoing values are the approximate strengths of the sewage from Minneapolis and St. Paul as determined in the latter part of 1927 and the early part of

1928, and because of their use in a number of economic studies to which reference will be made herein, have been used in this study. Except for the fact that the suspended solids content is rather low, the foregoing characteristics represent a sewage of normal strength, as evidenced by the fact that the average of the annual results at forty-two representative plants in the United States was approximately 239 ppm on a suspended solids basis and 198 ppm on a 5-day, B.O.D. basis.

Estimated Unit Construction Costs

The estimates of construction costs of the various treatment plants used in this analysis are intended to be reasonably close to, but in general slightly higher than, the actual costs, since in such a general investigation as this, where fully developed plans and specifications cannot be prepared, it is obviously difficult to make a detailed and complete list of quantities to which unit prices can be applied. Being mindful of this fact, contingency items were added to provide for inevitable omissions, so that the estimates will show costs which will probably be found to be slightly higher, rather than lower, than actual bid prices; 15% was added to the estimated construction costs for engineering, contingencies, and omissions.

Because of the impracticability of presenting detailed estimates of costs of all the structures and equipment which make up a complete treatment plant, the estimated costs used herein are presented as unit cost estimates (that is, per cubic foot of tank, per acre-foot of filter, etc). This manner of presentation lends itself to a comparison with substantiating costs at actual existing plants. The following unit costs have been used in the estimates, except in the case of smaller plants, where somewhat higher costs have been used:

1.—Settling Tanks.—Estimates are based on cost of 54 cents to 62 cents per cu ft of effective capacity, which includes cost of tanks, mechanisms, influent and effluent channels, and various appurtenances, the value depending upon whether it is used in connection with trickling filter or activated sludge plants, and whether preliminary or final settling tanks are under consideration. In substantiation of the foregoing costs, an average of eight plants, most of which were constructed from 1930 to 1934 was 51 cents per cu ft, the lowest unit cost being 35 cents and the most expensive, 66 cents.

2.—Trickling Filters.—Estimates are based on a cost of $14 000 per acre per ft of depth, exclusive of cost of covers. An average of five trickling filter plants constructed from 1923 to 1929 was $13 170 per acre per ft of depth, the lowest being $10 060 and the highest, $15 600.

3.—Aeration Tanks in Connection with an Activated Sludge Plant.—An estimated cost of 52 cents per cu ft of effective aeration tank capacity was selected, this cost including operating galleries, but excluding air compressors, return and excess sludge pumps, power plant, air filters, and appurtenances. In substantiation of this value the average of four plants constructed from 1927 to 1931 was 51 cents, the lowest being 45 cents and the highest, 57 cents.

4.—Power Plant, Air Compressors, Sludge Pumps, Air Filters and Appurtenances in Connection with Activated Sludge Plant.—The estimated cost of these items is $7 500 per million gallons capacity. The average of six plants con-

structed from 1927 to 1934 was \$8 800, the lowest being \$5 100 and the highest, \$12 200.

5.—*Sludge Digestion Tanks.*—Estimates are based on 35 cents per cu ft of effective capacity, including tanks, sludge pumps, galleries, heating plant, mechanisms, covers, etc. An average of nine plants constructed from 1928 to 1933 was 32 cents, the lowest being 21 cents and the highest, 44 cents.

6.—*Sludge Storage Tanks.*—Estimates are based on a cost of 20 cents per cu ft of effective capacity, including covers. An average of six plants constructed from 1928 to 1933, some of which were uncovered, was 18 cents, the lowest being 12.5 cents and the highest, 27 cents.

7.—*Sludge Drying Beds.*—Cost estimates are based on 60 cents per sq ft of open beds, the hand-cleaned type, including transportation facilities. In substantiation, the average of twenty-two plants, constructed from 1925 to 1933, was 55 cents per sq ft, the lowest being 26 cents, and the highest, \$1.03. Considering only the nine larger plants the costs averaged 48 cents per sq ft. Substantiating costs do not include hauling equipment.

Screens and Grit Chambers

General.—Although not productive of as high removals, proportionately, as other units of the treatment plant, the removal of the larger material by screens and the heavier material by grit chambers is none the less important. The efficient and economical operation of the remainder of the plant makes the inclusion of these units a necessary adjunct to a well designed plant, especially if the sewage is from a combined system.

The early installations of screens and grit chambers were simple, involving usually manual removal of screenings and grit; but recently mechanical and automatic equipment has become available and its use in connection with later plants has resulted in more efficient operation of these units.

Bar Screens

Removal Accomplished by Bar Screens.—A study has been made of the operating results of 117 bar-screen installations, in which the average removals varied with the size of the opening, somewhat as follows:

Clear opening, in inches	Average quantity of screenings, in cubic feet per million gallons
2.00	0.9
1.50	1.2
1.00	3.1
0.75	5.3
0.50	8.6

Usually the maximum peaks are 200% of the average quantity, but in some cases they range as high as 500 per cent.

With raw sewage containing approximately 200 ppm of suspended solids, approximately 1.5% of the suspended solids may be removed with screens with 1-in. clear openings and about 3.5% with screens of 0.5-in. openings.

Method of Disposing of Screenings.—In forty-nine plants concerning which data were available, sixteen plants disposed of screenings by incineration, eleven by grinding, twenty-one by burial and one by digestion. The costs for incineration range from \$1.07 to \$9.30 per ton, and by burial from \$0.50 to \$5.00.

Grit Chambers

Removal Accomplished by Grit Chambers.—In view of the varying designs of grit chambers, as well as of the characteristics of the sewage entering them, it is to be expected that the percentage of suspended solids removed will vary accordingly. The average quantity of screenings removed in thirteen plants investigated was 3.7 cu ft, or 0.14 cu yd per million gallons, the minimum removal being 1.16 cu ft, and the maximum, 7.0 cu ft per million gallons. The foregoing data are based on yearly, or at least on monthly, averages. Daily maximums have been observed as high as 80 cu ft per million gallons.

Based on the average removed of 3.7 cu ft per million gallons, and a raw sewage containing 200 ppm of suspended solids, grit chambers may be expected to remove about 5.5% of the solids contained in the sewage reaching them.

Total Quantity of Solids Removed by Screens and Grit Chambers.—With screens with 1-in. clear openings and grit chambers of usual length and cross-section proportionate to flow, it is reasonable to expect a total removal of about 7 cu ft per million gallons, corresponding to a total reduction in suspended solids of about 7% for a sewage of normal strength. The removal of B.O.D. is proportionately much less.

Construction Cost of Screens and Grit Chambers.—A wide range in the cost of building screens and grit chambers is indicated, dependent on the simplicity or elaborateness of the installation, the costs varying in twelve installations investigated from approximately \$150 to \$1 200 per million gallon capacity. The data indicate a total average cost, including mechanisms, approximating in the smaller installations, a value of \$1 000 per million gallon capacity, and ranging in the larger installations to \$600 per million gallon. A still further variation exists in the data on operation costs of screen and grit chambers.

In explanation of the foregoing data it should be emphasized that the flow capacities are those of the grit chambers or of the intercepting sewers entering the plant, and are usually several times the capacity of the remainder of the plant. The costs of screen and grit chambers are included in those of the basic processes mentioned subsequently.

Fine Screens

Removals Accomplished by Fine Screening.—Data have been collected on the removals of suspended matter accomplished by fifteen fine screen installations, with clear openings from 0.20 in. and smaller. The data indicate an average removal of suspended solids of 5% with screens having clear openings of 0.2 in., 6.5% for screens with $\frac{1}{8}$-in. openings, increasing to 10% for screens with $\frac{1}{16}$-in. openings, and to 20% from screens with 0.02-in. openings. Most of the fine screen installations have screens with openings from $\frac{1}{8}$ in. to $\frac{1}{16}$ in. with the latter size predominating.

Construction and Operation Cost of Fine-Screenings Plants.—A study of seven installations indicates a construction cost range of from \$5 000 per million gallon capacity in the smaller plants to \$4 000 per million gallon capacity in the larger plants, and an operating and maintenance cost ranging from \$2 per million gallon treated for smaller plants to \$1 per million gallon in the larger plants. For installations with $\frac{1}{16}$-in. clear openings, the total annual cost per million gallons per 1% reduction in suspended solids is as given in Table 1.

TABLE 1.—TOTAL ANNUAL CHARGES OF FINE SCREENING PLANTS

Sewage treated, in million gallons daily	Total annual charges	Total annual cost per million gallons per 1% reduction in suspended solids
10	\$10 000	\$0.26
25	21 300	0.23
50	38 000	0.21
75	53 000	0.195
100	66 200	0.18

SEDIMENTATION

Reductions Accomplished by Sedimentation.—It is well known that the reduction accomplished by sedimentation is influenced by a large number of factors. A more complete discussion of the effects of the various factors was published[2] in 1933. These factors may be summarized as follows: (1) The characteristics of the liquid, under which can be included specific gravity and viscosity; (2) the characteristics of the solids, which include size, shape, specific gravity, concentration, coagulation, etc.; (3) the characteristics of the design, comprising detention period provided, velocity of flow, depth, and ratio of length to depth, inlets and outlets, baffling, etc.; and, (4) miscellaneous effects, such as currents caused by winds, eddies, and difference of temperature, biological activities, etc.

TABLE 2.—PERCENTAGE REMOVAL OF SUSPENDED SOLIDS AND FIVE-DAY BIO-CHEMICAL OXYGEN DEMAND SETTLING FOR VARIOUS PERIODS

Detention period, in hours	RAW SEWAGE STRENGTH, IN PARTS PER MILLION*								REMOVALS FOR SEWAGE OF AVERAGE STRENGTHS USED IN THIS ANALYSIS	
	50 to 100		100 to 200		200 to 300		300 to 400			
	S.S.	B.O.D.	S.S.	B.O.D.	S.S.	B.O.D.	S.S.	B.O.D.	S.S.	B.O.D.
0.5	26	12	34	16	40	18	42	20	35.5	12.5
1.0	36	19	45	24	51	27	53	30	45.5	21.0
1.5	43	24	50	30	56	33	60	36	51.0	26.5
2.0	47	27	54	34	60	37	63	39	55.0	31.0
2.5	50	30	57	36	62	40	65	41	57.5	34.0
3.0	52	32	59	38	64	42	66	43	59.5	37.0

* S.S. = suspended solids; and B.O.D. = five-day bio-chemical oxygen demand.

For a particular sewage one of the most important factors affecting the reduction accomplished by sedimentation is the detention period. In Table 2 the relationship between detention period and reduction in suspended solids

[2] *Sewage Works Journal*, Vol. V, No. 2, March, 1933.

and 5-day B.O.D. for sewages of various strengths is shown. The reduction increases sharply for a period of time, depending on characteristics of the sewage, and then gradually diminishes until a point is reached at which an increase in detention period does not result in an appreciable increase in solids reduction. From an economic standpoint, a balance exists between the cost of treatment and the reduction accomplished beyond which it is uneconomical to provide settling capacity. Furthermore, there is the possibility that under adverse conditions too long a period of settling may result in a degradation of the effluent because of septic action and the resulting gas buoyance of particles.

The substantiating data used as a basis in preparing Table 2 consist of information available to the writer from more than thirty sedimentation plants scattered throughout the United States. A total of approximately 140 values was considered, each representing a monthly, up to a yearly, average. The settling periods shown are based upon theoretical, computed detention periods.

TABLE 3.—CONSTRUCTION, AND OPERATION AND MAINTENANCE, COSTS OF SEDIMENTATION PLANTS OF VARIOUS CAPACITIES

Plant capacity or sewage treated, in million gallons daily	CONSTRUCTION COST			OPERATION AND MAINTENANCE COST		
	Usual lower cost	Estimated in this analysis	Usual higher cost	Usual lower cost	Estimated in this analysis	Usual higher cost
10.....	$240 000	$325 000	$400 000	$17 000	$23 000	$29 000
25.....	550 000	725 000	900 000	38 000	50 000	62 000
50.....	980 000	1 300 000	1 600 000	68 000	90 000	110 000
75.....	1 350 000	1 800 000	2 250 000	94 000	125 000	160 000
100.....	1 720 000	2 300 000	2 900 000	120 000	155 000	200 000

Construction and Operating Costs of Sedimentation Plants.—In Table 3 estimates of construction costs and operation and maintenance costs are shown for plants of various capacities. These costs denoted as "estimated in this analysis" are based on a sedimentation period of 2.5 hr. Included also are usual lower and higher cost values which are approximately 25% below or above the estimated cost shown. Of twelve plants on which substantiating construction costs are available, the costs at one plant were less than the usual lower limit. Substantiating operation and maintenance cost data are available from ten plants of which four were less than the usual lower limit.

Economical Period of Sedimentation.—Because both the removals accomplished and the cost of sedimentation plants vary with the detention periods provided (but not in the same proportion) it is apparent that for a particular problem an economical period of sedimentation exists. A summary of data relative to this determination is shown in Table 4, in which the fourth and fifth columns contain the annual cost per million gallons per 1% reduction in B.O.D. and suspended solids, respectively. On a B.O.D. basis the economical period of settling is approximately 3 to 3.5 hr whereas, on a suspended solids basis, the economical period is approximately 1.25 to 1.50 hr. The foregoing statements apply to sedimentation in plants in which this is the sole process of treatment, and not to cases in which it is used in conjunction with secondary processes. Furthermore, it should be stated that for sewages with different

settling characteristics as affected by detention period, or where costs are materially different, the aforementioned indications must be modified; and there may be situations in which higher reductions than are indicated to be economical are necessary or desirable, or other cases where the minimum of treatment is indicated, in which cases the economic study, of course, has no application.

TABLE 4.—Construction Costs and Total Annual Charges; Sedimentation Plant of 100 Million Gallons Daily; Average Flow with Various Detention Periods

Detention period, in hours	Estimated construction cost	Estimated total annual charges*	Annual Charge per Million Gallons per 1% Reduction In: 5-day B.O.D.†	Annual Charge per Million Gallons per 1% Reduction In: Suspended solids
0.5	$990 000	$173 500	$0.380	$0.134
1.0	1 325 000	203 500	0.265	0.123
1.5	1 595 000	228 000	0.236	0.122
2.0	1 845 000	250 000	0.220	0.124
2.5	2 070 000	269 000	0.216	0.128
3.0	2 290 000	288 000	0.213	0.132
3.5	2 500 000	304 500	0.213	0.137
4.0	2 700 000	321 000	0.217	0.143

* Includes fixed charges @ 6 per cent.
† Bio-chemical oxygen demand.

As a general statement, however, it can be said safely that where the pressing need is simply removal of solids, a short period (1 to 1.5 hr, or less) is indicated as being economical, and in cases where higher B.O.D. removal is desired detention periods of 2.5 to 3.5 hr may be economical. It should be mentioned in the latter connection, however, that such higher reductions in B.O.D. can usually be secured more economically through a variety of secondary processes.

Trickling Filters

Reductions Accomplished by Trickling Filters.—As in other processes the reduction accomplished by trickling filters varies in the different plants, depending on the characteristics of the sewage and design features. Data have been collected on the over-all reduction accomplished by eighteen trickling filter plants, based generally on annual averages. The weighted reduction, discarding data on plants in which the sewage was of unusual characteristics or concerning which a duplication of data existed on the same plant, is 85% on a B.O.D. basis and 84% on a suspended solids basis. On a B.O.D. basis, the maximum reduction accomplished in any plant was 94.5% and the minimum, 80%; whereas on a suspended solids basis the maximum reduction accomplished was 92.3% and the minimum, 70.5 per cent.

It is interesting to consider also the efficiency of the filter itself. In 1931[3] data indicative of the reductions from the point of application on the filter (effluent of preliminary sedimentation) to the point of leaving the final tanks (if included), were presented in some detail by the late J. A. Childs, M. Am.

[3] For a more complete discussion of this matter the reader is referred to *Civil Engineering*, Vol. 1, No. 15, December, 1931, p. 1391 (see especially Figs. 1 to 4).

Soc. C. E., and the writer. This information was collected in an attempt to determine economical filter loadings and rates of application. Reference to the source will reveal the following general observations: (1) Within limits, the percentage reduction in 5-day B.O.D. accomplished by trickling filters is independent of the strength of the sewage applied to the filters; (2) also within limits, the efficiency is independent of the rate of application; and (3) the efficiency is not, within a reasonable range, influenced by increases in combined volume and strength (loading).

These statements must be tempered in their conclusiveness in that they apply only within the limits for which data were collected, which, however, covered a rather wide range in both volume and strength. It is obvious that there must be an upper limit to the values for which they apply, but on a B.O.D. basis, that limit had apparently not been reached at any of the plants investigated, as no reduction in efficiency was in evidence.

These deductions make economic studies very difficult, since they do not indicate the most economical loading. If it were possible to load a filter continuously at some of the higher values indicated, low cost would result. Other factors, however, such as depth of bed, climatic conditions, characteristics of the sewage, size and character of the filtering media, method of application, dosing and rest periods, as well as many others, affect the magnitude of the economical loading in varying degrees. The data collected from the fifteen plants[4] considered in this study indicate an average reduction of the 5-day B.O.D., by secondary treatment with trickling filters of 75%, the lowest efficiency being 49% and the highest, 88 per cent.

It may be of interest to mention that since the collection of the foregoing data in 1929, trickling filter plants have been operated on a large-scale experimental basis with rates up to 20 and even 25 million gallons per acre per day. If it is demonstrated definitely that such high sustained rates of application are practicable in actual operation, trickling filters will be in a most advantageous position as far as the economics of treatment are concerned.

Construction, Operation, and Maintenance Costs of Trickling Filter Plants.—The estimated construction cost of trickling filter plants is shown in Table 5.

TABLE 5.—CONSTRUCTION, OPERATION, AND MAINTENANCE COSTS OF TRICKLING FILTER PLANTS OF VARIOUS CAPACITIES

Plant capacity or sewage treated, in million gallons daily	CONSTRUCTION COST			OPERATION AND MAINTENANCE COST		
	Usual lower limit	Estimated in this analysis	Usual higher limit	Usual lower cost	Estimated in this analysis	Usual higher cost
10.....	$900 000	$1 200 000	$1 500 000	$22 000	$30 000	$38 000
25.....	2 130 000	2 850 000	3 550 000	47 000	62 000	78 000
50.....	4 130 000	5 500 000	6 850 000	83 000	110 000	138 000
75.....	6 000 000	8 000 000	10 000 000	115 000	155 000	195 000
100.....	7 900 000	10 500 000	13 200 000	145 000	195 000	245 000

The tabulation of estimated costs is based upon a filter rate of 2.0 million gallons per acre per day on a filter 8 ft deep.

[4] *Civil Engineering*, Vol. 1, No. 15, December, 1931, p. 1391 (see especially Fig. 3).

Of twenty-six plants on which substantiating construction costs are available, the cost of one plant was higher than the usual higher cost and the costs at seven plants were lower than the usual lower limit.

As an indication of the effect of higher rates of application on the construction cost of trickling filters, it is estimated that with a rate of application of 3.0 million gallons per acre per day the construction cost of a complete trickling filter plant will be approximately 85% of the cost with a rate of 2 million gallons, whereas with a rate of 5.0 million gallons, the construction cost will be approximately 70% of the cost with a 2.0 million gallon rate. Some reduction in operation and maintenance cost would also be effected.

The estimated operation and maintenance cost of trickling filter plants substantiated by actual costs at other plants is shown in Table 5. Of twenty-two plants on which operation and maintenance costs are available, the costs at two plants were higher than the usual higher limit and those at two other plants were lower than the usual lower limit. As an indication of the distribution of the annual operating and maintenance costs the average of six trickling filter plants, varying from a capacity of 5 to 55 mgd, was as given in Table 6.

TABLE 6.—Distribution of Annual Operating and Maintenance Costs; Average of Six Trickling Filter Plants

Item	Cost, in dollars per million gallons	Percentage of total cost
Labor and supervision	4.15	72
Maintenance, supplies, repairs, and renewals (includes power)	1.65	28
Total	5.80	100

Balance Between Various Processes.—Cost estimates have been prepared to determine the theoretical economical balance between the sedimentation and the secondary processes. It is evident that, unless the costs by both preliminary and secondary processes are identical, a variation in the detention period provided in the preliminary tanks will affect the over-all construction and operation and maintenance costs and the over-all reductions. Data relevant to this point, indicate that, at usual design rates, to increase the period of preliminary sedimentation from 1 hr to 2 hr results only in increasing the over-all reduction by about 2%, accomplished by raising the total annual charges by approximately 4% to 5 per cent. From a theoretical standpoint computations indicate that on a B.O.D. basis, the most economical period of sedimentation is the shortest one. However, there is a practical limit to the possibility of shortening the period of preliminary sedimentation as controlled by operation difficulties with filter nozzles and clogging of beds. It appears that a detention period in preliminary tanks of approximately 1.5 hr is a desirable and practical one for ordinary sewage. This has been demonstrated and is in accord with usual practice in design in this regard.

Activated Sludge Process

Reductions Accomplished.—The activated sludge process is capable of accomplishing somewhat greater reductions, at least on a B.O.D. basis, than other processes. Data representing, in most cases, yearly or longer averages from nineteen activated sludge plants have been collected indicating an average reduction of 91.7% of 5-day B.O.D. and in suspended solids of 89.5 per cent. A weighted average (excluding certain plants in which the sewage was of unusual characteristics or, for other reasons, the data were not considered representative) indicates an average removal of B.O.D. of 92.5% and of suspended solids of 91.0 per cent.

The foregoing data refer to reductions accomplished by complete activated sludge treatment, including preliminary treatment. The reductions accomplished by the secondary process alone, including aeration and final sedimentation, for various periods of aeration of mixed liquor (sewage plus return sludge) for sewage of normal strength and characteristics are as follows:

Aeration period, in hours	Percentage reduction of 5-day bio-chemical oxygen demand
3	74.0
4	84.5
5	91.0
6	93.0
7	95.0

The foregoing reductions are substantiated by the average reductions in seventeen activated sludge plants. A total of thirty-five values covering operating periods usually of 1 yr, as well as of test plants, have been considered in the determination of the average values given. The list indicates that up to a period of approximately 5 hr, the reductions in B.O.D. increase markedly, after which an increase in the period of aeration does not result in appreciable increase in removals.

TABLE 7.—Construction, Operation, and Maintenance Costs of Activated Sludge Plants of Various Capacities

Plant capacity or sewage treated, in million gallons daily	Construction Cost				Operation and Maintenance Cost			
	Usual lower limit	Average for Aeration: 5 hours	Average for Aeration: 6 hours	Usual higher limit	Usual lower limit	Average for Air: 0.6 cubic foot per gallon	Average for Air: 1.0 cubic feet per gallon	Usual higher limits
10	$ 560 000	$ 700 000	$ 800 000	$ 940 000	$75 000	$ 90 000	$110 000	$125 000
25	1 280 000	1 600 000	1 800 000	2 120 000	150 000	185 000	220 000	255 000
50	2 500 000	3 200 000	3 500 000	4 200 000	250 000	310 000	370 000	430 000
75	3 700 000	4 750 000	5 200 000	6 200 000	340 000	420 000	495 000	570 000
100	5 000 000	6 300 000	6 900 000	8 200 000	430 000	520 000	615 000	710 000

Construction, and Operation and Maintenance, Costs of Activated Sludge Plants.—The estimated construction costs of activated sludge plants along with the estimated construction cost of activated sludge plants with detention periods of 5 and 6 hr are shown in Table 7, together with the usual lower and

higher costs of this type of plant. An idea of the variation in construction costs is shown by the fact that of twenty-one plants on which construction costs are available, the costs at two plants were less than the usual lower limit and at one plant more than the usual higher limit shown.

Estimated operation and maintenance costs for activated sludge plants based on the use of 0.6 and 1.0 cu ft of air per gal are also shown. Of twenty-four plants on which operation and maintenance costs are available, the costs at five plants were less than the usual lower limit and at one plant, more than the usual higher limit shown. The operation and maintenance costs are based on a power rate of 1.0 cent for the larger plants, increasing for the smaller plants in accordance with usual schedules. In substantiation of the operation and maintenance costs shown in Table 7, and to furnish an indication of the composition of costs, the following costs at a number of representative activated sludge plants are shown: (1) Power costs: The average of fifteen plants in which power rates varied from 0.7 cent to 1.51 cents per kw-hr was \$6.50 per million gallon treated; (2) labor and supervision: The average of fourteen plants was \$6.35 per million gallon; and, (3) maintenance, supplies, repairs, and renewals: The average of fourteen plants was 1.74% of construction cost annually.

Economical Period of Aeration.—The fact that the reduction accomplished by activated sludge treatment is affected by the period of aeration, somewhat as shown previously, suggests the possibility of the existence of an economical period of aeration, at least from a theoretical standpoint. In Table 8 data of

TABLE 8.—CONSTRUCTION COSTS AND TOTAL ANNUAL CHARGES—SECONDARY TREATMENT* BY ACTIVATED SLUDGE PROCESS—PLANT CAPACITY 100 MILLION GALLONS DAILY WITH VARIOUS DETENTION PERIODS

Period of aeration, in hours	Percentage reduction in 5-day bio-chemical oxygen demand of influent	Estimated construction cost	Fixed charges @ 6%	Annual maintenance and operation cost	Total annual charges	Total annual charges per million gallons per 1% reduction in 5-day bio-chemical oxygen demand
3.0.....	74.0	\$3 445 000	\$207 000	\$402 000	\$609 000	\$0.226
4.0.....	84.5	3 845 000	231 000	425 000	656 000	0.213
4.5.....	88.0	4 005 000	240 000	436 000	676 000	0.210
5.0.....	91.0	4 235 000	254 000	447 000	701 000	0.212
6.0.....	93.0	4 640 000	278 000	471 000	749 000	0.221
7.0.....	95.0	5 040 000	303 000	494 000	797 000	0.230

* Preliminary treatment (that is, screen and grit chambers and preliminary settling) not included; disposal of sludge from secondary process included.

this nature are shown. The most economical period of aeration with normal sewage is calculated to be approximately 4.5 hr. The importance of determining the economical period of aeration from a cost standpoint, either in the design or in the operation stage, can be shown by the following: Considering secondary treatment alone (without preliminary treatment) an increase in the aeration period from 4.5 to 6 hr, increases the percentage reduction, 5%, the construction cost, 15%, and the total annual charges per unit of reduction, 5 per cent. At 4.5 hr, the total annual charges for secondary treatment are estimated to be \$0.210 per million gallon per 1% reduction in 5-day B.O.D.

A shorter aeration period, approaching that mentioned previously, is substantiated by recent designs and operating data. It was only a few years ago that plants were designed and operated with detention periods in aeration tanks of from 6 hr and to 8 hr. More recently, periods of 5 hr and even shorter are more usual in design. It is important to realize that, in some cases, longer periods are necessary or desirable either because of unusual characteristics of the sewage, or because of the controlling features of the installation.

Balance Between Preliminary and Secondary Treatment.—Data relative to a determination of the balance between various processes are shown in Table 9,

TABLE 9.—TOTAL ANNUAL CHARGES OF ENTIRE ACTIVATED SLUDGE PLANTS WHEN PRECEDED BY VARIOUS PERIODS OF SEDIMENTATION PLANT CAPACITY, 100 MILLION GALLONS DAILY

(Based upon detention period of 4.5 hours with 20% return sludge in aeration tanks, and 1 cubic foot of air per gallon of sewage)

Period of sedimentation preceding aeration, in hours	Total over-all reduction in 5-day biochemical oxygen demand	Total annual charge per million gallons per 1% reduction in 5-day biochemical demand	Period of sedimentation preceding aeration, in hours	Total over-all reduction in 5-day biochemical oxygen demand	Total annual charge per million gallons per 1% reduction in 5-day biochemical demand	Period of sedimentation preceding aeration, in hours	Total over-all reduction in 5-day biochemical oxygen demand	Total annual charge per million gallons per 1% reduction in 5-day biochemical demand
0.25	89.5	$0.264	1.50	91.9	$0.281	3.00	93.2	$0.294
0.50	90.3	0.269	2.00	92.5	0.285	3.50	93.4	0.298
0.75	90.7	0.273	2.50	92.9	0.290	4.00	93.6	0.302
1.00	91.1	0.276	....	....	......	....	....	......

based on various periods of preliminary sedimentation, and an aeration period of 4.5 hr. The data indicate that from a theoretical standpoint the most economical reduction of B.O.D. is accomplished with the shortest period of sedimentation. From a practical standpoint, there is a limit to the extent to which preliminary sedimentation can be reduced as affected or controlled by difficulties in aeration tanks and sludge handling. Substantiation for the conclusion that secondary treatment is more economical than the preliminary processes, where a high degree of treatment is necessary, can be found in practice by the fact that relatively short periods of sedimentation or even fine screening has been provided in the larger plants. It appears that a practical and desirable range in period of preliminary treatment is from 0.5 hr to 1 hr. The method of disposing of sludge affects the economics to a certain extent.

Air Requirements of Activated Sludge Process.—Data have been collected from fourteen plants on the quantity of air required in the activated sludge process. These data indicate that the quantity used for normal sewage varies from 0.4 cu ft to 2.5 cu ft per gal of sewage. The data show that of the total amount of air added, only 1% to 10% is actually used for oxidation of organic material in the sewage and even a portion of this may be derived from surface aeration. The remainder (from 90 to 99%) is used for agitation. The relation between influent strength in parts per million of 5-day B.O.D. and the cubic feet of air per pound of 5-day B.O.D. removed for the same data, is somewhat as follows:

Influent strength, in parts per million of five-day bio-chemical oxygen demand	Air required, in cubic feet of air per pound of five-day bio-chemical oxygen demand removed
100	1 700
200	900
300	650
400	500

In general, the quantity of air required per pound of 5-day B.O.D. removed, is greater for the weaker sewages than for the stronger ones. The possible economy of mechanical means of aeration, or a combination of air and mechanical methods is suggested from a study of the foregoing data for exceptionably weak sewages.

Period of Final Sedimentation.—The usual method of expressing the loading of final settling tanks following aeration is in terms of gallons of sewage per square foot of tank area per day. As an indication of the usual design practice, eleven activated sludge plants investigated, provide an average rate of approximately 900 gal of average sewage flow (not including return sludge) per sq ft per day, with the lowest design rate approximately 600 and the highest approximately 1 200. The average design rate at five of the larger plants was 830 gal per sq ft per day. Review of operating records indicates that rates from 800 to 1 000 gal per sq ft per day are practical and desirable rates for normal sewage. The rate selected should be based on a consideration of variations in flow (maximum and minimum with respect to average) and depth of tank. For example, with a 900-gal rate a 10-ft depth of tank provides a detention period of 2 hr; with a 12.5-ft depth, a period of 2.5 hr; and with a 15-ft depth of tank, 3 hr. In general, exceptionably shallow tanks should be designed with lower rates of application.

Chemical Treatment

Reductions Accomplished by Chemical Treatment.—One of the features of chemical treatment is its susceptibility to variation in reduction accomplished by varying the chemical addition. Data showing the reduction in suspended solids and B.O.D. from the results of experimental plants by various investigators and at six actual plants with different chemical additions have been collected. In these studies the chemicals added have been expressed in dollars per million gallons rather than parts per million because of the variety of chemicals used and their varying cost and effectiveness. Although a rather large range in reduction is indicated the data are believed to be sufficiently well substantiated and possible of attainment especially for well designed and well operated plants. Unusual sewages, of course, present an exception. The difference in effectiveness of chemical treatment at various plants should at least suggest the desirability of careful investigation before decision on the practicability of this type of treatment for a particular problem, as well as of the type of chemicals and various design features. The data collected (see Table 10) indicate the reductions in B.O.D. and suspended solids with various additions of chemicals for normal sewage and with reasonable detention periods in

TABLE 10.—REDUCTIONS ACCOMPLISHED BY CHEMICAL TREATMENT

Percentage removal of:	Percentage Reduction for the Following Chemicals Added, in Terms of Dollars per Million Gallons			
	3.00	6.00	9.00	12.00
Bio-chemical oxygen demand..........	52	63	72	77
Suspended solids.....................	69	77	84	88

the flocculating and settling tanks. Chemical costs used in comparing various plants are actual costs, except where data were not available, in which case actual chemical additions were used, applying the following chemical costs per ton:

Lime..............................	$10
Chlorine..........................	50
Copperas..........................	17
Ferric chloride...................	50

Based on the additions of chemicals at the various plants, and on the foregoing costs, the average chemical cost per million gallons in the various plants investigated is $8.35; and the average reduction in B.O.D. was 67.3%; and in suspended solids, 83.8 per cent.

Construction, Operation, and Maintenance Costs of Chemical Treatment.—The estimated construction costs of chemical treatment plants are shown in Table 11. Of thirteen plants on which costs are available the costs at two plants are less than the usual lower limit and at two plants more than the usual higher limit. The construction costs of chemical treatment plants is likely to vary widely depending on the provisions of the design.

TABLE 11.—CONSTRUCTION, AND OPERATION AND MAINTENANCE, COSTS OF CHEMICAL TREATMENT PLANTS OF VARIOUS CAPACITIES

Plant capacity or sewage treated, in million gallons daily	Construction Cost			Operation and Maintenance Cost*		
	Usual lower cost	General average*	Usual higher cost	Usual lower cost	General average	Usual higher cost
10.....	$ 380 000	$ 500 000	$ 620 000	$26 000	$35 000	$44 000
25.....	750 000	1 000 000	1 250 000	52 000	69 000	86 000
50.....	1 300 000	1 750 000	2 200 000	84 000	112 000	140 000
75.....	1 800 000	2 375 000	3 000 000	115 000	153 000	190 000
100.....	2 200 000	2 950 000	3 700 000	143 000	190 000	240 000

* Exclusive of chemical added.

Data on operation and maintenance costs are shown in Table 11. Of six plants on which operation and maintenance costs are available the cost at one plant was more than the usual higher limit. In view of the varying cost of chemicals depending on the reduction desirable or necessary, the costs with various additions of chemicals have been added to the costs shown in Table 11 to prepare Table 12. Including chemicals the total estimated costs to accomplish various reductions are as shown in Table 12.

The costs shown in Table 12 are subject to the same variations with respect to usual lower and upper limits as shown in Table 11.

TABLE 12.—ESTIMATED COST OF REDUCTION BY CHEMICALS

Reductions Accomplished		Annual Operation and Maintenance Cost				
Percentage of bio-chemical oxygen demand	Percentage of suspended solids	10 mgd	25 mgd	50 mgd	75 mgd	100 mgd
56	73	$48 000	$105 000	$185 000	$260 000	$330 000
69	83	65 000	140 000	255 000	360 000	475 000
79	89	80 000	180 000	330 000	465 000	625 000
82	91	85 000	190 000	60 000	515 000	680 000

EFFLUENT FILTERS

Removals Accomplished by Effluent Filters.—In the past several years, a number of installations of effluent filters consisting of beds of magnetite sand, usually of 3-in. thickness, cleaned by the action of a solenoid passing over the bed, have been made. Experimental data are also available on modified rapid sand filter beds, as used in water purification practice, and also on beds 12 in. deep, cleaned by jets of water; but to date (1938) no large-scale installations of these latter types have been made for which continuous long-time operating records are available.

Although filters of the previously described types may have a function as the principle method of treatment, their important application at present appears to be in conjunction with other basic processes, such as sedimentation, trickling filters, activated sludge, and chemical treatment, and only such use will be discussed herein. For this reason they are not considered in the comparison of basic treatment processes described subsequently. Data have been collected on the reductions accomplished by effluent filters (magnetite sand type) in certain plants. It will be observed from Table 13, which summarizes the data collected, that the reductions accomplished are a function of the strength of the filter influent, much the same as for trickling filters, except

TABLE 13.—REDUCTION ACCOMPLISHED BY EFFLUENT FILTERS FOLLOWING SEDIMENTATION AND CHEMICAL TREATMENT

Filter influent strength—bio-chemical oxygen demand and suspended solids, in parts per million	Reduction Accomplished, in Parts per Million			
	Following Sedimentation		Following Chemical Treatment	
	Bio-chemical oxygen demand	Suspended solids	Bio-chemical oxygen demand	Suspended solids
25	....	1	....	6
50	2	10	2	30
75	8	27	8	55
100	15	45	15	80
125	22	63	22	105

that the reduction is not a fixed percentage of the strength of the filter influent. The table is based on sewage of normal characteristics. As might be expected, the reductions in B.O.D. are somewhat lower than the reductions in suspended solids.

As in the case of trickling filters, the reduction accomplished does not appear for the same grain size to be affected by the rate of application, at least up to a limit of 4, or possible 5, gal per sq ft per min. A considerable variation occurs in practice in the filter rate varying in nine installations investigated from 1.8 to 3.0 gal per sq ft per min, the more usual rates being between 2.0 and 2.5. The rate to be chosen for design purposes should include a consideration of the reductions desirable or necessary, characteristics of filter influent, the sand size, variation in flow, and certain other factors.

Construction, Operation, and Maintenance Costs of Effluent Filters.—Data have been collected on the estimated construction costs of effluent filters, substantiated by costs of five installations and estimates from manufacturers. The cost of the equipment installed is estimated to be approximately 70% to 75% of the total installation cost. With upward flow filters constructed integrally with settling tanks, the extra cost of structures may be approximately 10% to 20% of the installed equipment cost. Based on rates of application of 2.5 to 3 gal per sq ft, the estimated construction cost per million gallons with separate structures vary approximately from $4 000 per million gallon of capacity for the larger installations to $5 000 per million gallon for the smaller installations.

Although data were available on some items of operation and maintenance costs, no actual substantiating data are available as to the total costs. They are believed, however, to be reasonably high for usual conditions. For various plant capacities the estimated operation and maintenance costs vary from approximately $1.00 per million gallon treated in the smaller installations to $0.70 per million gallon, in the larger installations.

Relative Costs as Compared with Other Processes.—This feature of cost comparison can best, and more accurately, be illustrated by application to a specific problem. A summary of data on an investigation of this subject prepared by the writer in connection with the Minneapolis-St. Paul project, is presented for this purpose. The design capacity of the plant is 134 mgd and the characteristics of the sewage are as assumed herein. The costs include only those chargeable to sewage treatment, and do not include the cost of sludge disposal, which was considered to be reasonably equal per ton of solids for sludge produced by other processes. On a suspended solids basis, the cost of filtration is approximately equal to chemical treatment if between 15 and 20 ppm are removed by filters, and to plain sedimentation if approximately 23 to 24 ppm are removed. On a B.O.D. basis, filtration is more economical than chemical treatment if between 12 and 15 ppm are removed, and as economical as sedimentation if approximately 16 to 17 ppm are removed. Based on the reductions by filters as shown in Table 13 and on the reductions expected by sedimentation and chemical treatment, it was estimated that the reductions accomplished by effluent filters for the Minneapolis-St. Paul project would be 18 ppm during chemical treatment and 31 ppm during plain sedi-

mentation on a suspended solids basis; and 8 ppm during chemical treatment and 21 ppm during plain sedimentation on a B.O.D. basis. As will be explained subsequently, chemical treatment is required in Minneapolis and St. Paul only about 10% of the time, and, therefore, the removals during plain sedimentation are of more interest. The addition of filters in the Minneapolis-St. Paul project is expected to reduce the quantity of suspended solids in the plant effluent by approximately 6 000 tons annually.

Comparison of Various Processes

General.—From what has previously been mentioned it is apparent that a wide variation in construction, operation, and maintenance costs, and in reductions accomplished, obtains for the basic processes described herein. It is evident also that it is possible to obtain removals to meet the requirements of a particular situation, from nothing to relatively complete treatment by some of the basic processes or combinations with other processes. Having arrived at what seem to be reasonable basic cost and reduction data for normal conditions, this section of the paper is devoted to a comparison of the various processes, and a discussion of certain factors that affect their relative economy and thus serve to alter the general indications presented in the comparisons.

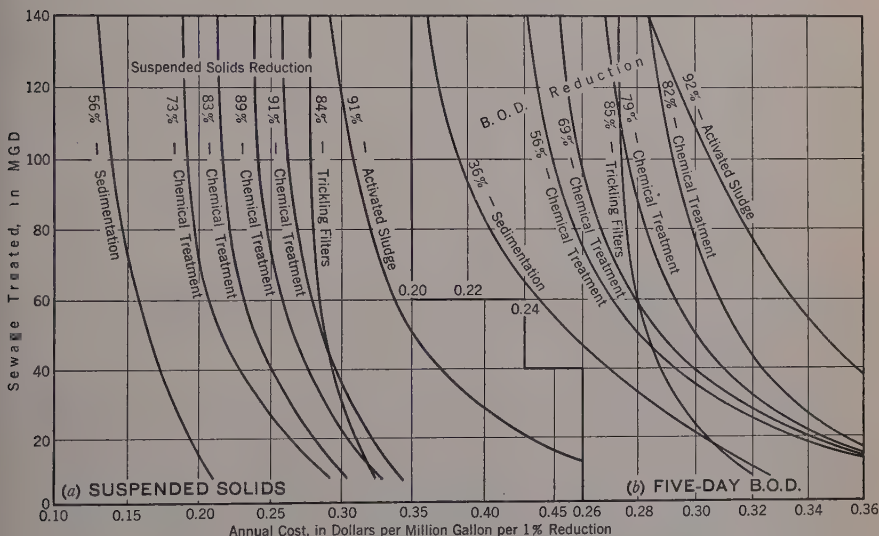

Fig. 1.—Annual Charges of Various Methods of Treatment

Comparison of Processes on Basis of Suspended Solids Reduction.—Using as a basis, the cost estimates and removals accomplished by the various processes, as already described, the estimated total annual costs per million gallons per 1% reduction in suspended solids, are shown for normal conditions, in Fig. 1(*a*). Selecting from this diagram three plant sizes for comparison, the costs may be summarized as shown in Table 14(*a*). It will be observed that on a suspended solids basis, the various processes, as applied to normal conditions,

may be arranged in order of ascendancy of costs per unit of reduction, as follows: (1) Sedimentation; (2) chemical treatment to various degrees; (3) trickling filters; and (4) activated sludge. This arrangement does not hold for the smaller plants in which trickling filters may be more economical than some degrees of chemical treatment.

TABLE 14.—COMPARISON OF VARIOUS PROCESSES

No.	Treatment process	(a) Suspended Solids Reduction				(b) Bio-Chemical Oxygen Demand Reductions				(c) Annual Cost, in Dollars, per Million Gallons per 1% Reduction of 5-Day Bio-Chemical Oxygen Demand with Fixed Charges of:		
		Percentage of suspended solids	Annual Cost, in Dollars, per Million Gallons per 1% Reduction for Plant Capacity (in Million Gallons Daily) of:			Percentage of bio-chemical oxygen demand	Annual Cost, in Dollars, per Million Gallons per 1% Reduction for Plant Capacity (in Million Gallons Daily) of:			5%	6%	7%
			20	50	100		20	50	100			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
1	Sedimentation	56	0.19	0.17	0.14	36	0.30	0.25	0.22	0.20	0.22	0.24
2	Trickling filters	84	0.31	0.28	0.27	85	0.30	0.28	0.27	0.24	0.27	0.30
3	Activated sludge	91	0.43	0.35	0.31	92	0.39	0.34	0.30	0.28	0.30	0.32
4	Chemical treatment	73	0.27	0.22	0.19	56	0.33	0.28	0.25	0.27*	0.28*	0.29*
		83	0.28	0.23	0.22	69	0.34	0.29	0.26			
		89	0.30	0.26	0.24	79	0.34	0.30	0.28			
		91	0.32	0.28	0.26	82	0.35	0.31	0.29			

* For a removal of 79% B.O.D.

Comparison of Processes on Basis of Bio-Chemical Oxygen Demand Reductions.—Data relevant to this comparison are shown in Fig. 1(*b*) and for various selected plant sizes in Table 14(*b*). It will be observed that, on a B.O.D. basis, the relative economy of the various processes is less marked. For a plant of 100 mgd capacity, the order of economy for normal conditions is as follows: (1) Sedimentation; (2) chemical treatment to a B.O.D. removal of 77%; (3) trickling filters; (4) chemical treatment with B.O.D. removal greater than 77%; and (5) activated sludge treatment. However, for a plant of 20 mgd capacity the indicated order is: (1) Sedimentation; (2) trickling filters; (3) chemical treatment to various degrees; and (4) activated sludge.

The foregoing statements are irrespective of reductions accomplished. For processes accomplishing approximately the same degree of treatment the order of economy is altered. It should be emphasized also that the processes capable of the highest reduction, in general, cost more per unit of reduction.

Effect of Various Influencing Factors on the Comparison.—In view of the fact that the cost of the various processes accomplishing substantially the same removal are remarkably close, the necessity of investigating a number of influencing factors which affect the comparison in a particular situation, becomes apparent. It is important also to call attention to certain of these

influencing factors which affect the comparison because, to this point, the analysis has of necessity been general, and the writer well realizes the impossibility of stating proper generalizations of many features of sewage treatment.

Effect of Degree of Treatment Capable of Accomplishment by the Various Processes.—The statement has already been made that the cost per unit of removal of the first 10% of pollution in a sewage or waste is less than the last 10%, indicating that processes capable of high removals, when compared on that basis will, in general, cost more per unit of removal than processes capable only of low removals. This fact can be shown very clearly by considering 35 (Fig. 1(*b*)) for a plant of 100 mgd capacity. For this condition, the costs can be summarized as in Column (9), Table 14(*b*). Thus, to effect the removals capable of accomplishment by activated sludge (that is, 92%) results in an increased cost per unit of removal of 35% over the cost by sedimentation. This fact must be borne in mind when considering the unit costs as presented herein.

It is because of the conditions stated herein that the degree of treatment necessary for a particular situation has such an important effect on the type of treatment selected and, therefore, upon the costs. The requirements of a situation may be such as to necessitate the adoption of a process or combination of processes capable of accomplishing high removals, in which case the cost of processes which result only in low removals is not pertinent.

Effect of Fixed Charges on Cost.—As already mentioned, this analysis has been based upon a uniform rate for fixed charges of 6 per cent. If, because of variation in interest rates or other considerations previously mentioned, these rates vary markedly, an important effect on the comparison of the various processes is observed. Table 15 illustrates the effect of this factor for a plant of 100 mgd capacity.

TABLE 15.—EFFECT OF FIXED CHARGES

Treatment	Total Annual Charges with Fixed Charges of:		
	5%	6%†	7%
Sedimentation	$260 000	$285 000	$310 000
Trickling filters	750 000	857 000	964 000
Activated sludge	955 000	1 024 000	1 093 000
Chemical treatment*	775 000	804 000	833 000

* For removals of 79% B.O.D. and 89% suspended solids.
† From Table 16 and used generally in this analysis.

A low rate of fixed charges reflects favorably on the cost of processes more expensive in first cost and lower in operation and maintenance costs. Thus, for the conditions assumed, a reduction in fixed charges from 6% to 5% reduced the total annual charges of the trickling filter process by 12.5%, whereas the reduction in the activated sludge processes is 6.7%, and in the chemical treatment process only 3.6 per cent. The total annual charges per million gallon treated per 1% reduction in B.O.D. shown in Fig. 1(*b*), with various rates of fixed charges, are as given in Table 14(*c*), for a 100 mgd plant. It will

be observed that whereas the cost per unit reduction of B.O.D. might be lower for the trickling filter process with fixed charges of 6%, with fixed charges of 7%, or more, the chemical treatment process becomes more economical. The effect of this factor in a particular installation must be considered in connection with studies of the economical form of treatment.

Effect of Sewage Characteristics.—The fact that the characteristics of the sewage may be such as to be more favorable to one type of treatment than to other types, must be considered in comparing processes for a specific situation. In fact, in some cases, the presence of inhibitive wastes makes some processes of the biological type impractical. Even in more normal sewages, strength and other characteristics affect both construction and operation costs of sewage treatment and sludge disposal. This influencing factor can be viewed more clearly by considering the individual processes—sedimentation, trickling filters, etc.

Sedimentation.—A number of considerations of sewage characteristics, such as its settleability, temperature, concentration, etc., are likely to affect the reductions capable of accomplishment and, therefore, for a given reduction, the period of detention necessary. It is well known, for example, that one plant with a detention period, of, say, 1.5 hr, may accomplish as great or a greater reduction than another plant providing a period of 2.5 hr. At least, part of this difference in behavior may be attributed to the sewage characteristics.

The processes following sedimentation, such as effluent filters, trickling filters, activated sludge, affect the detention period which should be provided, as already mentioned.

Considering equal reductions as being accomplished, a decrease in the detention period from 2.5 hr to 1.75 hr, reduces the construction costs of the entire plant by approximately 15 to 20 per cent. Assuming a decrease in construction cost of 15% and no reduction in operation and maintenance costs, such a decrease in detention period reduces the total annual charges of a plant of 100 mgd capacity by 8%, or the cost per unit of reduction in B.O.D. from 22 cents to 20 cents per million gallon.

Trickling Filters.—It is likely that, within limits, the rate of application of sewage on a trickling filter can be increased without affecting the reduction accomplished. Thus, it is conceivable that under both favorable design conditions and sewage characteristics it is possible to increase the average rate of application from 2 million gallon per acre per day as considered herein to as high as 5.0 million gallon in which case the construction costs will be reduced by approximately 30 per cent. Without considering possible reductions in operation and maintenance costs, the total annual charges will be reduced by approximately 22 per cent. In other words, the total annual charges per million gallons per 1% reduction in B.O.D. shown in Fig. 1(*b*), might be reduced from 27 cents to 21 cents. The effect of such a decrease in costs in the comparison of various processes is apparent.

Activated Sludge Process.—It may be possible under favorable conditions to reduce the detention period from 6 hr to 5 hr, and the quantity of air from 1.0 cu ft of air per gal to 0.6 cu ft. Under these conditions the construction costs might be reduced by as much as 9% and the total annual charges by as

much as 12%, in which case the total annual charges per million gallons per 1% reduction in B.O.D., would be reduced from 30 cents to 26 cents.

Chemical Treatment.—In view of the varying effect of the addition of chemicals as affected by the characteristics of the sewage observed at various plants, it is apparent that, with a sewage particularly amenable to chemical treatment, the quantity of chemicals necessary to produce a given reduction may be materially less than for another less favorable sewage. If a reduction in chemical costs of $4 per million gallons can be effected for a sewage to which chemicals can be applied advantageously, a saving in annual operation and maintenance charges of approximately $150 000 annually for a plant of 100 mgd capacity will result, equivalent, in a condition where the chemical costs are reduced from $12 to $8 per million gallon for a 79% reduction in B.O.D., to a decrease in total annual charges approximating 19%, or to a decrease in the cost per million gallons per 1% reduction in B.O.D. from 28 cents to 23 cents.

Effect of Seasonal Variations in Degree of Treatment.—The preceding discussions are based on year-round treatment by the various processes. In certain situations, either because of variation in pollution, dilution capacity requirements, and possibly for other reasons, the necessity of a high degree of treatment is seasonal in character. Preliminary treatment may suffice for a

TABLE 16.—Construction Costs and Total Annual Charges of Various Treatment Processes: Secondary Treatment for Various Periods of Time; Plant Capacity 100 Million Gallons Daily

No.	Type	(a) General				(b) Total Operation and Maintenance, with Secondary Treatment for:			(c) Total Annual Charges, with Secondary Treatment for:		
		First cost, in thousands of dollars	Fixed charges @ 6%, in thousands of dollars	Percentage Reduction: Suspended solids	Percentage Reduction: B.O.D	12 months	6 months	2 months	12 months	6 months	2 months
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
1	Plain sedimentation	2 500	150	56	36	135	135	135	285	285	285
2	Trickling filters	10 700	642	84	85	215	175	148	857	817	790
3	Activated sludge	6 900	414	91	92	610	375	215	1 024	789	629
4	Chemical Treatment; Cost of Chemicals per Million Gallons:	2 900	174	..	..	....	....	....	....	....	....
4(*a*)	$4.00	....	....	73	56	335	235	170	509	409	324
4(*b*)	8.00	....	....	83	69	485	310	195	659	484	369
4(*c*)	12.00	....	....	89	79	630	385	220	804	559	394
4(*d*)	14.00	....	....	91	82	700	420	230	874	594	404

part of the time, but for certain critical periods a higher degree of treatment is necessary. For such a condition, depending upon the percentage of time that such secondary treatment is necessary, it may be stated as a general principle that treatment processes with low initial costs and higher, proportionately, operation costs are the most economical. It is clearly evident, for example, that in a case in which a secondary treatment is indicated for 10% of the time

(as in Minneapolis and St. Paul), a process with high fixed charges on construction is definitely uneconomical as compared with a process with low first costs and proportionately higher operation costs for a short period of time. Data indicative of this variation for a plant of 100 mgd are shown in Table 16. With secondary treatment for twelve months, chemical treatment (at least with lower chemical additions) is the most economical, with trickling filters next and activated sludge most expensive. On a basis of two months' secondary treatment, however, chemical treatment is still more economical, activated sludge is next in cost, and trickling filters the most expensive. Here, again, the effect of fixed charges on construction on the economics is clearly demonstrated. Sedimentation during the entire year is included in the foregoing estimates, and no allowance is made for the maintenance of key men for the secondary treatment processes on the payroll, or for the periodic operation and checking of equipment as may be necessary in a number of cases. This has been done in a later estimate applicable to the Minneapolis-St. Paul situation.

The degree of treatment necessary during critical dilution conditions here, again, plays an important part in the determination of the type of treatment.

Effect of Commodities Cost.—As has already been mentioned construction costs vary under different conditions. In addition, operation and maintenance costs may vary depending upon cost of power, chemicals, labor, etc. In some cases, this variation may assume controlling importance in the selection. The effect of power production from sludge gas may be an important factor, especially in the selection of the type of sludge disposal. For reasons clear to the reader, especially the large variation possible in these factors and their relative effect on the various processes, the effect of commodity cost variation on the economics of treatment will not be discussed in detail, but has been mentioned in view of its important effect, in some cases, in the selection of the economical treatment process.

Effect of Hydraulic Head Available.—In certain instances, circumstances may be such as to make certain processes uneconomical when consideration is given to the hydraulic head available for operation of the treatment plant. As an example, the head necessary for operation of trickling filters may not be available in a specific problem in which case pumping of the sewage may be necessary. Even if sufficient head is available, its value in reducing the cost of intercepting sewers which would be possible in connection with other processes must be considered in the analysis. For example, in connection with the Minneapolis-St. Paul project it was determined that every foot of head saved in the plant was worth approximately $50 000 in reducing sewer sizes, when applied to the intercepting sewer system.

Effect of Design Features.—A rather wide range in the construction cost of a treatment plant can be expected as affected by a number of design features, such as provisions made for: Odor control, flexibility and dependability of operation, mechanical and automatic operation of equipment, etc. The personal opinions of the designer with respect to the conservativeness and the care with which the structural design is made, may affect the comparison between the processes. Furthermore, as knowledge has advanced (as it very markedly has during the 10 years, 1928–1938), economies have been affected in

both design and operation. In the activated sludge process for example, the commonly accepted width of aeration tanks until 1930 was about 15 to 18 ft, whereas in recent years widths as great as 30 ft and more have been adopted, resulting in reduced structure costs and in air equipment and requirements. Similar economies have been, and will continue to be, made in other processes of sewage treatment.

Miscellaneous Factors Affecting the Comparison.—The writer has attempted to mention some factors of general application that affect the comparison of the economics of various treatment processes, but has by no means covered all such factors. The practical consideration that funds for operation are ordinarily difficult to procure may be considered sufficient justification in certain cases to affect a theoretical selection, and lead to the choosing of a process costing comparatively more to construct with attendant lower operation and maintenance costs. The method of sludge disposal selected or determined to be economical should be considered, together with the type of treatment, and, in certain cases, may exert an important influence on the selection. The size of the plant constructed as related to present requirements is a factor to be considered in the comparison. In special cases climatic and foundation conditions may have an influencing or even controlling effect on the selection. For various reasons all such factors cannot be covered in a paper of this nature and, therefore, the writer wishes to point out summarily that the generalizations made herein must be considered in the light at least of the major factors that affect economics of sewage treatment.

An idea of the effect of the various factors may be gained from a consideration of the substantiating data on construction and operation costs. Considering first construction cost: Of the 71 plants for which substantiating data are available 53, or 75%, are within a range of 20% above and below the estimated curves of costs shown in the curves. Of the 62 plants in which operation and maintenance costs are shown, 38, or 61%, are within the same range. Although this would indicate a fair substantiation of the estimates prepared for average conditions, it also serves to reveal the effect of a large number of factors that affect the costs in individual cases.

Economics of Various Processes as Applied to Minneapolis-St. Paul Project.—As a result of the relatively long period available for preliminary investigations in the "Twin City" project, both by the Metropolitan Drainage Commission over a period of five years, and the Minneapolis-St. Paul Sanitary District for two years, an unusual opportunity was presented for a careful determination of the relative economy of various processes and degrees of treatment. In various stages of the investigations, sedimentation, trickling filters, activated sludge, and chemical treatment were considered progressively. It was determined early that a high degree of secondary treatment was not necessary, and that preliminary treatment would suffice for all but a relatively small proportion of the time. The almost continuous record of river conditions over a 9-yr period was an invaluable aid in these determinations.

As a result of the investigations it was determined that preliminary treatment represented by sedimentation should be provided during the entire year, and that provision should be made to accomplish a reduction of as much

as 70% of B.O.D. during certain critical periods estimated to be approximately 10% of the time. For periods of ten to fifteen years no secondary treatment whatever may be necessary, based on present river standards. The estimated costs as determined for the "Twin City" project are shown in Table 17. The

TABLE 17.—Estimated Construction Costs and Total Annual Charges of Various Types of Treatment Plants for Minneapolis-St. Paul, Minnesota

Treatment No.	Description of Treatment in Terms of Million Gallons Daily Treated: Chemical	Sedimentation	Activated sludge	Percentage reduction in five-day biochemical oxygen demand	Construction cost	Total Annual Charges (Fixed Charges at 6%), with Secondary Treatment for: Two months	Six months	Twelve months
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1...	134	....	..	70	$3 376 000	$521 100	$659 100	$937 600
2...	94	40		70	5 201 800	625 600	748 600	976 900
3...	....	134	84	70	6 149 700	677 900	747 900	902 000
4...	....	134	..	33⅓	2 796 300	413 000	413 000	413 000
5...	....	134		92	7 342 000	767 800	845 400	1 032 000

estimates indicate the relative lower first cost of the chemical treatment process as compared with the activated sludge process. Based on year-round treatment the total annual charges of the activated sludge process (Treatment No. 3, Table 17) is $35 000 less expensive than the chemical treatment process (Treatment No. 1, Table 17). However, on a two-months basis (which is more than is estimated as necessary) chemical treatment will cost about $155 000 less annually than activated sludge treatment. Under different conditions the effect of the various influences previously mentioned may be such as to alter the determination.

Summary and Conclusion

In an attempt to arrive at a solution of some of the economic problems involved in sewage treatment, the writer has presented data on the construction, operation, and maintenance costs, and reductions accomplished by some of the basic processes of sewage treatment, and has attempted to draw attention to certain factors which, in a particular situation, may alter such generalizations which of necessity have been made herein. Certain processes and combinations of processes have not been discussed because of space limitation and the writer wishes expressly to emphasize that this has not been done because such processes are not, in his opinion, of economic importance.

In conclusion, it should be stated: (1) That, although the writer has attacked the problem in a general manner, the effect of influencing factors in affecting conclusions relative to a definite project must be considered; (2) that in the writer's opinion the effect of such factors dictates that almost every project be analyzed separately; (3) that the fundamental and controlling economic factor is the degree of treatment necessary in a particular project; (4) that sufficient and well proved processes are available to satisfy, economically, the require-

ments of almost any problem that may arise; and (5) that the advances in the last five years, and the improvements that are certain to occur in the future, are of such magnitude as to require careful study in each project in order to reach an economical solution.

Acknowledgments

Because it has been found impractical to acknowledge information and data furnished by others at its particular place of use, the writer wishes, collectively and summarily, to acknowledge his indebtedness for all such data used in this analysis. Data have been extracted from the engineering publications on the subject; they have been secured by direct visits to a number of treatment works; and by correspondence with the officials in charge, for all of which the writer expresses his gratefulness.

Special acknowledgment is due to the Metropolitan Drainage Commission and to Mr. Childs, its former Chief Engineer, whose assistance in connection with the preliminary studies, is noteworthy; and to the Board of Trustees of the Minneapolis-St. Paul District (Mr. C. C. Wilbur, Chief Engineer), for information relative to the "Twin City" project used in this paper.

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PAPERS

DEOXYGENATION AND REOXYGENATION

By C. J. Velz,[1] Assoc. M. Am. Soc. C. E.

Synopsis

Too frequently, sewage disposal is thought of as synonymous with discharge n to the nearest body of water. Such is not the case. Ultimate disposal (that is, final oxygen equilibrium) is inescapable and if it does not occur, at least partly, before the effluent is discharged, the sewage becomes a stream liability. The public is fast becoming impressed with the fact that it must be concerned with this problem beyond the sewer outlets. There is a general awakening and a demand that something be done about the menace and nuisance of stream pollution.

With advances in the knowledge and art of treatment permitting the reduction of almost any waste burden, the remedial problem is largely one of procedure and financing. If economy in efforts and funds were of no consequence, the problem could be solved simply by the invariable application of the highest and most elaborate treatment known. A sound and economical solution, however, requires an evaluation of pollution liabilities and stream assets, in advance of building, to permit the determination of the effect upon the stream of any proposed remedial program.

In design it is highly important to give careful consideration to plant details and mechanical efficiency, but in the enthusiasm over the development of the "means" one should not lose sight of the "end." First and foremost in importance is the effect of the design on the stream.

A technique for stream analysis is presented in this paper which is easy and rapid to apply and should serve as an aid in determining where, when, and how much to build, and what degree of treatment to provide.

Notation.—The introduction of some change in letter symbols is due to an effort to avoid conflict with standard symbols in allied fields of study.

Note.—Presented at the meeting of the Sanitary Engineering Division, New York, N. Y., January 21, 1937. Written comments are invited for immediate publication; to ensure publication the last discussion should be submitted by **August 15, 1938.**

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Introduction

Nuisance in streams is a combination of noxious conditions. Although no single measure fully reflects all these conditions, oxygen content of the stream is the most inclusive and best single index of the degree of pollution. Deoxygenation is a complex bio-chemical process influenced radically by time, temperature, and the nature of waste. Reoxygenation, no less complex, is primarily a chemical process, influenced radically by concentration of dissolved oxygen, time, temperature, depth, and turnover. Professor Earle B. Phelps and H. W. Streeter, M. Am. Soc. C. E., have integrated these factors into a single expression for resultant oxygen, as defined by the well-known formula:[2]

$$O_t = \frac{k\, O_{0d}}{K - k}\left(e^{-kt} - e^{-Kt}\right) + O_0\, e^{-Kt} \qquad (1)$$

in which: O_0 = initial dissolved oxygen saturation deficit of the water, in parts per million; O_t = saturation deficit, in parts per million, after time, t; O_{0d} = initial oxygen demand of the organic matter of the water, in parts per million; k = coefficient defining rate of deoxygenation; K = coefficient defining rate of reoxygenation; t = elapse of time, in days; and e = Naperian base.

An integrated formula such as Equation (1) has many advantages but, owing to its complexity and the many factors involved, is subject to the dangers of misunderstanding and misapplication. In certain instances, a better understanding and wider application of the fundamentals can be obtained by breaking down these complexities and considering deoxygenation and reoxygenation separately.

Reoxygenation

The most important asset of a stream is the natural process of reoxygenation—that is, its absorption of oxygen from the atmosphere. Without it the entire world would long ago have been faced with complete treatment, or sewage everywhere would have to be disposed in deep sea water. Fortunately, reoxygenation is a gift of Nature. It is important in the pollution problem that this natural asset be evaluated because economics demands that it be used wisely.

As early as 1910, in his studies of the pollution of New York Harbor, Professor Phelps[3] established a fundamental basis for an evaluation. He considered reoxygenation as a separate phenomenon governed by two fundamental laws: (1) The law of rate of solution; and (2) the law of diffusion. The rate of absorption is inversely proportional to the concentration of dissolved oxygen; and the rate of diffusion through water and between two proximate points is proportional to the difference in concentration at these two points. With these two fundamental laws and his experimental determination of the coefficient of diffusion in quiescent water, Professor Phelps developed the formula for absorption of oxygen:

$$O_t' = 100 - \left(1 - \frac{O_{0d}'}{100}\right) \times 81.06\left(0.779^{6} + \frac{0.105^{c}}{9} + \frac{0.0019^{c}}{25}\cdots\right) \ldots (2)$$

[2] "A Study of the Pollution and Natural Purification of the Ohio River," U. S. Public Health Service, *Public Health Bulletin No. 146*.

[3] "Location of Sewer Outlets and Discharge of Sewage into New York Harbor," Rept. by the late Maj.-Gen. William Black, U. S. Army (*Retired*), and Prof. Phelps, 1910.

in which O_{0d}' = initial dissolved oxygen expressed as the average content in percentage of the saturation value; O_t' = final dissolved oxygen expressed as the average content in percentage of the saturation value; and c = a constant for each set of conditions and equal to,

$$c = \frac{\pi^2 \, a \, t}{4 \, d^2} \quad \text{(3)}$$

in which t = time of exposure, in hours; d = depth of water, in centimeters; and, a = a diffusion coefficient which, in turn, is a function of temperature.

From an analysis of Professor Phelps' experimental determination of diffusion coefficient, the writer develops the relationship between the coefficient and temperature represented by the formulas:

$$\log a = 0.04125 \, T - 0.672 \quad \text{(4)}$$

or,

$$a = 1.42 \times 1.1^{(T-20)} \quad \text{(5)}$$

in which T = temperature, in degrees Centigrade. It is apparent from the graphical representation shown in Fig 1 that there is linear relationship between the log of the coefficient of diffusion and temperature.

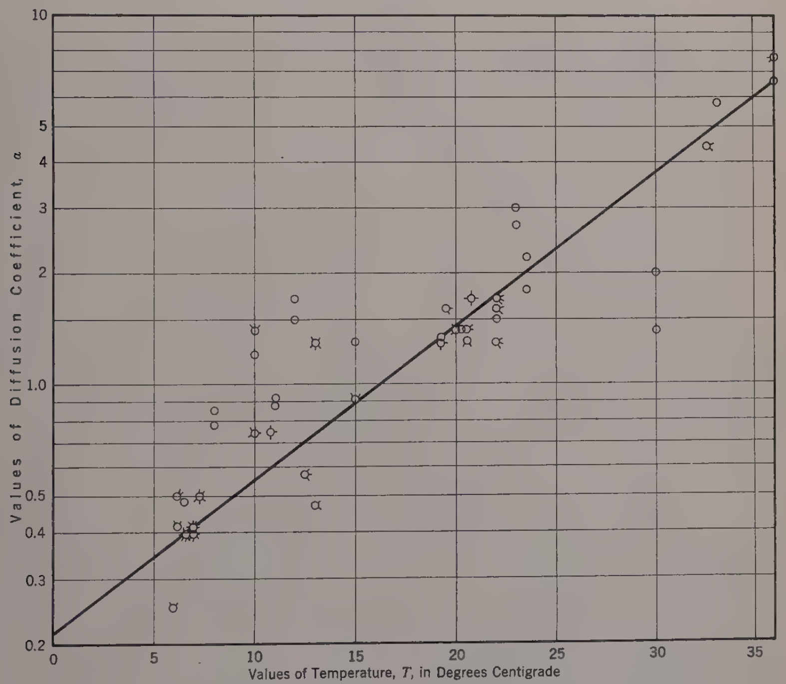

Fig. 1.—Relationship Between Diffusion Coefficient and Temperature (Degrees Centigrade), Based on Phelps' Experimental Data

The fundamental relationships established mathematically and experimentally are applicable to actual stream conditions. For example, consider a flowing stream: The integration of the average oxygen content of a stretch of

the stream is analogous to mixing the entire volume prior to exposure to the atmosphere. To maintain the integrated average oxygen content at a constant value (that is, along given profiles and contours) by the equilibrium established between deoxygenation and reoxygenation, is analogous to considering the body of water quiescent in so far as movement on a horizontal plane is concerned. As long as that equilibrium is maintained the initial oxygen deficit remains constant, deoxygenation factors utilizing oxygen just as fast as, but no faster than, the water absorbs oxygen from the atmosphere. Under such conditions the quantity of oxygen absorbed in any given time is dependent only upon the frequency of vertical mixing—the stream turnover. Thus, by substituting the factors in Equation (2), using time of exposure, t, as the interval between vertical mixes, the oxygen absorbed by the river stretch is obtained. Under the conditions of equilibrium maintained, the same quantity will be absorbed in each succeeding mixing interval. Hence, Professor Phelps' integrated equation expressing total quantity, in actual application becomes a differential equation expressing rate of absorption. The only factor not readily measurable in the stream is frequency of vertical mixing, which, for the most streams, falls between relatively narrow limits.

Equation (2) permits direct evaluation of the influence of depth, which, as is apparent from Equation (3), is a dominating factor. In Equation (1) all reoxygenation influences are combined into a single constant, K. The comprehensiveness of K is likely to lead to error when it is applied to streams other than the one for which it was evaluated, whereas, in Equation (2) cognizance is taken of the particular physical characteristics of each stream. Furthermore, Equation (1) is limited in application to inland waters. The complexities of tide-water flow which preclude the application of Equation (1) to brackish water, have been indicated in a paper on the pollution of New York Harbor written in 1932.[4] In practical application, the disadvantage of Equation (2) is the mathematics necessary to solve it. A large number of terms of the converging series, involving Exponent c must be used in order to obtain accuracy (40 to 60 terms, when the value of c is small). The writer has developed a graphical means of solution from which oxygen absorption, under any set of conditions, can be obtained accurately at a glance. By mathematics the process would require much painstaking effort.

Standard Reoxygenation Curve

From Equation (2) it is apparent that oxygen absorption is inversely proportional to initial concentration, or directly proportional to oxygen deficit $\left(1 - \frac{O_{0d}'}{100}\right)$. Furthermore, it is a function of the summation of a converging series involving time, depth, and a specific diffusion coefficient as an exponent, c, in each term of the series. By making O_{0d}' equal to zero (beginning with water devoid of oxygen), the summation of the series,

$$\Sigma s = 81.06\left(0.779^c + \frac{0.105^c}{9} + \frac{0.0019^c}{25} \cdots\right) \ldots\ldots\ldots\ldots (6)$$

represents the deficit at any time; or the deficit becomes a function of c.

[4] "Report on the Pollution of New York Harbor," by Earle B. Phelps and C. J. Velz, *Sewage Works Journal*, January, 1933.

A solution of the series for various arbitrary values of c provides a set of control points indicating the relationship between c and the deficit which, when plotted on natural scales, produces Fig. 2. The section of the curve most frequently applicable to actual stream conditions lies between values of c from

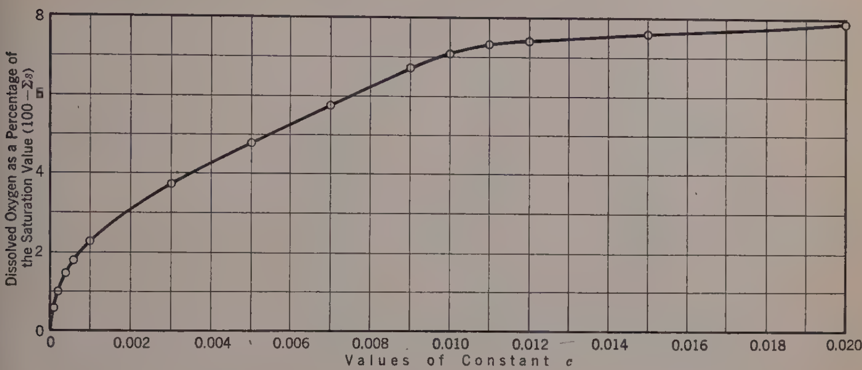

FIG. 2

0.000001 to 0.001, for which accurate interpolation is not permitted by Fig. 2. In order to emphasize the range of the curve and particularly to provide a convenient means of evaluating c in terms of the factors by which it is influenced, the writer has developed the graphical representation shown in Fig. 3.[4a] It is at once apparent from this chart that for the most useful range of the curve there is linear relationship between the dissolved oxygen and the log of c, a very convenient relationship for interpolation and for conversion

In the evaluation of c in Equation (3), in terms of time, depth, and diffusion coefficient, including temperature, the writer has used the characteristics of a logarithmic scale adaptable in the principle of a slide-rule: Constant c is inversely proportional to the square of the depth, d, and directly proportiona to time, t, and the diffusion coefficient, a. The diffusion coefficient, a, is a function of temperature as defined by Equation (5); and, since c is directly proportional to a, for each unit change in temperature, T, a fixed rate of change occurs in c. The use of the standard reoxygenation curve is illustrated by examples.

Example 1.—Given a stretch of stream in which the average depth, d, is 5 ft; the average temperature, T, is 24° C; the dissolved oxygen profile of the stretch is maintained constant at an integrated average of 40% of the saturation value; and the flow conditions were such as to produce a time of mix, t, of 15 min.

Referring to Fig. 3: (1) Opposite 5 ft on Scale (4) read 0.000151 on Scale (3); (2) slide the control point of Scale (8) to coincide with 0.000151 on Scale (3); (3) opposite $\frac{1}{4}$ hr on Scale (8) read 0.000038 on Scale (3); (4) slide the control point of Scale (7) to coincide with 0.000038 on Scale (3); (5) opposite 24° on

[4a] Double-sized copies of Fig. 3 and Fig. 4 can be secured for 25 cents per set by addressing the Secretary of the Society, 33 West 39th Street, New York, N. Y.

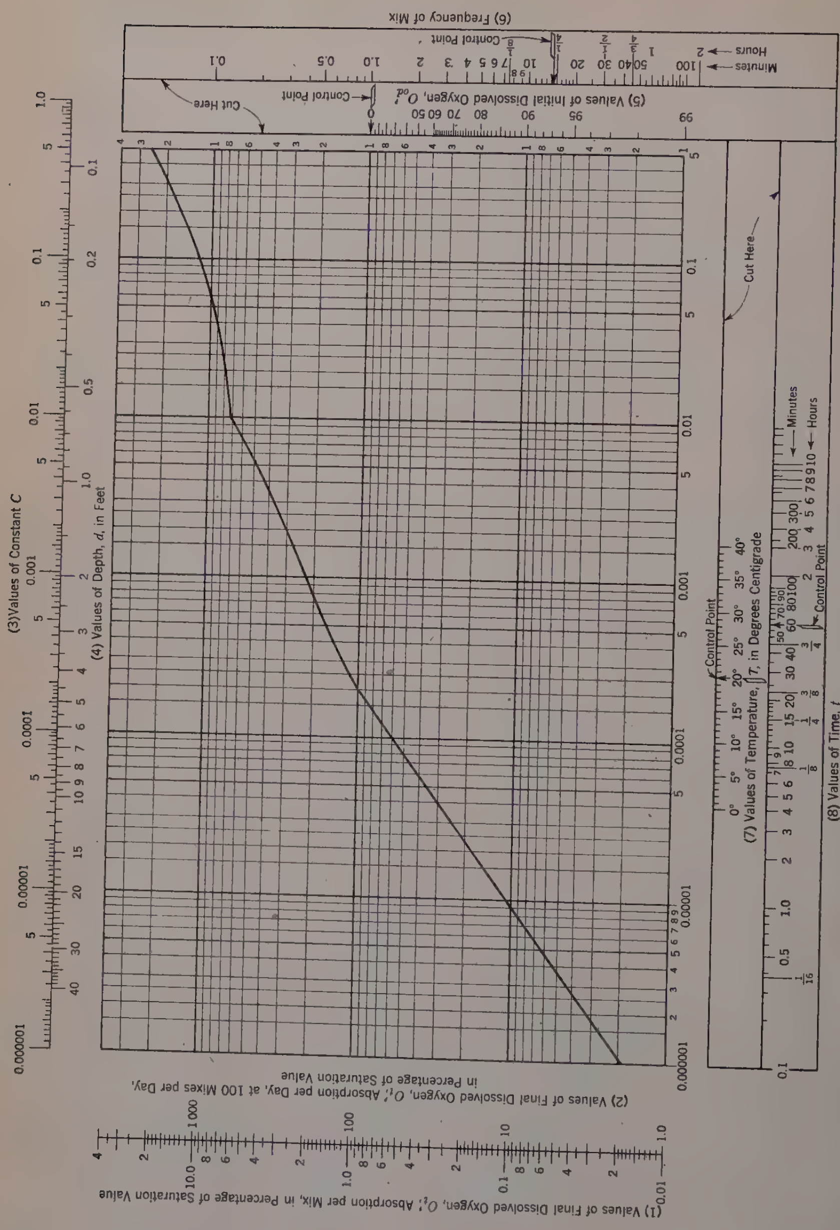

FIG. 3.—STANDARD REOXYGENATION CURVE, BASED ON BLACK AND PHELPS' FORMULA

Scale (7) read 0.000055 on Scale (3) (this gives the value of the constant, c, at a depth of 5 ft, a mixing time of $\frac{1}{4}$ hr, and 24° C); (6) descend vertically from 0.000055 on Scale (3) to an intersection with the standard reoxygenation curve; (7) extend horizontally to Scale (1) and read 0.38% (the quantity absorbed under the conditions in one mix period if water were initially devoid of oxygen); (8), slide the control point of Scale (5) to coincide with 0.38% on Scale (1), and opposite 40% on Scale (5) read 0.228% on Scale (1) (this represents the percentage of oxygen absorbed in each $\frac{1}{4}$-hr interval between mixes under the given conditions); and (9) to convert the result of Step (8) to a percentage of saturation absorbed per day, slide the control point of Scale (6) to coincide with 0.228% on Scale (1), and opposite $\frac{1}{4}$ hr on Scale (6) read 22.0% per day on Scale (2). Thus, under the conditions given, and as long as the oxygen balance remains constant at an average of 40% of saturation, this body of water will absorb oxygen each day equal in amount to 22% of its saturation value.

Some workers prefer to express reoxygenation in terms of pounds of oxygen absorbed per million gallons per day. A convenient graphical conversion from the "percentage of saturation" value absorbed per day to "pounds per million gallons per day" is readily effected by utilizing Fig. 4, which the writer constructed by adapting the principles that govern the development of Fig. 3.

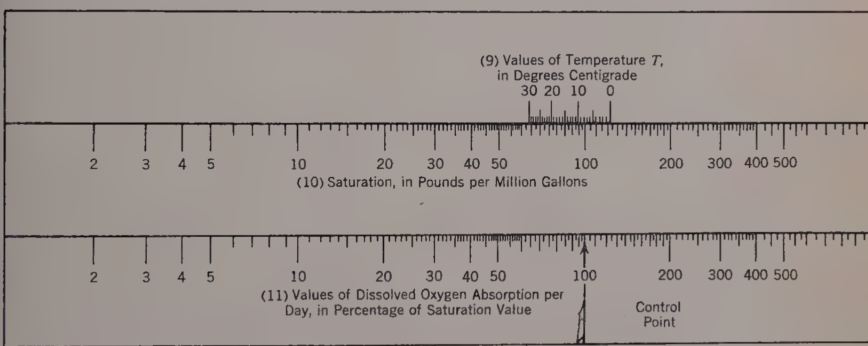

FIG. 4.—CONVERSION OF PERCENTAGE OF SATURATION VALUE PER DAY TO POUNDS PER MILLION GALLONS PER DAY

Example 2.—Assume the same conditions as in Example 1. Opposite 24° C on Scale (9) read 71.1 lb per million gallons (the saturation value at 24° C) on Scale (10); then slide the "control point" of Scale (11) to coincide with 71.1 lb per million gallons, on Scale (10); then opposite 22.0% per day, on Scale (11), read 15.6 on Scale (10), which represents the conversion from 22% of saturation value per day into pounds per million gallons per day.

Reoxygenation temperature has a double influence, first in the rate of diffusion and again in the limit of the saturation value, with opposite effects which, however, do not necessarily balance each other. Increase in temperature increases the rate of diffusion and, consequently, the rate of absorption, but the limit of quantity absorbed (namely, the saturation), decreases with increase in temperature.

Verification of Equation (2)

An excellent check on Equation (2) and Fig. 3 is afforded by the experiments reported by H. W. Streeter and Chilton Wright, Members, Am. Soc. C. E., and Mr. R. W. Kehr in 1936.[5] By applying depth, time, and temperature values from these experiments to Fig. 3, a close agreement is noted between the dissolved oxygen computed and that observed in the experiments. Furthermore, if curves in the foregoing report[6] are replotted on a basis of percentage of saturation, beginning with initial oxygen devoidance, the time of turn-over, or frequency of mix, is obtained at the point where the dissolved oxygen coincides with that obtained by Fig. 3 at the same time interval.

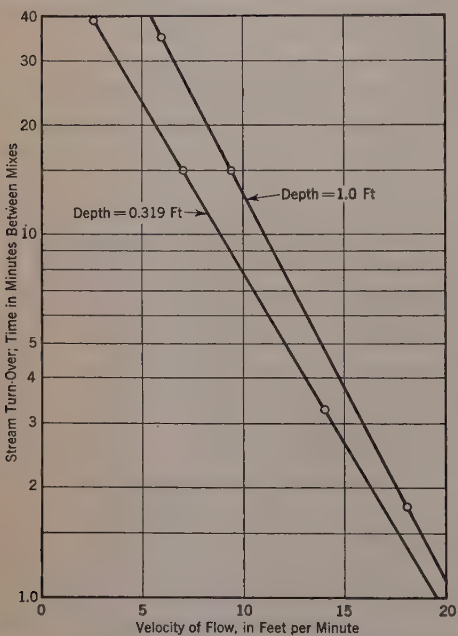

Fig. 5.—Relationship Between Stream Turn-Over, Velocity, and Depth

The frequencies of mix computed by this method, at depths of 0.318 ft and 1.0 ft, indicate a relationship with the velocity shown in Fig. 5. It will be noted that the logarithm of the frequency of mix bears a linear relationship to velocity and, logically, as depth increases, the frequency of mix diminishes. Further controlled experiments extended to depths normally encountered in streams will doubtless lead to a set of curves similar to Fig. 5. In the absence of such a set of curves the writer has obtained satisfactory checks against observed dissolved oxygen profiles by using an average frequency of $\frac{1}{4}$ hr per mix, ranging from $\frac{1}{8}$ hr for shallow, rapidly flowing streams to $\frac{1}{2}$ hr for deep, slowly flowing streams.

The experimental confirmation of Equation (2) and Fig. 3 under controlled conditions of flow and the close agreement between computed and observed dissolved oxygen values when applied to actual stream conditions, substantiate Professor Phelps' original fundamental assumptions. It is hoped that the advantages of simple, direct evaluation of reoxygenation for both inland and tidal waters will assist in routine stream-analysis technique as applied in the design of pollution remedial work.

Deoxygenation

The major pollution liability in streams is deoxygenation resulting from the satisfaction of the natural oxygen demand of organic waste matter. In

[5] *Sewage Works Journal*, March, 1936.

[6] *Loc. cit.*, p. 298.

the expression of his law, namely, that "the rate of biochemical oxidation of organic matter is proportional to the remaining concentration of unoxidized substance measured in terms of oxidizability," Professor Phelps changed the concept of pollution from that of a simple, static, chemical dilution phenomenon to one of changing, bio-chemical complexity. The law has been checked repeatedly by experiment and has been confirmed in actual observation of stream pollution studies; and can be incorporated, safely and advantageously, as routine technique in the design of remedial works. It is recognized that there are modifying elements, such as immediate demand, nitrification stage

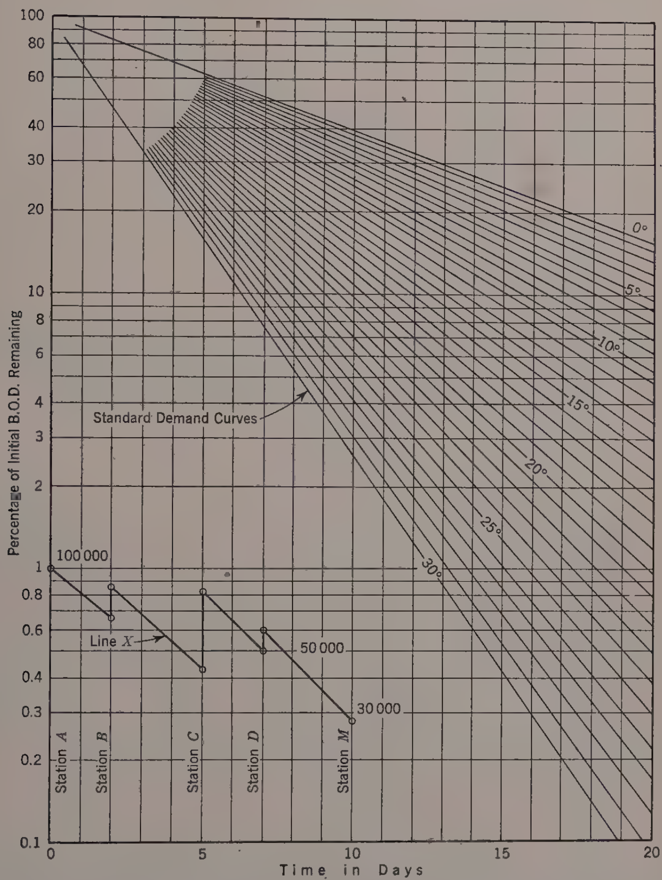

Fig. 6.—Relationship Between Bio-Chemical Oxygen Demand and Time at Temperatures, 0°C to 30°C

demand, sedimentation, and scour, which are highly important in a theoretical scientific analysis; but, in so far as routine technique for the design of remedial works is concerned, until such time as these refinements are firmly established, the writer prefers to include them in the factor of safety used in design.

The technical literature of the ten years, 1927–1937, is replete with discussion of the fundamentals of bio-chemical oxidation, but these fundamentals are not so widely used in actual practice. As a stimulus to their further application to practical problems, the writer presents a convenient graphical representation of the well-known time-temperature functions. By plotting the logarithm of oxygen demand expressed as a percentage of the total initial demand against time, for rates at temperatures from 0° C to 30° C, the nest of Standard Demand Curves shown on Fig. 6 is obtained. These curves signify that, at a given temperature in successive intervals of time, the proportion of the demand satisfied is always a constant percentage of that remaining at the end of the previous interval—a very convenient relationship. The influence of temperature upon the rate of oxidation (by coincidence) is similar to that determining the rate of diffusion in reoxygenation; namely, the log of the rate is directly proportional to temperature. With the logarithmic slopes of the standard demand curves known, and with the distance between stations expressed in time of passage, the temperature of the water and the quantities of pollution contributed (expressed in any convenient term such as population equivalents), at each station, known, it is a simple graphical operation to integrate the resultant demand and determine the relative responsibility of polluters at any place in the course of the stream.

Line *X*, Fig. 6, represents such a resultant demand curve for the hypothetical conditions given in Table 1. This technique for integrating resultant demand affords a rapid, direct means of evaluating deoxygenation, which is the "load factor" in the design of projects to remedy pollution problems.

TABLE 1

Station	Mean temperature of water between stations, in degrees Centigrade	Time of Passage in Days		Pollution added at station, expressed in population equivalents
		Mean time between stations	Total time from Station A	
(1)	(2)	(3)	(4)	(5)
A	18	2.0	0.0	100 000
B	20	3.0	2.0	20 000
C	22	2.0	5.0	40 000
D	22	3.0	7.0	10 000
M*	....	....	10.0	0

* M = Mouth.

Oxygen Balance

Oxygen balance is the algebraic summation of satisfied demands of pollution and stream assets. A scientific guide in planning measures to correct stream pollution is afforded in a series of oxygen-balance profiles computed for variations in the three major factors: Predicted pollution loads, probable stream flows, and proposed combinations of remedial plans. Evaluation of the relative effect of these factors is essential to safeguard natural resources and to insure the greatest return from remedial works with the least expenditure of funds. For the convenience of the computer the writer has indicated an oxygen balance procedure in Tables 2, 3, and 4.

Variation in pollutional load is a function of population and, except for unusual concentrations of industrial waste, is directly proportional to it. How much to build now, how much excess capitalization to provide for growth, when to add units, or to increase degree of treatment—these questions depend

TABLE 2.—FORM FOR ANALYZING THE PHYSICAL CHARACTERISTICS OF A STREAM

(At —% Chance Flow, — Cubic Feet per Second per Square Mile)

Station (location of major sources of pollution and dilution) (1)	Drainage area tributary to station, in square miles (2)	Run-off, at the station, in cubic feet per second (3)	Mean depth between stations, in feet (4)	River volume between stations, in cubic feet (5)	Time of passage, in days, between stations (computed from volume displacement) (6)	Estimated frequency of stream turnover; in hours per mix (7)	Mean temperature of water between stations, in degrees Centigrade (8)

TABLE 3.—FORM FOR ANALYZING DEOXYGENATION

(At —% Chance Flow, — Cubic Feet per Second per Square Mile and Predicted Load as of ——— (Year))

Station (1)	Mean temperature of water, in degrees Centigrade (2)	Time of passage, in days from initial station (3)	Bio-Chemical Oxygen Demand, in Population Equivalents*: Tributary to station (no treatment) (4)	Removed by treatment prior to discharge (5)	Discharged into stream at station (6)	Residual from stations up stream (7)	Total at station (tributary residual plus residual from up stream) (8)	Satisfied in stream between stations (9)	Total satisfied above station (10)

* Bio-chemical oxygen demand of industrial and municipal waste expressed as population equivalents (0.22 lb per day per person).

to a large extent upon the prediction of future loads. A well co-ordinated long-range program permits a more conservative allowance for these factors in the basis of design. Restriction of immigration increase in the average age of the population, a declining birth rate, and redistribution of population, are

retarding influences tending toward stabilization in numbers, particularly in urban centers where the disposal problem is most acute.

Variation in stream flow is a paramount factor in oxygen balance by virtue of its influence on volume and, more important, on the element of

TABLE 4.—Form for Analyzing Reoxygenation and Oxygen Balance (At —% Chance Flow, — Cubic Feet per Second per Square Mile and Predicted Load as of ———) (Year)

Station	Run-off at station, in cubic feet per second	Mean temperature, in degrees Centigrade	Mean depth, in feet	Frequency of turnover, in hours per mix	River volume, in cubic feet	Total demand satisfied above station, in population equivalents	Dissolved Oxygen, in Population Equivalents*								Percentage saturation at station (Column (15) divided by Column (10))
							Added by run-off between stations	Total added by run-off above station	Total run-off at station at saturation	Net, at station (Column (9) minus Column (7))	Added by reoxygenation between stations per million gallons†	Total, added by reoxygenation between stations	Total, added by reoxygenation above station	Net oxygen balance at station (Column (11) plus Column (14))	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	12)	(13)	(14)	(15)	(16)

* Dissolved oxygen expressed in population equivalents (0.22 lb per day per person).

† Per million gallons per day from Fig. 2 and Fig. 3 converted to population equivalents.

time in deoxygenation and reoxygenation. Uncontrollable disturbance of equilibrium in Nature's oxygen balance is almost entirely associated with variations in the rate of stream flow. Of run-off so little is known that it is considered a chance phenomenon, and, consequently, it is evaluated by standard statistical methods, a determination of probability of occurrence based upon recorded past behavior. With the possible exception of winter when an ice cover closes the stream to reoxygenation, the coincidence in summer of high temperature and low run-off provides the critical stream condition. For most problems, therefore, an analysis of the summer monthly average run-off records is sufficiently accurate for purposes of design.

A convenient system for establishing the run-off probability curve is afforded by the graphical method developed by the late Allen Hazen, M. Am. Soc. C. E.[7] If a duration flow curve is made of monthly average run-offs, showing the relative frequencies of flows exceeding any given value, a curve of the logarithms of these flow values against percentage frequencies of occurrence

[7] "Flood Flows," by Allen Hazen, John Wiley & Sons, Inc., 1930.

plotted on probability paper approaches a straight line. Fig. 7 represents a typical probability curve. From such a curve the most probable minimum flow for any desired percentage chance of occurrence is readily obtained.

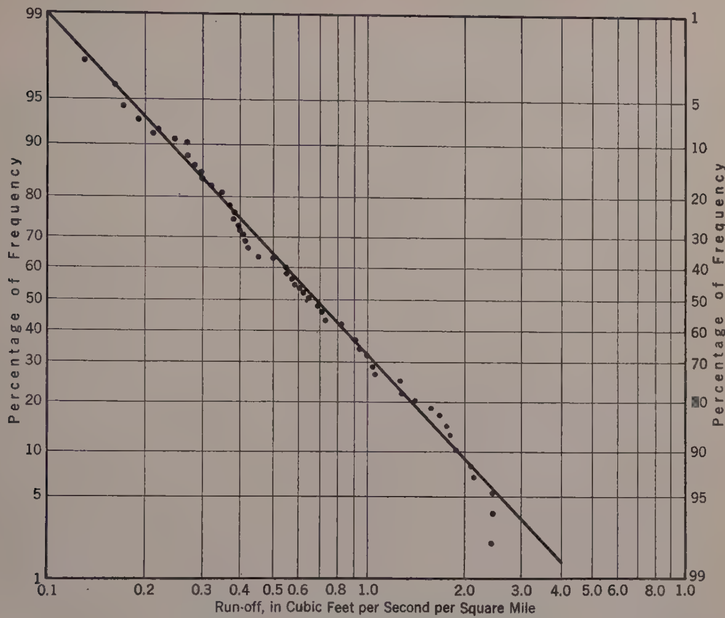

FIG. 7.—RUN-OFF PROBABILITY CURVE

Although any particular oxygen-balance profile is possible of reasonably exact determination from the fundamental laws governing deoxygenation and reoxygenation, load factors and run-off probabilities are not definitely predictable. However, by computing a series of oxygen-balance profiles over a range of loads and run-off, accurate limits of stream condition are defined. Obviously, these defined limits afford a foundation work from which to measure the necessity and effectiveness of any arrangement of treatment plants designed to relieve the stream of portions of the pollution load. The subjection of various remedial combinations to such evaluation insures the proper protection to the stream and to the taxpayer.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

THE PASSAGE OF TURBID WATER THROUGH LAKE MEAD

Discussion

By Messrs. R. E. Redden, and Raymond A. Hill

R. E. Redden,[46] Esq. (by letter).[46a]—The Engineering Profession should indeed be grateful to Messrs. Grover and Howard for the presentation of facts and conclusions relative to the transportation of silt through reservoirs and the worth-while suggestion that a study be made of the little understood phenomena. It is in answer to the authors' invitation for discussion and related information that the writer submits data and conclusions on the passage of a flood through Echo Reservoir in Utah.

On August 6, 1934, a mud flow produced by a heavy local storm which hit at Rockport, Utah, about thirteen miles up the Weber River from Echo Reservoir, entered that body of water at 10:30 P.M., and 10 hr later, after apparently following the river channel, passed through the needle-valve gates below the dam. The flood water passing through the gates was reddish-brown in color, very thick, with suspended matter, and gave off a putrid odor.

From the automatic water-stage recorders on the river, and from data gathered by witnesses (including the writer) average velocities for the flood in the river above the reservoir and through the reservoir have been computed. The average velocity of the flood water above the reservoir was 4.4 ft per sec., whereas the average velocity through the clear water in the reservoir was 0.22 ft per sec.

The greater specific gravity of the flood water, due to the character of the suspended matter, is ascribed for its not mixing with the clear water of the reservoir. The intensely putrid odor of the flood water as it poured into the stilling-basin from the gates, is attributed to the gases locked up in the mass of

Note.—The paper by Nathan C. Grover, M. Am. Soc. C. E., and Charles S. Howard, Esq., was published in April, 1937, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: June, 1937, by Messrs. O. A. Faris, Paul A. Jones, Carl E. Scofield, and Ivan E. Houk; September, 1937, by Messrs. William P. Creager, Harold K. Palmer, Morrough P. O'Brien, John C. Page, John H. Bliss, and B. H. Monish; October, 1937, by Messrs. D. M. Forester, A. D. Lewis, G. C. Dobson, and William W. Rubey; and November, 1937, by Messrs. J. C. Stevens, and C. S. Jarvis.

[46] Professional Engr. and Geologist, Coalville, Utah.

[46a] Received by the Secretary February 21, 1938.

suspended matter and is additional evidence that the two waters remained unmixed.

Floods are not uncommon in the mountainous valleys of Utah. They are of rather short duration, carry variable loads of débris and, in general, do no great damage; however, a few mud flows have taken human lives and caused considerable property damage. In the main, these mud flows have not been caused by the much over-worked word, "over-grazing." Burned over and over-grazed areas add to the intensity of the mud flow if centered within the local area of maximum precipitation. Field observations of mud flows suggest that they may occur on any steep area during a general or local storm where a localized belt within the general storm area receives a maximum precipitation in the form of hail, which is characteristic of flood-producing storms. Hail, as such, is not readily absorbed by the soil mantle; hence it piles up as temporary immobile precipitation, which is partly melted and lubricated by the "tapering-off" rain, a few minutes later, and rushes down the slopes, dragging with it a portion of the soil mantle to join together below in one mad rush down the local canyons.

Floods entering reservoirs built on streams in Utah have generally deposited their loads on the bottom of the reservoir; however, many instances are known in which the silt-laden waters have appeared at the outlet gates. These records show that this phenomenon is more noticeable on certain drainage areas, which gives rise to the question of the difference in the soil mantle, and which, in turn, is dependent upon the geology of the water-shed area.

It is expected that future studies will show that flood loads from certain formations of a particular geological age have a large influence upon the phenomenon of floods passing through reservoirs of clear water without intermixing.

Weber River and Echo Reservoir.—Weber River heads in the Uinta Mountains in Utah and drains an area of 732 sq miles above Echo Reservoir. It has a maximum discharge of about 4 000 cu ft per sec. The reservoir was built across the channel of Weber River and completed for a storage capacity of 74 000 acre-ft in 1930. On the day of the flood the reservoir held 4 100 acre-ft. The depth of the water at the dam and outlet was 30 ft.

During the spring run-off, the river, at times, carries considerable material in suspension. However, during these periods of high-water run-off, the water discharging from the needle-valve gates during the filling of the reservoir has remained clear, with few exceptions. Only part of the high water entering the reservoir is discharged for irrigation rights below the dam. The remainder is stored in the reservoir. The old river channel meanders over the floor of the reservoir and the length of the submerged channel, when the reservoir is at full stage, is about 4.5 miles. The temperature of the water discharging from the reservoir on July 26, 1934, ten days before the flood, was 60° F, whereas the temperature of the water in the reservoir, 0.5 ft below the surface, was 65° F. The maximum temperature of the atmosphere was 94° F, and the minimum was 44° F. No temperature tests were made on the day of the flood. During the late summer, when the reservoir is being emptied, the discharge water is slightly turbid.

Record of the Flood.—Between 3:30 and 3:50 P.M., on August 6, 1934, a typical thunder storm centered 1.5 miles west of the valley at Rockport. This heavy storm was part of a general storm which covered an area of about 100 sq miles. Floods came down eight short canyons west of the State highway and spread out over the narrow river valley. These mud flows lost their bed load before entering the Weber River, but retained their suspended load down the river and, for the most part, through the reservoir.

The distance from the center of the flood area on the Weber River to the U. S. Geological Survey water-stage recorder at Coalville, Utah, was 12 miles. The recorder showed the first abrupt rise at 8:05 P.M. The record of stream flow for 12 hr, beginning at 8:00 P.M., is as shown in Table 7.

TABLE 7.—Flood Run-Off, Weber River and Chalk Creek, Utah

August 6, 1934	Discharge, in Cubic Feet per Second		August 7, 1934	Discharge, in Cubic Feet per Second	
	Weber River, at Coalville, Utah	Chalk Creek		Weber River, at Coalville, Utah	Chalk Creek
8:00 P.M.	26	....	2:00 A.M.	73	9.0
8:05 P.M.	227	....	4:00 A.M.	63	....
9:00 P.M.	246	1.5	6:00 A.M.	56	....
9:30 P.M.	....	56.0	8:00 A.M.	49	....
10:00 P.M.	201	40.0	9:05 A.M.	33	....
12:00 P.M.	105	14.0	9:30 A.M.	....	3.0

The average velocity of the flood water in the river over the distance of 12 miles was 3 miles per hr, or at the rate of about 4.4 ft per sec. The distance covered by the river from the gaging station to the upper end of the reservoir was nearly 4 miles, making the time of the entry of the flood into the clear water of Echo Reservoir at 10:30 P.M.

A storm of similar intensity struck on Chalk Creek, 8 miles east of Coalville, a short time after the heavy storm at Rockport (see Table 7). Chalk Creek enters the Weber River 0.5 mile below the gage, making very little difference in the time at which both floods entered Echo Reservoir.

The thick muddy water entered the clear water of the reservoir at 10:30 P.M., and passed through the needle-valve gates below the dam at 8:30 A.M. The length of the submerged river channel was about 7 900 ft, which would establish an average velocity of 730 ft per hr, or nearly 0.22 ft per sec for the flood through the reservoir.

The quantity of flood water, as determined from the water-stage recorders, which flowed into the upper end of the reservoir before any of it appeared below the dam, was 102 acre-ft. A body of water 7 900 ft long by 110 ft wide by 5 ft deep would contain approximately 102 acre-ft. The channel of the Weber River before the reservoir was built would average very close to 110 ft wide and 5 ft deep. It seems not far wrong to state that the flood water followed the old river channel through the reservoir.

The color of the flood water above and below the reservoir was reddish-brown. When it was whorled out of the needle-valve gates and into the stilling-basin the violent agitation seemed to liberate a very putrid odor which

was stifling near the stilling-basin. It was noted that clay particles, which apparently had remained in suspension through the reservoir, settled out near the lower end of a protected section of the stilling-basin. Evidence was against these particles being in the stilling-basin before the flood water came through. The unlocking of the gas and the breaking up of whatever affinity these clay particles had for each other may shed some light on some of the factors affecting these phenomena.

Conclusions.—When the thunder-storm hit at Rockport, the downpour of hail and rain fell on soil weathered from the thin-bedded clay shales of the Upper Jurassic Age which are reddish-brown in color and contain a high percentage of colloidal clay. This fine material absorbs a large quantity of water and remains in suspension for many hours. The rolling, slipping action which the clay, sand, and coarse material received on its way down from the mountain sides and steep hollows, produced a mud flow having a maximum density of fine particles suspended in an emulsion of colloidal clay and water, which, in turn, supported the bed load of coarser material. Only the suspended load was carried down the river. The flood lost little of its load down the river, as was noted the following day. This dense liquid mass, more or less tied together, seemed to confine itself to the submerged river channel through the reservoir and failed to mix to any extent with the clear, less dense water, notwithstanding the marked decrease in the velocity of the flood water through the reservoir as compared to the velocity in the river above it.

Had the mud flow been composed of different materials, a product of a different geologic age, it would have lost its identity in the reservoir as some other floods have done.

The writer agrees with the authors that further study on the flow of water through reservoirs will be of practical significance with respect to engineering problems. It seems reasonable that some changes in design and position of the outlet works may be needed where culinary water is to be taken from reservoirs to be built on streams which are subject to seasonable floods.

Raymond A. Hill,[47] M. Am. Soc. C. E. (by letter).[47a]—Judging from similar conditions at Elephant Butte Reservoir on the Rio Grande, little is to be gained by the passage of silt through Lake Mead. From the former reservoir, water carrying large quantities of suspended matter has been discharged for several days in almost every year, but the total quantity of silt so removed is only a small part of the accumulation in the reservoir. Even if the life of Elephant Butte Reservoir thus becomes greater than would have been the case if no silt flows had occurred, it will make little difference whether the lost capacity is replaced, for example, in 1960, or a year or two later. Furthermore, only the finest particles are held in suspension and these tend to seal the soil when such water is used for irrigation. Likewise, to increase the quantity of silt that may naturally pass through Lake Mead would probably not be welcomed by irrigation water users, and definitely would be disadvantageous to those communities that will take their domestic supplies from the Colorado River.

[47] (Quinton, Code & Hill-Leeds & Barnard), Los Angeles, Calif.

[47a] Received by the Secretary March 2, 1938.

Although the passage of silt through Lake Mead, therefore, is of itself unimportant, the phenomena which produced such silt flows are important. The impression might be gained from the authors' presentation that water flowed through Lake Mead without mixing only at times when the discharge at Boulder Dam was turbid. Actually, it is evident from the chemical analyses of the water entering the reservoir and discharged from it that the flow of saline water through Lake Mead is of usual, rather than unusual, occurrence. The writer, therefore, would re-phrase the first conclusion of the authors to read: "The passage of water in considerable quantities through a reservoir without mixing has much more than scientific interest and importance."

TABLE 8.—QUALITY OF WATER, COLORADO RIVER

Period	Discharge, in acre-feet	Total Dissolved Solids		Percentages		Milligram Equivalents per Liter						U. S. Geological Survey Laboratory No.
		In parts per million	In tons per acre-foot	$Na + K$	$Cl + NO_3$	Ca	Mg	$Na + K$	$CO_3 + HCO_3$	SO_4	$Cl + NO_3$	
(a) At Grand Canyon Station												
June 1–10	985 000	335	0.46	30.6	14.0	2.60	1.23	1.69	2.52	2.14	0.75	14 616
11–20	1 680 000	289	0.39	26.2	11.8	2.60	0.90	1.24	2.62	1.62	0.55	14 661
21–30	1 320 000	243	0.33	26.3	12.4	2.20	0.77	1.06	2.18	1.35	0.48	14 662
July 1–10	726 000	281	0.38	30.5	16.9	2.25	0.99	1.42	2.07	1.77	0.76	14 663
11–20	451 000	378	0.51	34.9	18.8	2.75	1.15	2.09	2.34	2.44	1.09	14 695
21–31	338 000	571	0.78	37.2	19.3	3.79	1.73	3.27	2.72	4.43	1.70	14 696
August 1–10	214 000	692	0.94	41.8	23.6	4.14	1.97	4.40	2.90	5.27	2.48	14 697
11–20	179 000	834	1.13	44.6	22.9	5.14	2.22	5.91	3.52	6.56	2.97	14 805
21–31	188 000	1 099	1.49	37.7	20.0	6.89	3.45	6.25	3.57	9.66	3.27	14 806
September 1–10	178 000	1 173	1.59	39.7	19.4	7.49	3.21	7.04	3.66	10.85	3.46	14 871
11–20	124 000	1 124	1.53	42.4	22.8	6.49	3.37	7.26	3.29	9.95	3.87	14 872
21–30	152 000	1 107	1.50	42.6	23.7	6.14	3.37	7.06	3.28	9.54	3.94	14 873
October 1–10	172 000	1 184	1.61	45.8	20.3	6.69	3.13	8.30	3.75	10.56	3.62	14 976
11–20	103 000	1 114	1.51	43.6	25.0	6.19	3.54	7.50	3.39	9.43	4.21	14 977
21–31	110 000	1 232	1.67	45.0	26.3	6.44	4.03	8.55	3.54	10.33	4.89	14 978
(b) At Willow Beach Station												
August 1–10	202 000	299	0.41	32.4	17.8	2.20	0.99	1.53	1.87	2.08	0.83	14 700
11–20	202 000	292	0.40	29.7	16.9	2.40	0.99	1.43	1.84	2.06	0.75	14 801
21–31	220 000	284	0.39	29.6	16.2	2.20	0.99	1.34	1.75	2.04	0.70	14 802
September 1–10	198 000	380	0.52	22.5	14.6	3.24	1.32	1.33	2.28	2.77	0.84	14 803
11–20	198 000	361	0.49	25.9	16.9	2.94	1.23	1.45	2.00	2.73	0.83	14 804
21–30	197 000	297	0.40	28.8	15.0	2.40	0.99	1.37	1.74	2.27	0.68	14 874
October 1–10	196 000	466	0.63	33.3	15.5	3.49	1.40	2.44	2.61	3.58	1.12	14 973
11–20	195 000	621	0.84	38.9	18.9	4.14	1.73	3.73	2.54	5.29	1.80	14 974
21–31	214 000	457	0.62	37.3	18.6	3.14	1.40	2.69	2.05	3.79	1.31	14 975
November 1–10	193 000	479	0.65	36.5	18.7	3.29	1.48	2.74	2.10	4.04	1.39	14 988
11–20	192 000	654	0.88	39.2	21.6	4.19	1.97	3.97	2.39	5.62	2.17	14 989
21–30	187 000	797	1.08	40.5	22.6	4.84	2.47	4.98	2.69	6.85	2.76	14 990

Variations in Salinity.—For several years the U. S. Geological Survey has taken daily samples of water from the Colorado River at Grand Canyon Station above Lake Mead and at Willow Beach Station below the reservoir. The percentage of sulfates has been determined for each sample and complete water analyses have been made of composites of each consecutive 10-day group of samples. Table 8(*a*) contains the analyses of the composites made up from daily samples taken at Grand Canyon Station during the months of June to

October, 1935, inclusive; Table 8(*b*) contains similar data for the waters at Willow Beach Station from August 1 to November 30, 1935, inclusive. The constituent ions are recorded in milligram equivalents per liter, rather than in parts per million, for convenience of geochemical classification. The total discharge of the Colorado River at each station for the successive 10-day periods is also recorded in acre-feet.

The data from Table 8 are shown graphically in Fig. 7, which is an enlargement of part of a geochemical chart. This form of plotting is not in general use and some explanation of it is probably warranted. The small inset is an outline of a geochemical chart which esentially consists of two trilinear diagrams and a diamond-shaped diagram adjoining them. The left-hand triangle is used for plotting the percentage reacting values of the cations; the right-hand triangle is similarly used for plotting the percentage reacting values of the anions. Since the percentage reacting values of the cations and of the anions always total unity, the cations in any water can be represented by a single point on one trilinear diagram, and the anions by a single point on the other equilateral triangle. When these are projected to an intersection on the diamond-shaped figure, the resulting position represents the geochemical grouping characteristic of the water. The diamond, itself, is made up of two trilinear diagrams having a common base, and the sum of the perpendiculars from any point likewise equals unity. For comparison, the range of the points plotted at the enlarged scale is indicated on the inset.

It may be demonstrated that: (*a*) On each trilinear diagram, and on the diamond-shaped diagram, the mixture of two waters must plot on the straight line connecting the plotted positions of each of the waters; and (*b*) the plotted position of the mixture must be at the center of gravity when all quantities are expressed in ton-equivalents. Ton-equivalents bear the same relation to total tons as milligram equivalents bear to parts per million, and the magnitude is fixed by the following relation:

$$T_e = \frac{Q \times m_e}{736} \qquad (11)$$

in which T_e = ton-equivalents; Q = quantity, in acre-feet; and m_e = milligram equivalents per liter.

The points at the left hand of Fig. 7 (in Zone I) thus lie in a part of the cation triangle with the sodium vertex at the upper left, the calcium vertex at the lower left, and the magnesium vertex at the right; the points in Zone III thus lie in the anion triangle with the chloride vertex at the upper right, the bicarbonate vertex at the lower right, and the sulfate vertex at the left. The center group (Zone II) lies in a part of the diamond-shaped chart at the intersections of the projections from corresponding points at the left and at the right.

The values for each of the analyses given in Table 8(*a*) are plotted as circles designated by the month and a letter, A, B, or C, depending upon whether the analysis was of samples taken in the first, second, or third 10-day period, respectively. The points shown by squares are from Table 8(*b*) and the designation, "Aug. C," for example, represents the composite of samples

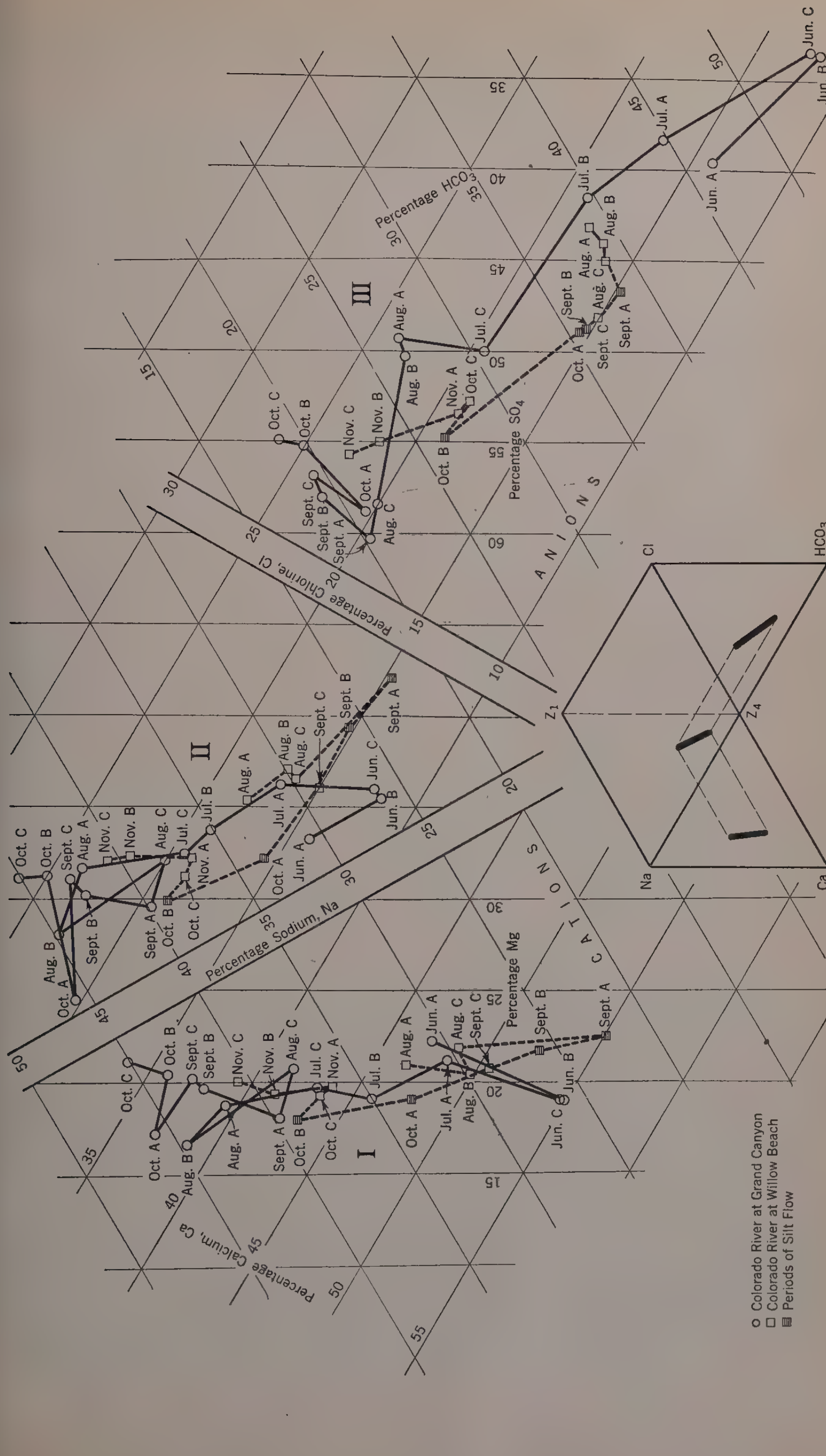

FIG. 7.—RELATIVE VARIATIONS IN SALINITY OF WATER ENTERING AND LEAVING LAKE MEAD, 1935

taken at Willow Beach Station during the last ten days of August, 1935. The silt flows particularly discussed by the authors, occurred within the first two periods in September and, again, in each of the first two periods of October, and are distinguished by solid black squares.

From the geochemical plotting, it will be noted that the waters entering the reservoir at the time of run-off from melting snow had the highest calcium percentage and the greatest bicarbonate percentage of any of the waters sampled; furthermore, as the flow of the Colorado River lessened in the fall, the proportion of calcium progressively decreased from about 55% to 34%, and the bicarbonates decreased from almost 55% to less than 20% of the anions. The general trend of the changes in the proportions of the cations was reversed late in August just prior to the discharge of turbid water from Lake Mead in September; also, the "October A" sample showed a sudden departure from the trend. On the anion triangle a pronounced increase in the sulfate percentage occurred late in August, and a sudden reduction in chlorides took place early in October. These departures furnish clues to the character of water that carried silt through Lake Mead.

The change in position of the points corresponding to the analyses of water sampled below Lake Mead followed somewhat the same pattern as those for the water entering, but with significant differences. On the cation triangle the proportion of sodium reduced from more than 30% to less than 25% in September, and then increased rapidly until after the discharge of turbid water in October. On the anion triangle there was a gradual shift toward the sulfate vertex until the first of October; then a pronounced change took place coincident with the October flow of turbid waters, and the sulfate and chloride percentages both increased.

Characteristics of Turbid Waters.—Material quantities of silt were carried by the water at Willow Beach in both the "September A" and "September B" periods, and likewise during each of the first two 10-day periods in October. For ease of computation, the "September A" and "September B" waters at Willow Beach may be grouped, and likewise the "October A" and "October B" waters. The composition of these waters is given in Table 9, Columns (2) to (5).

The authors have concluded from other data that about eight days elapsed between the time that very silty water passed Grand Canyon Station and the time when this water reached Willow Beach. Accordingly, water that passed Grand Canyon late in August and early in September should have been characteristic of the water leaving the reservoir during the time of the turbid flow in September; but from Fig. 7 it is obvious that either the "August C" water at Grand Canyon Station did not produce the turbid outflow from Lake Mead in September, or considerable mixing or dilution took place. Furthermore, with reference to the cation plotting, it is apparent that part of the turbid water at Willow Beach in September had a very low percentage of sodium, because the average sodium percentage of the September turbid water was less than the sodium percentage of even the spring flood waters.

These apparent anomalies make possible the calculation of the quantity and character of the water that carried silt through the reservoir in September.

After repeated trial and error, it was found that a mixture of 355 000 acre-ft of spring flood water, carrying 2 080 ton-equivalents of salt, with 41 000 acre-ft of water, loaded with 1 010 ton-equivalents and having a high calcium sulfate content, would produce the quantity and quality of water measured at Willow Beach from September 1 to September 20, 1935. The spring flood water was evidently itself a mixture, in equal quantities, of the water that entered Lake Mead in the last ten days of June and the first ten days of July, 1935. The composition of these waters is shown in Table 9, Columns (6) to (9).

TABLE 9.—ANALYSIS OF COLORADO RIVER WATER

Description	Average Quality of Water at Willow Beach, in 1935, From:				Spring Flood Water in Reservoir Release		Water That Carried Silt Through Lake Mead, in 1935, From:			
	September 1 to September 20, inclusive		October 1 to October 20, inclusive				September 3 to September 14, inclusive		October 6 to October 14, inclusive	
	Milligram equivalents per liter, m_e	Percentage reacting values	Milligram equivalents per liter, m_e	Percentage reacting values	Milligram equivalents per liter, m_e	Percentage reacting values	Milligram equivalents per liter, m_e	Percentage reacting values	Milligram equivalents per liter, m_e	Percentage reacting values
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Cations:										
Calcium	3.09	53.8	3.82	45.0	2.22	51.2	10.64	59.1	6.73	37.4
Magnesium	1.27	22.1	1.56	18.4	0.87	20.2	4.70	26.1	2.84	15.8
Sodium	1.38	24.1	3.10	36.6	1.24	28.6	2.66	14.8	8.43	46.8
Total	5.74	100.0	8.48	100.0	4.33	100.0	18.00	100.0	18.00	100.0
Anions:										
Carbonates	2.13	37.1	2.57	30.3	2.12	49.0	2.22	12.3	3.80	21.1
Sulfates	2.75	47.9	4.44	52.3	1.56	36.1	13.04	72.5	10.67	59.3
Chlorides	0.86	15.0	1.47	17.4	0.65	14.9	2.74	15.2	3.53	19.6
Total	5.74	100.0	8.48	100.0	4.33	100.0	18.00	100.0	18.00	100.0
Quantity of water, in acre-feet	396 000		391 000		....	....	....	....	....	....
Quantity of salts, in ton-equivalents	3 090		4 500		....	....	....	....	....	....

In October, 1935, the passage of silt through Lake Mead was produced by a water of markedly different characteristics from those of the water which produced the silt flow in the preceding month. From Fig. 7 it will be noted that there was a marked increase in sodium percentage at the time of turbidity in October, as distinguished from a decrease when silt was discharged in September. The direction of shift of the "October A" and the "October B" points for Willow Beach on divisions of Fig. 7 is toward the "October A" point for Grand Canyon Station. These directions likewise depart from those for the ten days immediately preceding and immediately following, which also is indicative of some change in conditions coincident with the passage of silt through the reservoir. There thus seems to be justification for the assumption that the water which carried the silt through Lake Mead in October was, in part at least, closely similar to the "October A" water at Grand Canyon.

Actually, the water which produced the silt flow was found to be that given in Table 9, Columns (10) and (11).

A simple mixture of this water and of the spring flood water would not satisfy the criterion that the mixture of "October A" and "October B" water at Willow Beach Station should lie at the center of gravity of the points corresponding to the spring flood water and the October silty water entering the reservoir. It was observed that the clear water released from Lake Mead between the times of the two silt flows had substantially the same proportion of anions as the turbid water released in September. This indicated that some of the calcium sulfate water which carried silt through the reservoir in the first part of September was still passing through the reservoir without its silt load in the latter part of that month.

This assumption is borne out by the fact that the various criteria can be satisfied by mixing with the water entering the reservoir in October, some of the water characteristic of the September run. Actually, the 4 500 ton-equivalents of salts carried past Willow Beach from October 1 to October 20 were evidently made up of 1 630 ton-equivalents of salts of the character carried by the water which entered the reservoir about July 1, 510 ton-equivalents of salts characteristic of those in the water which came into Lake Mead late in August, and 2 360 ton-equivalents of salts carried by water which passed Grand Canyon about the end of September, 1935. The corresponding quantities of water are as follows: "June C" water, 137 000 acre-ft; "July A" water, 137 000 acre-ft; September silt-run water, 21 000 acre-ft; and 96 000 acre-ft of the water which directly caused the flow of turbid water in October, 1935.

Minimum Silt Percentage.—In September, roughly 40 000 acre-ft of water carried 2 000 000 tons of silt through Lake Mead, equivalent to an average of 50 tons per acre-ft. It would appear from this that the water which brought the silt to Willow Beach Station was that which passed Grand Canyon Station on August 26 and on September 4, and that little of the suspended load at Grand Canyon on the intervening days found its way through the reservoir. Again, in October, roughly 2 500 000 tons of silt were carried through by less than 100 000 acre-ft of water, or more than 25 tons per acre-ft. The days of high silt percentage at Grand Canyon were September 30 to October 3, inclusive, during which time the discharge of the Colorado River was almost as great as the quantity of water that apparently passed through the reservoir with its silt load. In general, the data indicate that silt was not carried through Lake Mead when the water entering the reservoir contained less than 2.0% of suspended matter, and then only if the proportion of fine particles was relatively large.

Passage of Clear Saline Water.—Reference was made in the foregoing discussion to the passage of clear saline water through Lake Mead without material mixing except at the outlets. In the writer's opinion, this is of far greater practical importance than the removal of silt that might otherwise deposit in the reservoir.

With reference again to Fig. 7, and particularly to the points on the segment of the anion triangle, it is apparent that the clear waters released from the reservoir in August, in September, in October, and in November, 1935,

were not characteristic of the great body of water that entered during the flood run-off in the spring. The points for the flow at Willow Beach in the month of August lie roughly on a line between the "June C" and the "July C" water at Grand Canyon Station, indicating that the water in the river several miles below Boulder Dam was a mixture of spring flood water and of the flow entering the reservoir late in July. The clear water at Willow Beach in the last ten days of September, 1935, definitely must be a mixture of the spring flood water and some other water having a much higher sulfate percentage. Actually, the data indicate that some of the calcium sulfate water which produced the turbid flow early in September was continuing to flow under the lake without its silt load in the latter part of that month. The water at Willow Beach near the end of November appears to be somewhat similar to the water entering the reservoir in September, but the concentration of salts in this water is less than two-thirds of the concentration in the water entering. There must have been, therefore, dilution or mixing of the water characteristic of that entering Lake Mead late in September, with other water which came into the reservoir during spring floods.

It is not necessary that this mixing should take place in the reservoir, and probably this was not the case. The outlets at Boulder Dam presumably drew both from the saline water which was flowing along the bottom toward the dam and from the large mass of water lying above the saline flow. This could be true, of course, regardless of whether one outlet or a series of outlets were in service, although it would be more likely to occur with outlets open at different elevations.

This phenomenon of the passage of saline water along the floor of the reservoir is also suggested by the upper graph in Fig. 2 of the paper, in which it is shown that the quantity of sulfates at Willow Beach steadily increased from September to the end of November, 1935, approaching the concentration of sulfates in the water at Grand Canyon Station, regardless of the fact that most of the water in Lake Mead entered there during months of run-off from melting snow.

Various estimates have been made of the quality of water that will be available for domestic and other uses from the Colorado River after regulation in Lake Mead. Such estimates have naturally been based upon the assumption that there would be complete mixing of the relatively pure flood waters with the saline low-water flow of the Colorado River. Analysis of the data now available (1938) indicates, on the other hand, that the saline waters tend to flow through the reservoir rather than to mix, and that the spring flood waters tend to "ride on top." Since May 1, 1936, the tunnel gates at Boulder Dam have been closed and the lowest outlet is far above the original stream bed. The question naturally arises: Will the normal release from Lake Mead be materially more saline than the weighted average inflow and will the best water be wasted at the time of spill or release for flood control?

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

ESSENTIAL CONSIDERATIONS IN THE STABILIZATION OF SOIL

Discussion

BY DONALD M. BURMISTER, ASSOC. M. AM. SOC. C. E.

DONALD M. BURMISTER,[8] ASSOC. M. AM. SOC. C. E. (by letter).[8a]—An excellent and comprehensive summary of soil stabilization, particularly the important physico-chemical surface phenomena, has been presented in this paper. The authors have shown that the grading and proportions of the fine and coarse materials, admixtures and chemical treatment, and temperature and film character have an important effect on the density, optimum moisture content, and stability of compacted soils.

The primary purpose of such investigations is to build a fund of knowledge on soil stabilization, so that the experiences and knowledge gained may be of practical value, and serve as a guide in solving such problems. In order to accomplish this purpose more completely, soil behavior must be translated into terms of soil character, which means that the soils in each case must be described, accurately and systematically, on the basis of a few quantitatively measured physical properties, so that they can be identified unmistakably, and classified. This would make possible comparisons and reliable correlations with soils from other localities.

In addition to the basic principles of soil compaction established by Mr. Proctor (see heading, "Bibliography," Reference (20)[9]), and the principles emphasized by the authors, there are a number of important physical relations which the writer believes are fundamental, because they bring into a more unified and consistent pattern many of the physical factors governing the behavior of soils. Five of these relations are as follows:

(1) There is a general tendency for a decrease in the maximum density and an increase in the optimum moisture content with increasing fineness of the

NOTE.—The paper by C. A. Hogentogler, Assoc. M. Am. Soc. C. E., and E. A. Willis, Esq., was published in June, 1937, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: October, 1937, by Jacob Feld, M. Am. Soc. C. E.

[8] Asst. Prof., Dept. of Civ. Eng., Columbia Univ., New York, N. Y.

[8a] Received by the Secretary March 1, 1938.

[9] *Proceedings*, Am. Soc. C. E., June, 1937, p. 1054.

soil, the degree of fineness being designated by a mean grain size or a size factor[10] which correlations show to be a practical and useful measure of fineness.

(2) The important influence of the grading of soils and the proportions of mixtures on density (that is, the grading-density relations[11]) can best be expressed in terms of the type of grading curve and the range of particle sizes,[10] there being consistent relationships at the optimum condition of compaction even to the finest materials.

(3) The plastic characteristics of the fine fraction of the soil, expressed in terms of the liquid and plastic limits, and the plasticity index relations, give a valuable clue to the physical character of the soil, because the plastic characteristics are influenced by the same physical factors that determine density and stability—namely, wetting characteristics, plasticity, mineralogical nature, binder, or cohesive qualities, etc.

(4) The proportion and the characteristics of the coarse fractions of the soil, particularly the wetting characteristics and the grading-density relations,[11] have an important influence on the moisture-density relations and on the stability of the whole soil. If the proportion of coarse materials is large, the fine fraction acts as a filler and binder, and the character of the coarse material predominates. If the soil is predominantly fine, the moisture-density relations and stability are determined almost entirely by the plastic characteristics of the fine fraction of the soil.

(5) There appears to be a fairly narrow range of clay content (finer than 0.005 mm) necessary to obtain both maximum density and stability, particularly for stabilized earth roads, where permanent stability is essential in both the wet and dry condition. Where there is a deficiency of clay (that is, less than about 5%) and where the soil is non-plastic, with a plasticity index less than about 4, the density at the optimum moisture content is materially less, and the soil has insufficient binder qualities. The optimum moisture content may be less, but there is a larger quantity of entrapped air. Clay content in excess of 35% to 40% and a plasticity index greater than about 15 to 20 are undesirable from the standpoint of stability of rolled-earth dams and fills, although the compacted soil may be very dense. The requirements for stabilized earth road soils, of necessity, must be defined more closely because of the extreme service conditions.

A study of compaction tests on different soils and an analysis of the compaction test data from a great many sources[12] show that these fundamental relations and those stated by the authors form the basis for correctly interpreting and evaluating the influence of many physical characteristics of soils on their density and stability. The writer wishes to suggest the following simple, systematic method of presentation and analysis of the data given by the authors in Fig. 5 and Table 3, which is intended to reveal and focus attention on the characteristic features of the soils under investigation, or soils from any given locality, so that the influence of their individual similarities, or differences,

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[10] *Proceedings*, Am. Soc. C. E., February, 1938, p. 397.

[11] *Loc. cit.*, p. 365.

[12] "The Physical Characteristics of Soils, with Special Reference to Earth Structure," by D. M. Burmister, Assoc. M. Am. Soc. C. E. (Not published.)

and qualities can be learned; and so that the inter-relationship of all physical factors can be studied in an attempt to trace their influence on density and stability. Although general statistical relationship is valuable, it is believed that the emphasis should be placed on the particular character of the individual soils. Differences in physical characteristics then become of equal importance with similarities of soils as reflected in density.

In Fig. 8, the mean grain size or size factor,[10] as a significant physical characteristic of soils, is used as the plotting argument. All related data for each soil are then plotted on a vertical line through the mean grain size in the different component parts of the diagram. This method of plotting serves to focus

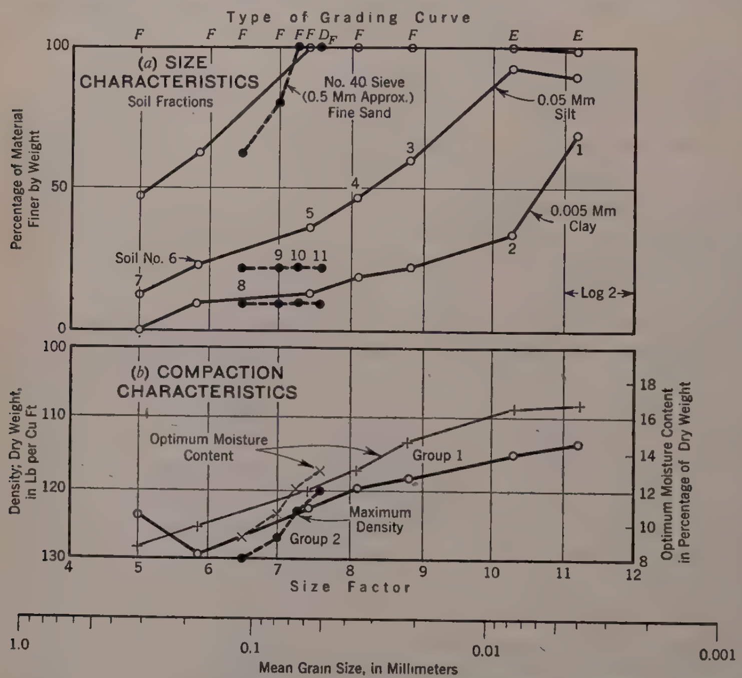

FIG. 8.—THE GRADING-MOISTURE-DENSITY RELATIONS OF FINE-GRAINED SOILS

attention on the particular characteristics of the soils, and the confusion of superimposing a large number of grading curves is eliminated.

Size Characteristics.—The distribution of grain sizes is indicated for each soil by projecting and plotting certain soil fractions as, for example, percentages finer by weight than 5.0, 0.5 (approximately No. 40 sieve), 0.05, and 0.005 mm on to a vertical line through the respective size factors or mean grain sizes. A closer sub-division such as log 6 or log 2 intervals might have been chosen. The distribution of these fractions, or grouping (bunching above or below the 50% size) is characteristic and serves to indicate both the type of grading curve, which is designated by letter in Fig. 8, and the range of particle sizes.[10] A broken line traced through the respective fractions shows clearly the tendency

in these soils for increasing clay content and decreasing coarse material with increasing fineness. The peculiarities of any particular soil with respect to the distribution of grain sizes are now indicated by the departures from the general trend of these fractions.

It is to be noted that for the soils in Group 1 the range of particle sizes decreases with decrease in mean grain size, and the type of grading curve changes from the wide-range, coarse F-type to the narrower range F-types (almost the D-type) and, finally, to the fine E-type. Soils in Group (2) have a constant proportion of fine material and vary only in the coarseness and range of the coarse fraction, and they change from the typical F-type to the D_F-type.

Compaction Characteristics.—These characteristics of the soils are now reflected in the maximum density and optimum moisture content. First, there is a decrease in density with decreasing mean grain size; and, second, this decrease is also associated with a decrease in the range of grain sizes and a change in the type of grading curve for these soils. The importance of the range of grain sizes is shown in Group (2), where change in the range of the coarse fraction alone causes a marked decrease in density. The decrease in density of Material No. 7, due to deficiency of clay content (or decrease in the range in the fine sizes), is very noticeable, but the optimum moisture content is also less, showing that there is a larger quantity of entrapped air.

However, the size characteristics alone are insufficient to describe and identify soils. The plastic characteristics give much more valuable and specific information on their true character. Fig. 8, therefore, is extended for the analysis of certain groups of soils in Fig. 9 to include the plastic characteristics of the fine fractions of the soil passing the No. 40 sieve (0.5 mm, approximately), which are defined in terms of the liquid limit and the plasticity index.

A study of the density relations at the optimum condition discloses consistent relationships with the specific characteristics of each soil. This method of plotting the density against the mean grain size or size factor, as an argument, brings compaction data into a pattern having a definite trend. The slope of the band of density, its position, and width—that is, the variation of density within the band for any given group of materials—are determined primarily by the five physical relations enumerated in the first part of this discussion.

A comparison of the plastic characteristics with the clay line (finer than —0.005 mm) shows that the clay fraction has an important influence on the general character of the soil.

The consistent relationship or mirroring of density and the plastic characteristics for Group (4) materials is more than a coincidence, but reveals the direct influence of the particular characteristics of each soil on density, and indicates the importance of the plastic characteristics. The influence that the fine fraction exerts on the character of the soil, will depend upon its proportion of the whole soil, as indicated by the No. 40 sieve line.

Again, it is important to note the general trend of increasing density with increasing mean grain size and range of sizes. The importance of the type of

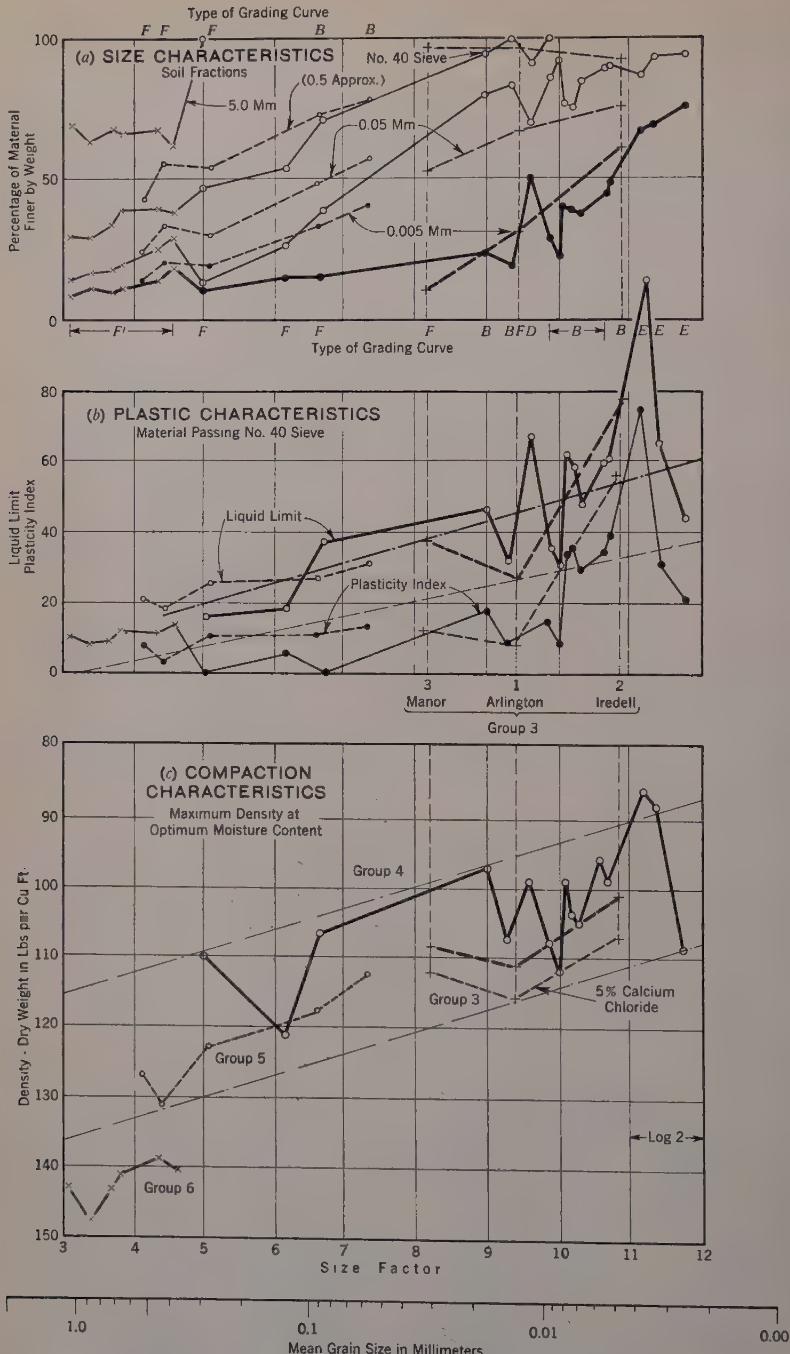

FIG. 9.—THE COMPACTION CHARACTERISTICS OF SOILS SHOWING THE INFLUENCE OF SOIL CHARACTER ON DENSITY

grading curve is shown by the marked increase in density for the Group (6) materials, representing the ideal type, F', grading for maximum density—the Talbot grading, which conforms to the requirements for stabilized earth-road soils. The typical grouping of the soil fractions of this group, the wide range of particle sizes, and particularly the relatively high clay content indicate clearly the physical character of Type F' soils. These relations, together with the relatively high plasticity index of the fine fraction (passing the No. 40 sieve), represent the ideal requirements that must be satisfied in order to obtain maximum density. In general, where the plasticity index is lower than the trend the density is lower, and where it is higher, the density is greater, an inverse relationship, which is quite consistent.

Equally consistent relations are indicated for the Iredell, Arlington, and Manor soil of Group (3), which the authors used in their investigations (see Fig. 5). In view of the fact that the plastic characteristics reflect, consistently, the influence of the physical factors of the fine fraction that determines density and stability, the specific influence of admixtures, electrolytes, and chemical treatment, except bituminous materials, may be determined and correlated by their specific effects on the liquid limits and plasticity indices of the particular soils, because these substances affect primarily the wetting characteristics, film character and thickness, and physico-chemical surface phenomena of the fine fraction of the soil. In other words, the influence of the 5% calcium chloride solution on density could have been estimated fairly reliably in Fig. 9 if the effect on the liquid limit and plasticity index were known.

With a few control compaction tests on a group of soils to define the density band, and with the physical character of the soil indicated by the size and plastic characteristics, a reliable estimate of the density of a large number of soils can be made more economically and rapidly on the basis of the simpler liquid and plastic limit tests by making departures within the density band proportional to the departures of the plastic characteristics from their trend, and by giving the proper weight to the proportion of the fine fraction of the whole soil, as revealed in Fig. 8 for soils of Group (2).

Additional sub-divisions may then be added to the diagram showing the quantity and kind of admixture or chemical treatment. Where it is deemed advisable, Fig. 9 may be extended to include the Proctor penetration resistance at the optimum moisture content, or the results of other tests, such as crushing strength tests to show the binder qualities of the soil and slaking tests to show how the soil reacts to variations in moisture content with consequent changes in stability and cementing properties, shear tests, etc.

It is believed that much useful and valuable information can be made of more general and permanent value as a guide for rolled-earth dams and stabilized earth-road constructions; but the information should be as complete as possible with a careful and systematic description and identification of soils so that the experience gained by investigation or by construction in one locality may be of practical value in solving the problems in another. The soil diagram affords certain advantages because it summarizes and correlates the information in a single diagram and focuses attention on the characteristic features of soils that have an important influence on density and stability.

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DISCUSSIONS

NATIONAL ASPECTS OF FLOOD CONTROL

A SYMPOSIUM

Discussion

BY FRANKLIN F. SNYDER, JUN. AM. SOC. C. E.

FRANKLIN F. SNYDER,[73] JUN. AM. SOC. C. E. (by letter).[73a]—In the part of their paper discussing the U. S. Weather Bureau relation to the 1936 flood at Pittsburgh, Messrs. Morse and Thomas compare a stage prediction made at 8:00 A.M. on March 17, 1936, for the forenoon of March 18, with the observed stage at 8:00 A.M., March 18, and arrive at an error of 10 ft. From the data now available it appears that the forecast made on 8:00 A.M., March 17, was not more than 3 ft in error. The aforementioned type of comparison and conclusion is often made and illustrates the care that must be taken to make clear the basis for any particular forecast. Studies for the improvement of existing flood forecasting methods, made co-operatively by the Pennsylvania Department of Forests and Waters, the U. S. Weather Bureau, and the U. S. Geological Survey, were begun in 1937, and, in this connection, some interesting aspects of the 1936 flood at Pittsburgh, Pa., have been developed.

Procedures of river forecasting on head-water areas are based on data accumulated only a short time before the forecast is released. Progress is being made in weather forecasting, and particularly in air-mass analysis, which indicates possibilities of forecasting rainfall quantitatively, but, until such developments reach a state of practical application, forecasts, as issued, must be understood to apply to flow from rain already fallen.

The interval between the time of critical rainfall and that in which the resulting flood can be identified in the principal channel system is appreciable in

NOTE.—This Symposium was presented at the Fall Meeting of the Society and at the meeting of the Waterways Division, Pittsburgh, Pa., October 13 and 14, 1936, and published in March, 1937, *Proceedings*. Discussion on this Symposium has appeared in *Proceedings*, as follows: June, 1937, by Messrs. F. C. Scobey, Howard T. Critchlow, T. T. Knappen, M. C. Tyler, Gordon R. Williams, Arthur T. Safford, W. G. Hoyt, J. D. Arthur, Jr., John H. Meursinge, H. K. Barrows, E. D. Hendricks, and Edward W. Bush; September, 1937, by Messrs. H. K. Barrows, Ivan E. Houk, and John E. Field; October, 1937, by Messrs. C. S. Jarvis and Joseph Jacobs; December, 1937, by Messrs. W. M. Dawley, and Howard M. Turner; and January, 1938, by Ralph W. Powell, M. Am. Soc. C. E.

[73] Hydr. Engr., Co-Operative Hydrologic Investigations, Commonwealth of Pennsylvania, Dept. of Forests and Waters, U. S. Weather Bureau, and U. S. Geological Survey, Harrisburg, Pa.

[73a] Received by the Secretary March 17, 1938.

most cases. The unit graph procedure provides an opportunity to demonstrate this point.

In analyzing the March, 1936, flood at Pittsburgh, the Allegheny River was studied at Dam No. 3 (drainage area, 11 530 sq miles) which is 14.5 miles up stream from the junction. On the Monongahela River, Dam No. 2 (drainage area, 7 340 sq miles), 11.2 miles up stream from the junction, was used. The surface run-off hydrographs were computed at these two stations and added to the Pittsburgh base flow to obtain the computed hydrographs of Fig. 25.

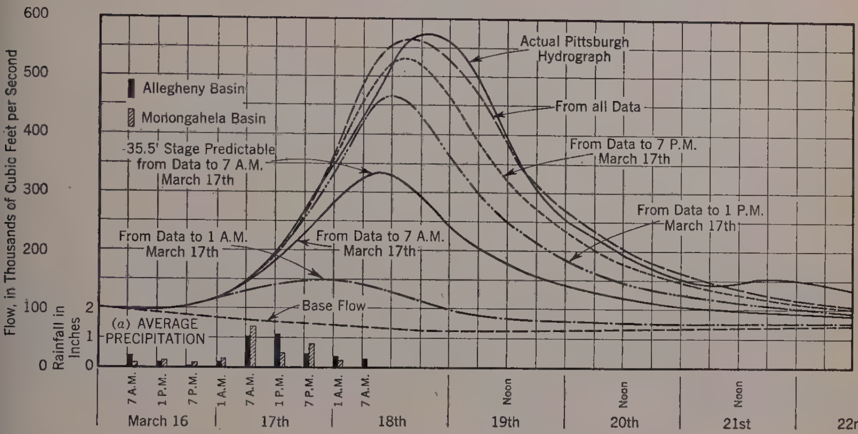

FIG. 25.—FLOOD OF MARCH, 1936, AT PITTSBURGH, PA.

Nineteen selected precipitation stations were used on the Allegheny Basin. They gave an average depth of 3.94 in. of precipitation for the period, March 16–19, 1936, of which the last 0.63 in. was classified as snow and was eliminated from the study for the period after 7:00 A.M. on March 18. The unit graphs used were designed for 6-hr rainfall data, the basin precipitation being divided into 6-hr intervals according to the precipitation recorded automatically at the Piney Plant of the Pennsylvania Electric Company, on the Clarion River (see Table 14).

Snowfall equivalent to 0.50 in. of water was added and assumed to melt in proportion to the precipitation for lack of a better method. This gave 3.81 in. of water depth to be accounted for and, with an initial loss of 0.25 in. deducted from the rainfall of the first period, left 3.56 in. The estimated volume of surface run-off was 1.95 in., giving a surface run-off factor of 54.8 per cent. This factor is not a real one since only enough run-off occurred from the precipitation above Franklin, Pa. (mostly melted snow) to maintain a recession flow of 42 000 cu ft per sec from the previous rise to give a peak of 50 000 at midnight, March 17–18.

Another way to express the run-off factor, more in accord with the actual conditions (that is, the area above Franklin eliminated), is based upon a depth of precipitation plus water equivalent of snow of 5.00 in., with a surface run-off volume of 3.75 in., giving a run-off coefficient of 75% for the lower area. It will be found for this illustration that either procedure will give about the same

result, the controlling factor being the 6-hr distribution of rainfall as established by the Piney record. However, since the depths used are those for the entire area, the unit graph applied is that designed for conditions prevailing when the greater run-off comes from the lower basin.

TABLE 14.—PRECIPITATION AND RUN-OFF DATA, IN INCHES, MARCH 16–19, 1936

	SIX-HOUR PERIODS ENDING AT:									Total
	March 16			March 17				March 18		
	7:00 A.M.	1:00 P.M.	7:00 P.M.	1:00 A.M.	7:00 A.M.	1:00 P.M.	7:00 P.M.	1:00 A.M.	7:00 A.M.	
Allegheny Basin:										
Piney precipitation	0.50	0.18	0.05	0.17	1.32	1.43	0.54	0.41	0.33*	4.93
Basin precipitation	0.39	0.14	0.04	0.13	1.02	1.10	0.42	0.32	0.25*	3.81
Initial loss	0.25	...	...	...	...	...	...	...	...	0.25
Run-off	0.08	0.08	0.02	0.07	0.56	0.60	0.23	0.18	0.13	1.95
Monongehela Basin:										
Pittsburgh precipitation	0.28	0.35	0.16	0.20	0.85	0.39	0.28*	0.09*	*	2.60
Elkins precipitation	0	0	0.08	0.23	1.50	0.41	1.06	0.26	*	3.54
Average precipitation	0.14	0.18	0.12	0.21	1.18	0.40	0.67	0.17	...	3.07
Basin precipitation	0.16	0.21	0.14	0.25	1.37	0.46	0.78	0.20	...	3.57
Initial loss	0.16	...	...	...	...	...	...	...	...	0.16
Run-off	0	0.17	0.11	0.20	1.11	0.37	0.63	0.16	...	2.75

* Snow.

TABLE 15.—SIX-HOUR ORDINATES OF THE UNIT GRAPHS, AND CORRESPONDING DISTRIBUTION GRAPHS

Period	ALLEGHENY RIVER		MONONGAHELA RIVER		Period	ALLEGHENY RIVER		MONONGAHELA RIVER	
	Rainfall Concentrated in Lower Valley		Rainfall of Average Distribution Over Valley			Rainfall Concentrated in Lower Valley		Rainfall of Average Distribution Over Valley	
	Unit graph, in thousands of second-feet	Distribution graph, percentage of total flow	Unit graph, in thousands of second-feet	Distribution graph, percentage of total flow		Unit graph, in thousands of second-feet	Distribution graph, percentage of total flow	Unit graph, in thousands of second-feet	Distribution graph, percentage of total flow
1	8.0	0.63	2.9	0.36	15	20.8	1.68	22.3	2.81
2	31.2	2.50	10.8	1.36	16	16.8	1.35	19.2	2.42
3	66.5	5.35	23.0	2.90	17	13.4	1.08	16.7	2.11
4	118.0	9.51	40.3	5.09	18	10.7	0.86	14.5	1.83
5	176.3	14.22	64.7	8.16	19	8.3	0.67	12.5	1.58
6	177.7	14.35	90.7	11.45	20	6.3	0.51	10.7	1.35
7	155.5	12.54	92.8	11.70	21	4.6	0.37	9.1	1.15
8	118.0	9.52	79.3	10.00	22	3.0	0.24	7.7	0.97
9	88.5	7.14	65.2	8.22	23	1.9	0.15	6.3	0.80
10	66.6	5.38	53.3	6.72	24	0.6	0.05	5.0	0.63
11	50.4	4.06	44.0	5.55	25	....	....	3.8	0.48
12	39.4	3.18	36.5	4.60	26	....	....	2.7	0.34
13	32.0	2.58	30.6	3.86	27	....	....	1.6	0.20
14	25.8	2.08	26.0	3.28	28	....	....	0.6	0.08
Totals	....	....	....	....	..	1 240.3	100.00	792.8	100.00

Twenty-five precipitation stations were used on the Monongahela Basin and, for the period, March 16–19, 1936, gave an average of 3.54 in., of which the last

0.47 in. was classified as snow assumed to have fallen after midnight of March 17–18. The water content of melting snow was assumed at 0.50 in. With an initial loss of 0.16 in., the remaining 3.41 in., together with a surface run-off factor of 80.5%, provided the desired volume of 2.75 in. of surface run-off. The precipitation period was divided into 6-hr intervals according to the average of the recorded rainfall records of the U. S. Weather Bureau stations at Pittsburgh, and at Elkins, W. Va., as shown in Table 14. The unit graph applied was that designed for use with a fairly uniform distribution of run-off.

Six-hour ordinates of the unit graphs and their corresponding distribution graphs are given in Table 15. The first value is to be plotted at the middle of the first 6-hr run-off period and the successive values at 6-hr intervals.

No time correction was applied to the computed surface run-off data for the time lag between Dam No. 2 and Dam No. 3 and Pittsburgh. The actual Pittsburgh hydrograph is also shown, and is based on a Pittsburgh rating obtained by gage relations from the U. S. Geological Survey Station at Sewickley, Pa.

The foregoing example is presented to illustrate the situation that must be taken care of when forecasts are to be issued at regular intervals. Certainly this must be done on head-water areas where, if the forecast were delayed until the cessation of rainfall, the opportunity for preparation would be largely lost. The illustration is not intended as a demonstration of the unit graph procedure nor as a justification or criticism of any actual forecasts. A considerable part of the study was prepared by R. J. MacConnell, of the Pennsylvania Department of Forests and Waters.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
DISCUSSIONS

PRESSURES BENEATH A SPREAD FOUNDATION

Discussion

BY MESSRS. FREDERICK J. CONVERSE, AND D. P. KRYNINE

FREDERICK J. CONVERSE,[46] ASSOC. M. AM. SOC. C. E. (by letter).[46a]—The method, described by Professor Krynine, for obtaining the pressure of any point within the soil below a loaded area, contains three important viewpoints which should be included in any method of pressure determination. In the first place, recognition of the fact that the soil cannot always be considered as an elastic isotropic medium is given by the use of such formulas as Equations (1) and (9), containing the "concentration factor," n. Secondly, the method can be easily adapted to non-uniformly distributed pressures at the surface. In the third place, the method can be used with equal facility for the case of single spread footings and for groups of foundations.

The work involved in the graphical solution presented by the author may be shortened somewhat by the following procedure: Consider a unit pressure to be applied at Point O', Fig. 2, and construct contours of equal vertical pressure in the soil below Point O', using Equation (8) with $\bar{p}\, dA$ equal to unity. Move the "transformed area," $M\ T''\ N$, vertically downward until the horizontal through $M\ N$ passes through Point O, as shown in Fig. 21. The percentage of the unit pressure, indicated by the position of Point T' on the iso-pressure chart, multiplied by the height, $T'\ T''$, gives the vertical component of the pressure at Point O due to a concentrated load, $T'\ T''$, at the surface. This may be plotted vertically above Point T' and represents a point on the "pressure curve" (see Fig. 22). Its value corresponds to that of $T'\ y$ in Fig. 2, multiplied by $p \dfrac{n}{2\,\pi\, z^2}$. In a similar manner, other points on the "pressure curve" may be found and the curve drawn. The area between this curve and $M\ T'\ N$ gives the total vertical pressure at Point O.

NOTE.—The paper by D. P. Krynine, M. Am. Soc. C. E., was published in April, 1937, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: September, 1937, by Messrs. O. K. Fröhlich, Donald W. Taylor, and Jacob Feld; October, 1937, by Messrs. G. P. Tschebotareff, and A. Hrennikoff; November, 1937, by Messrs. Robert G. Hennes, T. A. Middlebrooks, and A. A. Eremin; and December, 1937, by M. M. Buisson, Esq.

[46] Asst. Prof. of Civ. Eng., California Inst. of Technology, Pasadena, Calif.

[46a] Received by the Secretary February 21, 1938.

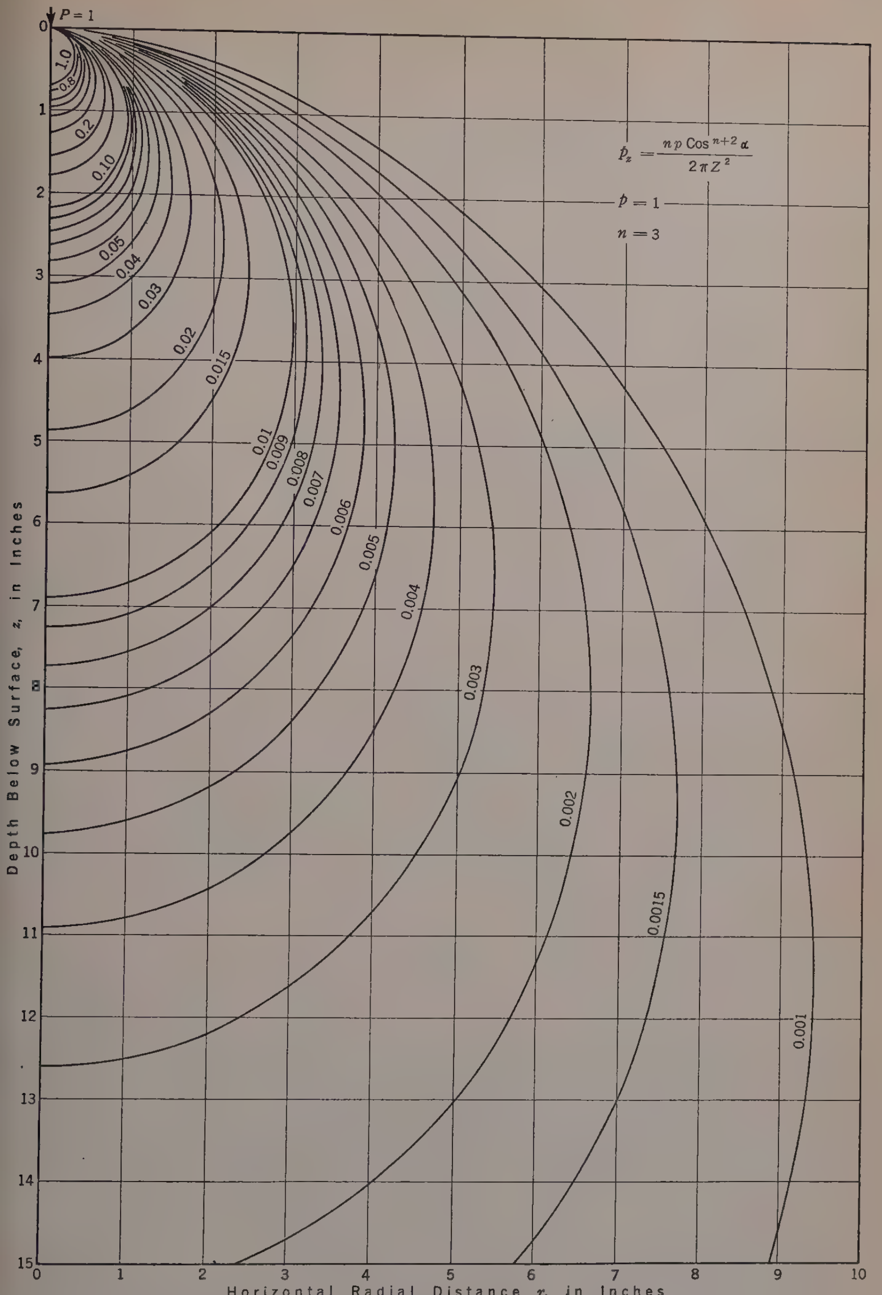

FIG. 21.—ISO-PRESSURE LINES OF VERTICAL COMPONENT OF PRESSURE WITHIN THE SOIL DUE TO A CONCENTRATED UNIT LOAD AT THE SURFACE

It is apparent that if an iso-pressure chart for a unit load at the surface is available, it is only necessary to draw the transformed area on transparent paper and place it over the chart at the desired position, using the same scale for depth that was used in the plan. In this manner the pressure curve may be readily constructed at any depth, once the "transformed area" has been obtained. If several iso-pressure charts are available, each representing a different value of n in Equation (8), the engineer may exercise his judgment in choosing the proper chart to fit the particular soil conditions.

Still more time may be saved if the data are presented in tabulated form without actually drawing the "transformed area" and the "pressure curve."

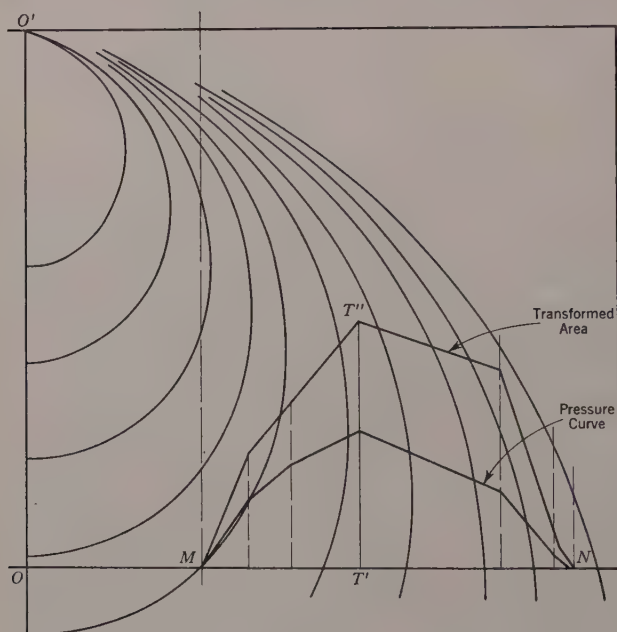

FIG. 22

In fact, even the circles used by the author to establish points on the "transformed area" need not actually be drawn if a transparent polar co-ordinate chart is at hand. With the center of the polar chart placed on the footing plan at the point below which the pressure is desired, the concentric circles will divide the footing areas into zones, and the number of degrees of arc swept by each zone may be read from the chart. It is not even necessary to redraw the foundation plan, if it is drawn to scale. Blue-prints may be used if the actual scale of the drawing is determined—that is, if the scale of the original drawing is corrected for shrinkage.

The author's example supported by Fig. 6 has been used as an illustration of the work involved when the graphical constructions are entirely eliminated. The plan shown in Fig. 6 was used without redrawing, the scale being considered at 80 ft per in. Table 3 shows all the calculations for pressure at

Point x at depths of 100 ft and 320 ft. The iso-pressure chart used was constructed from Equation (8) with $n = 3$. It is shown in Fig. 21.

TABLE 3.—CALCULATIONS FOR AUTHOR'S EXAMPLE, POINT x

(Scale, 1 in. = 80 ft.; Formula, $p_z = \frac{p\, n \cos^{n+2} \alpha}{2\, \pi\, z^2}$; $n = 3$)

Zone No.	Zone width, in inches	Radius of center line, in inches	LENGTH, CENTER LINE OF ZONE		Average unit pressure at surface, in tons per square foot	Product, Columns (2), (5), (6)	100-FOOT DEPTH = 1.25 INCHES ON CHART		320-FOOT DEPTH = 4 INCHES ON CHART	
			Degrees	Inches			Unit pressure (Chart)	Unit pressure, in tons per square foot	Unit pressure (Chart)	Unit pressure, in tons per square foot
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
1	0.20	0.10	270	0.47	2.5	0.235	0.30	0.0705	0.0300	0.0071
2	0.20	0.30	270	1.41	2.5	0.705	0.28	0.1970	0.0295	0.0208
3	0.20	0.50	270	2.36	2.5	1.180	0.20	0.2360	0.0285	0.0336
4	0.15	0.675	270	3.18	2.5	1.191	0.155	0.1845	0.0275	0.0328
5	0.15	0.825	133	1.92	2.5	0.720	0.125	0.0900	0.0265	0.0191
6	0.1	0.95	74	1.23	2.5	0.308	0.095	0.0293	0.0258	0.0080
7	0.1	1.05	31	0.57	2.5	0.142	0.082	0.0116	0.0250	0.0036
8	0.1	1.15	8	0.16	2.5	0.040	0.065	0.0026	0.0245	0.0010
Total	....	....	....	....	....	....	....	0.8215	....	0.1260

There is an important advantage in being able to use an independent iso-pressure chart, aside from the simplification of calculations. At present, Boussinesq's formula, as modified by the "concentration factor," gives the best available information as to pressure distribution in soils. If the soil is stratified within the pressure range, still further modifications must be made. The iso-pressure chart may be modified to suit the particular conditions of the site, and to take advantage of advancing knowledge of stress distribution, without affecting the foregoing method of analysis in any manner.

D. P. KRYNINE,[47] M. AM. SOC. C. E. (by letter).[47a]—The discussion of this paper has brought to the attention of the profession several important ideas and has permitted the elucidation of several obscure facts in a more or less adequate manner.

Difference Between the Boussinesq and Fröhlich Stress Distribution.—The writer always has been of the opinion that there is no strain at a certain distance from the loaded area; and this opinion, although contrary to that held by most engineers is in accordance with that of Boussinesq who states:[48]

"* * * besides it is evident that the sections of the body located at infinity, or rather beyond a certain distance from the loaded portion [contact area] keep their shape and their dimensions."

Thus, what conventionally is meant by "infinity" may be a rather short distance, according to the properties of the soil and the value of the load. Ob-

[47] Research Associate in Soil Mechanics, Dept. of Civ. Eng., Yale Univ., New Haven, Conn.

[47a] Received by the Secretary February 16, 1938.

[48] "Application des Potentiels," p. 51, Lines 8–11.

viously, this statement should be checked experimentally. Furthermore, Boussinesq visualizes the semi-infinite body as a hemisphere of a radius, r, tending toward infinity;[49] but he states categorically that his analysis does not apply to the total loading of the horizontal plane bounding this body from above,[50] since in this case the settlement is infinite. To make this fact clearer, it should be remembered that according to a general belief the settlement of a loaded rigid or non-rigid circular disk, as measured at its center, is proportional to its diameter. Admittedly, the body upon which the disk is placed is assumed to be elastically isotropic. If the diameter of this disk becomes infinite, the settlement likewise will be infinite. Such a situation is not covered by the Boussinesq analysis as stated by Boussinesq himself; and this leads to an opinion that there should be a certain limit of loaded area to which this analysis (particularly the method of superposition) may be applied. Furthermore, according to one of the basic requirements of the theory of elasticity, strains under consideration must be small. In using the principle of superposition in the case of larger areas, this fundamental condition is not satisfied. In other words, application of the principle of superposition to larger areas does not seem to be justified; and, as far as earth masses are concerned, this statement may be proved by experience.

If a part of the horizontal surface of an earth mass is loaded, earth under a rigid structure is "disturbed," except when the unit load is very small. Experiments and some field observations prove this statement.

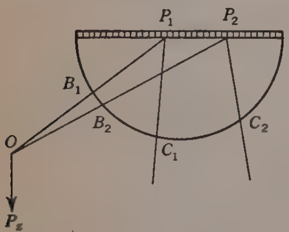

FIG. 23

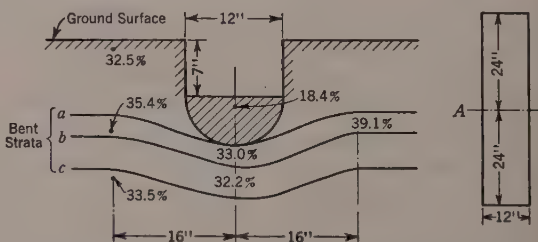

FIG. 24.—LOADING EXPERIMENT AT EAST HARTFORD, CONN. PERCENTAGES SHOW MOISTURE CONTENT AFTER EXPERIMENT

When the Boussinesq formula is applied (see Fig. 23), it is assumed that the "disturbed zone" has no influence on the value of the stresses at Points B_1 and B_2 and that pressures at Point O may be added by superposition. This procedure is not quite accurate. Dr. Fröhlich attempted to increase or to concentrate all the stresses that radiate rather vertically (as at Points C_1 and C_2) and to decrease those that radiate rather horizontally (see Fig. 23). Pressures at Point O are added by superposition.

Beyond question, the Boussinesq formula is the only scientific tool to use for computing stress distribution within a mass. Other formulas proposed are simply practical corrections of the Boussinesq formula which, however, does not fit small-scale loading experiments on sands. The Fröhlich-Griffith formula

[49] "Application des Potentiels," p. 53, Lines 1–10.
[50] *Loc. cit.*, p. 191, Lines 17–27.

fits the latter experiments, but there are objections against its application to larger areas. These objections, as revealed in the discussion, are examined subsequently.

Formation of a "Disturbed Zone."—The "disturbed zone" possesses physical properties that are different from the earth mass. It is a kind of pedestal for the structure and moves down with it during the entire settlement period. Professor Hennes states that the "disturbed zone" needs more investigation. The writer welcomes this statement and believes, in addition, that this study should follow a certain order, beginning, for instance, with the observations of pressure at the base of full-sized structures as formulated in one of the writer's conclusions. It is generally assumed that the basic cause of the formation of the "disturbed zone," in dry cohesionless sands, is the lateral escape of particles. The edges thus become unloaded and the central part stressed. This is the opinion expressed by Professor Tschebotareff in his able discussion. Sometimes it is believed that in certain cases there is no "disturbed zone" at all. For instance, Mr. Hrennikoff thinks that such is the case when the foundation bed is situated between the levels of capillary and gravitational water; and, generally, that the formation of the "disturbed zone" is due to mutual sliding of particles.

To these two causes of the formation of a "disturbed zone" (lateral escape and mutual sliding) the writer wishes to add "packing in" of the particles if there is a more or less considerable settlement and if the structure is wide. It is obvious that if a foundation is 200 ft wide, particles at the center will not travel 100 ft in either direction to escape at the edges. They will stay where they are and will tend to penetrate downward, forming the "disturbed zone." The phenomenon of "packing in" may be observed even in the case of small loaded areas as may be seen from the following field experiments made by the writer.

A wooden platform, 4 ft by 1 ft in size, was loaded uniformly with 12 tons of pig iron that remained on the platform for more than two weeks. The experiment was made in East Hartford, Conn., the soil being fine, cohesive, silty sand.[51] An excavation through the middle of the loaded area, where the stress distribution could be considered plane, revealed a clear-cut, semi-circular "disturbed zone" (Fig. 24). Thin natural sand layers of a color different from the remainder of the mass were bent (but not broken) by the action of the load, and from their shape it could be concluded that the "disturbed zone" was formed by the packing in of soil particles from beneath the loaded platform. There was also some bulging around the load. Another experiment in Stratford, Conn.,[52] was made at the bottom of a rectangular excavation, 13 ft deep, the soil being fine sand with the water-table 1 ft below the bottom of the excavation. Two posts, A and B (Fig. 25), 1 sq ft in cross-section, were loaded to 5 tons each, and the settlement of the posts and of the adjacent area was observed. Post A was simply placed on a sand surface, whereas previous to placing Post B, the corresponding 1 sq ft area was enclosed in a coffer-dam of $1\frac{1}{2}$-in. boards driven 3 ft deep. The earth from this coffer-dam was not excavated. There was bulging arount Post A, whereas the entire area surrounding Post B simply

[51] *Engineering News-Record*, Vol. 109 (1932), p. 782.

[52] Annual Rept., Connecticut Soc. of Civ. Engrs., 1934, pp. 70–81.

settled. Since in the latter case there was no bulging, and no possibility of the particles escaping laterally, it was concluded that the entire volume of soil corresponding to the settlement sank until it was packed both into the coffer-dam and into the "disturbed zone" in the same manner as in the East Hartford experiment.

The writer believes, furthermore, that in the case of a wide structure the "disturbed zone" is not geometrically similar to that under a smaller model. Other opinions as to the formation of a "disturbed zone" below a rigid loading plate, are: (*a*) Fröhlich's "plastic" zone [53] with an elastic nucleus inside (Fig. 26) which, in the belief of the writer, may occur as an exception only when

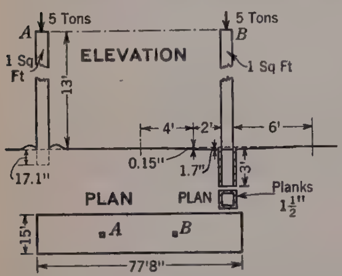

FIG. 25.—LOADING EXPERIMENT AT STRATFORD, CONN.

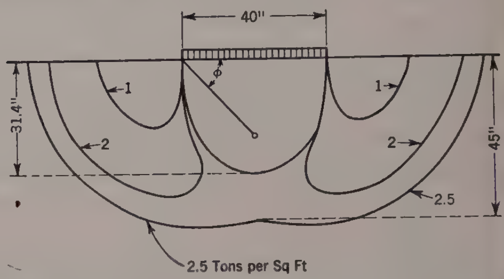

FIG. 26.—GRADUAL FORMATION OF A DISTURBED ZONE AFTER FRÖHLICH

there is no settlement and the loading plate itself is non-rigid; (*b*) Terzaghi's rough mechanical model of the "disturbed zone," [54] composed of sand grains glued with asphalt, the bonds between asphalt and sand being broken as the increasing load reaches a certain limit (Fig. 27(*a*)); (*c*) A. Nádai's disturbed zone [55] in a metallic plate which is formed when a rigid metallic punch is applied to the plate and is revealed upon heating and re-crystallization (Fig. 27(*b*)). A "disturbed zone" under a rigid tool, working in metal, has been reported on several occasions. It follows that the main cause of the formation of a "disturbed zone" is not the lateral escape of particles (which is merely a possibility), but rather "packing in" as mentioned previously.

Concentration Factor.—This item appears in almost every discussion. It is mentioned by Dr. Fröhlich, who is the original author of this theory. Mr. Taylor rightly states that the free use of the "concentration factor" is permitted only when the extent to which it is applicable is held clearly in mind. The writer wishes that such sound judgment might also be the guiding principle in applying the Boussinesq formula. "Under the footing," states Mr. Taylor, "progressing from the edge to the center, this factor will decrease from point to point and also it will decrease with increasing depth. Thus, the 'concentration factor' at different points for the given case may vary as much as from 8 to 4 * * *." Mr. Feld indicates that the broad application of Equations (1) and (6) containing the "concentration factor" is possibly open to criticism.

[53] "Druckverteilung im Baugrunde," p. 76 (1934).

[54] "Theorie der Setzungen von Tonschichten," von Charles Terzaghi, M. Am. Soc. C. E., and O. K. Fröhlich, p. 13, Fig. 8 (1936).

[55] "Plasticity," by A. Nádai, p. 249, Fig. 312 (1931).

Professor Tschebotareff discusses the matter in a very comprehensive manner and comes to the conclusion that a "concentration factor" changes in value with the distance from the center to the foundation toward its edges. At a certain depth of foundation beneath the soil surface the resistance to the lateral yielding would become so great that a sand would behave like a solid material with a "concentration factor," $n = 3$. Mr. Hrennikoff states: "It seems improbable that the same value of n should properly describe the state of stress both near the foundation where the soil is disturbed and outside the zone of disturbance." Professor Hennes expresses an analogous opinion stating that it is difficult to understand how the "concentration factor" can be made to cover the situation which arises in the "disturbed zone."

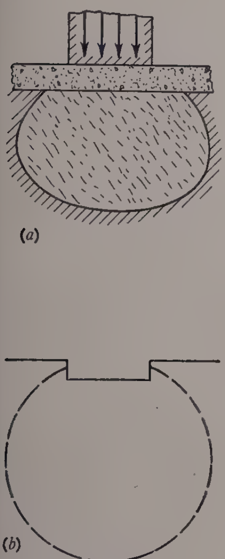

FIG. 27.—DISTURBED ZONE: (*a*) TERZAGHI; (*b*) NÁDAI

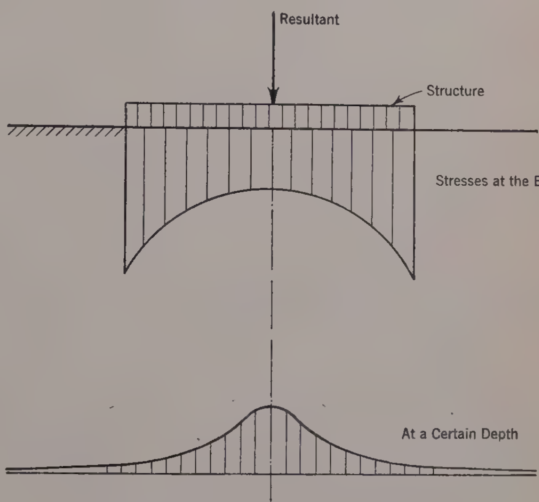

FIG. 28.—APPLICATION OF SAINT VENANT'S PRINCIPLE TO THE COMPUTATION OF STRESSES IN AN EARTH MASS

The writer wishes to make it clear that he never intended to suggest that stresses within the "disturbed zone" could be computed using Fröhlich's formulas. It is impossible to do so since these formulas have been designed for homogeneous and isotropic bodies as carefully specified in the paper, and the "disturbed zone" represents nothing but a foreign body within a relatively homogeneous and isotropic mass. As a rule, stresses never are computed close to sections where forces are applied. In textbooks on elasticity, points of application of forces are generally circled to show that at that section there is a zone that does not obey the formulas developed. In an analogous manner the progress report of the Committee of the Society on Earths and Foundations, leaves a blank space close to the ground surface when tracing a pattern of isobars.[56]

[56] *Proceedings*, Am. Soc. C. E., May, 1933, pp. 782–783.

As to the value of the "concentration factor" which changes from 8 at the edge of a foundation to 4 at its center, the writer would be very willing to examine this statement if he knew precisely what the center of a foundation is. To clarify this idea, consider Point X in Fig. 6. It practically coincides with the centroid of the plan of the structure; and the vertical pressure under this point, if computed according to the Boussinesq formula, is greater than under any other point of the structure. Hence, Point X is the center of the foundation shown in Fig. 6; on the other hand, it is located at the very edge. Is the "concentration factor" at that point a maximum or a minimum?

Undoubtedly, there is an "edge action" or "edge disturbance" along the perimeter of a round foundation; and this is shown by curves in Fig. 11(*b*) and 11(*d*). Owing to the phenomenon of "packing in," it is also very probable that there is stress concentration at the middle of such a foundation and Fig. 10 was prepared to show this phenomenon. It may be seen therefrom that the value of the "concentration factor" at the middle of a round experimental plate was more than 6 and not 4. Fig. 10 shows also that the "concentration factor" decreases with the depth. The writer was the first to establish this fact.[57] His general opinion concerning the "concentration factor" was published in 1936, as follows:[58] "The physical nature of this new conception is not very clear as yet; and perhaps this is simply a temporary tool which will be put aside when the theory has advanced in this province." In this paper the writer recommends field research as the only way to solve this problem. Apparently, mere speculation or mathematical formulas are of no help in clarifying the bases of such a theory.

In designing the "reduced area method" the writer carefully avoided recommending any numerical value of the "concentration factor," leaving its determination to the eventual field research as stipulated in the "Conclusions." The two values of the "concentration factor" given at the beginning of the paper have been secured in small-scale experiments and cannot be used in practice.

If field research shows that variable values of the "concentration factor" should be used in computations (and this is not certain as yet) the "reduced area method" will still hold. Mr. Feld rightly remarks that the graphical method described must be repeated for each point at which the knowledge of the stress is desired. It is equally easy to apply the method in question to different points either using the same value of the "concentration factor" or changing it from point to point. In any event, a single value of the "concentration factor" will suffice for smaller areas; and it should be noted that curves in Fig. 9, to which Mr. Taylor objects, have been traced for this case.

Summary.—It is premature to argue about the law that governs the distribution of the "concentration factor" around the base of a structure before the corresponding field research is done. Even the incomplete field evidence at the disposal of the writer does not agree in any respect with the speculations advanced by most of the discussers.

Test of Fröhlich's Formulas by Saint Venant's Principle.—Stresses and strains may be distributed arbitrarily within the disturbed zone without influencing the

[57] *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 1089.

[58] *Proceedings*, International Conference on Soil Mechanics, Vol. 3, p. 74 (Paper E–15).

general trend of stress distribution in deeper strata which always shows overloading close to the center, if the foundation is uniformly loaded and round or rectangular in shape. Actually, according to Saint Venant's principle, stresses at a point remote from the loaded part at the surface of a body are practically the same as if that part were replaced by a statically equivalent system. Therefore, in all cases of loading, the pressure distribution curve tends to have its major ordinate close to the direction of the resultant of forces acting at the given loaded area although stresses at the base of the structure or close to it may be thrown toward the edges (see Fig. 28). Upon application of the principle of superposition to a uniformly loaded circular disk both the Boussinesq and Fröhlich formulas give satisfactory results; namely, in both cases stresses are concentrated under the center of the disk as they should be according to Saint Venant's principle. This means that in both cases the principle of superposition holds, at least approximately.

Principle of Superposition.—It has been already shown that, apparently, the principle of superposition always involves an approximation, and the discussion which follows is a proof of this statement. Generally, the problem of superposition as applied to soils is not new. Its origin may be traced to the work of Bastian,[59] about 1906. Kögler and Scheidig [60] discussed the problem of superposition in soils and came to the conclusion that it applies approximately. In numerous "loading experiments" made in the past fifty years, the principle of superposition was used freely. The test procedure was to: (1) Measure the pressures caused by the unloaded earth (mostly sand) at the bottom of the experimental box (Reading A); (2) measure the pressures caused by both the earth and the load (Reading B); and (3) subtract Reading A from Reading B to obtain the pressure caused by the load alone. It is obvious that this subtraction is the application of the principle of superposition in an implicit form. An exception to this procedure is noted in the experiments by Faber which have been introduced in the discussion by Professor Tschebotareff. However, the results of Faber's tests check with others, quantitatively at least. Model experiments show that small disks settle practically proportionally to the square root of their areas; or (which is the same), proportionally to their diameters. This rule cannot be applied, however, in comparing contact settlements obtained in small loading tests with those observed under actual structures. A large structure settles less than it should according to that rule; and, in the writer's opinion, application of the principle of superposition to any earth mass furnishes stress excess at some points of the foundation. The writer greatly appreciates the able discussion of the principle of superposition by Mr. Hrennikoff. Especially fascinating is his statement that the principle of superposition does not hold in the case of earth disturbance, because the boundary of the disturbed zone is not the same for the three loads: Loading 1, Loading 2, and Loading 1 + Loading 2. This very potent argument, which appears to contradict Fröhlich's theory of "concentration factor," in reality proves that the principle of superposition never holds, except perhaps for very light loadings. This is because the disturbed zone is being formed in all cases of heavy loadings, including the case of metals (see Fig. 24 and Fig. 27(b)).

[59] "Organ für die Fortschritte des Eisenbahnwesens," 1906.

[60] "Die Bautechnik," 1927–1929.

The mistake made by Fröhlich and all others (including the writer) who have written about the "concentration factor" was the lack of a clear-cut statement that the use of the principle of superposition in this case (as in others, incidentally), is an approximation.

Principal Stresses: Coulomb Formula.—At a greater depth, soil particles are compressed by the action of principal stresses; no shear failure is possible there except small movements and changes in the shape of individual grains. Hence, Mr. Middlebrooks' question, as to whether the major principal stress or the vertical pressure is to be taken into consideration in computing consolidation, is to be answered in favor of the action of the major principal stress; and these stresses are to be computed at all points where their action may produce a more or less considerable effect. A doubt as to why vertical pressures are used instead of principal stresses in computing settlements, is expressed also by Mr. Hrennikoff. In this connection the writer wishes to state that principal stresses are to be computed from the combined action of the weight of the earth mass and the given load; and this furnishes lines of major principal stresses which at greater depths possess almost vertical tangents so that the actual error in replacing the major principal stress with the vertical pressure may be insignificant in many cases. The action of the minor principal stress is thus disregarded.

Some explanations as to Equation (35) should be included under this caption. This formula states the "condition of plasticity" of a granular mass expressing the least ratio of principal stress as a function of the angle of friction. Without arguing about the angle of friction (which, after all, is indefinable), suppose that ϕ is the correct value of the angle of friction at a given point located at a great depth and that Equation (35) is not satisfied. Then, according to Mr. Hrennikoff, a very small sliding should occur; but this sliding would not modify the principal stresses, s_1 and s_2, at the given point since they depend only on the value of the given load and its position. Hence, the situation still remains the same, and the sliding should continue indefinitely. What happens in reality is not so much the mutual slip of particles, as the change in shape of individual grains due to the shearing stress developed in each of them considered as a separate body. As to the Coulomb formula (Equation (34)), it is valid beyond question, at a given point, but only under given circumstances, since the actual value of c is also variable. Both coefficients, f and c, have been introduced to represent, in a more or less tangible form, some elaborate phenomena; and it is inadvisable to interpret them too literally and to base categoric statements on formulas containing these values.

Discussion of Fig. 17.—Mr. Taylor is to be commended for preparing Fig. 17 which conveys a number of useful ideas. At the top, Fig. 17(*a*) shows that there is no continuity of strains at the upper layers of a sand mass loaded with a non-rigid load. The break at the edges of the loaded area indicates that this is not yet a body in the sense used in mechanics, but merely an agglomeration of particles. The upward movement shown in the sketch is simply a dynamic process, or movement of particles which have not been deprived of all degrees of freedom as in a body. Hence, such terms as "modulus of elasticity" or "modulus of deformation" cannot be applied to the upper strata of a sand mass such

as that shown in Fig. 17(a). This mass becomes a body only at a certain depth. The stress distribution at the bottom of Fig. 17(a) is that which the writer obtained by using Saint Venant's principle. The most ardent defender of the validity of the principle of superposition in sands would not be able to prepare a more convincing sketch than Fig. 17(a). The remainder of Fig. 17 contains still more useful information as to settlement in deeper strata. Stresses in a rigid structure are always influenced by this settlement, whether or not it is even. Investigators in soil mechanics must correct their views on the given subject to accord with this valuable statement by Mr. Taylor.

Small Strains; Stress-Strain Compatibility.—Admittedly, the theory of the semi-infinite elastically isotropic body is based on the assumption that strains are very small, as already stated at the beginning of this discussion. In turn, stresses may be very great and even infinite, as at the edges of an absolutely rigid plate (Equation (16)), provided, however, that the corresponding strains are kept below a reasonable limit and are continuous. Apparently, the latter conditions are not satisfied in some papers on soil mechanics[26, 27] mentioned by Dr. Fröhlich.

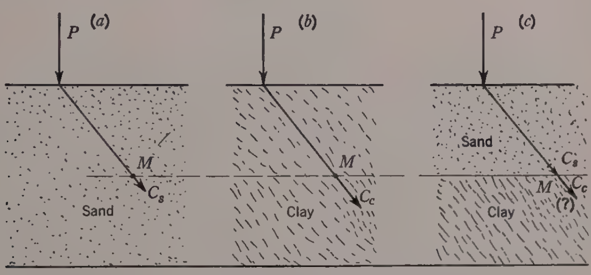

FIG. 29.—STRESS-STRAIN COMPATIBILITY

An example of the lack of stress-strain compatibility is given in Fig. 29 which, for the sake of simplicity, is treated as a plane problem. The plane stress distribution does not depend on the elastic constants of the material; hence, the stress at Point M is the same whether the earth mass is of sand or of clay (Fig. 29(a) and Fig. 29(b)). The situation is quite different if Point M is in a plane that separates sand and clay (Fig. 29(c)). Since the elastic constants of these two materials are different, there should be two different displacements, C_s and C_c, at Point M. Since in reality there is only one displacement, this means that there should be some stress re-arrangement at the plane in question. This circumstance may be of special interest to Messrs. Middlebrooks and Buisson who are working on the stress determination at a surface separating two different materials.

Equivalent Load at the Center of Gravity of a Loaded Portion.—Mr. Feld proposes to sub-divide the loaded area and to apply equivalent loads at the center of gravity of each sub-division. Apparently, this method may indicate some

[26] "Druckverteilung unter einem gleichmässig belasteten, elästischen Plattenstreifen, welcher auf der Oberfläche des elastisch-isotropen Halbraumes liegt," von H. Borowicka, International Assoc. for Bridge and Structural Eng., Second Congress. Final Rept., 1937.

[27] "Die auf dem elastische-isotropen Halbraum aufruhende, zentral symmetrisch belastete elastische Kreisplatte," von A. Habel, *Der Bauingenieur*, 1937, H. 15–16.

excess in pressure if the point where Pressure p_z is determined is not far from the vertical line passing through the center of gravity of the uniformly loaded foundation. Such is the case of the example in Fig. 6, in which the difference in values computed by Mr. Feld and by the writer is about 14% taking the writer's value as 100. If the point where p_z is to be determined, is outside the foundation, Mr. Feld's results may be smaller than those of the writer, although the difference may be practically negligible. For the sake of simplicity, the latter statement will be proved for the case of a uniformly loaded strip of a length, a, situated at the boundary of an elastically isotropic body ($n = 3$, $n_1 = 2$). Assume that the radius vector to the equivalent line load, $\bar{p} = p\,a$ (Fig. 30), makes an angle, α_0, with the vertical. Using Equation (33) by Mr.

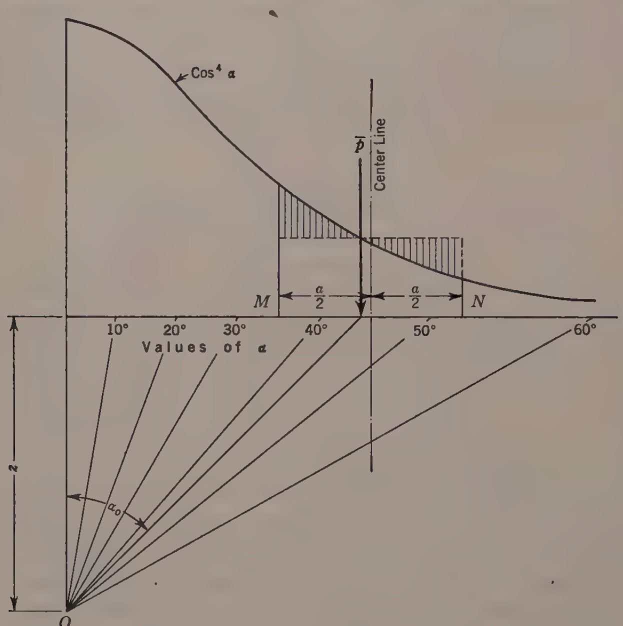

Fig. 30.—Comparative Action of Concentrated and Distributed Load

Feld, substituting $\alpha = \alpha_0$; making p_z equal to the pressure of the strip load; and remembering that $dA = da$:

$$\frac{2\,\bar{p}}{\pi\,z}\cos^4\alpha_0 = \frac{2\,p}{\pi\,z}\Sigma\,da\cos^4\alpha \quad \text{.....................(40)}$$

Multiplying and dividing the right side of Equation (40) by a:

$$\cos^4\alpha = \frac{1}{a}\Sigma\,da\cos^4\alpha \quad \text{.....................(41)}$$

The right side of Equation (41) is the average ordinate of the sub-divided area, $\Sigma\,da\cos^4\alpha$, corresponding to the loaded portion, $M\,N$. Equation (41) shows that a line load, $\bar{p} = p\,a$, and the distributed strip load, $p\,a$, produce equal

vertical pressures at Point O, if the line load, $\tilde{p}$, passes through the centroid of the part of the area, $\Sigma\, da \cos^4 \alpha$ (corresponding to the loaded part, $M\,N$) (see Fig. 30). The vertical passing through this centroid may not pass through the middle of the loaded part, $M\,N$, however.

Graphic Integration.—Graphic integration of expressions of the type of $\Sigma\, d\omega\, f(\alpha)$, (in which $d\omega$ is an element of area and $f(\alpha)$, a trigonometric function), is a simple operation if the given area, ω, is sub-divided into circular rings, and if these rings are represented as "transformed areas," in the manner described in this paper. An example is given by Mr. Eremin. Since the angle designated by Kögler and Scheidig as ϕ is assumed to be constant, the right side of Equation (39) must be broken into two integrals:

$$p_z = c\,[\Sigma\, dA \cos^5 \alpha - \cot \phi\, \Sigma\, dA \sin \alpha \cos^4 \alpha] \dots\dots\dots (42)$$

in which c is a constant to be determined by Equation (39). The integral, $\Sigma\, dA \sin \alpha \cos^4 \alpha$, may be obtained graphically from the integral, $\Sigma\, dA \cos^5 \alpha$, by multiplying each ordinate of the curve which bounds the area expressing the latter integral, by the variable, $\tan \alpha$; thus:

$$[dA \cos^5 \alpha] \tan \alpha = dA \sin \alpha \cos^4 \alpha \dots\dots\dots\dots\dots (43)$$

Professor Converse proposes an interesting simplification of the writer's "reduced area method." Instead of drawing the "reduced area," Professor Converse draws a "pressure curve" (which, after all, is the reduced area), all ordinates of which are multiplied with the coefficient, $\frac{p\,n}{2\,\pi\,z^2}$. The possibility of a complete elimination of graphical construction also merits attention. Simplifications of the "reduced area method" are always welcome because this method may be applied not only for determining vertical pressures in an earth mass, but also for solving several other problems of mechanics. Mr. Buisson's remark that the method of graphical integration has been known for years is true, of course; but the principal feature of the writer's method consists not in the application of the graphical integration as such, but in its combination with the transformation of the loaded area, and this considerably simplifies the procedure.

Failure Lines.—The writer has been much interested in Mr. Buisson's studies of "failure lines" which according to his statement (see following Equations (42)) are close to the trajectories of maximum shearing stress. In a personal conversation with the writer, Hardy Cross, M. Am. Soc. C. E., expressed the following idea: Since the shearing stress and the shearing value of the earth material are not constant along such a trajectory, equilibrium will be broken first at some part of the theoretical "failure line." Then, this part of the "failure line" will be "out of service," and the body under consideration will therefore be transformed to a different condition of stress. Subsequent failure will be along a line or surface which may well be quite different from the original line. In about 1922 the writer visited a locality in Russia where previously many landslides had occurred, and was shown a mass of land about to slip. There was a crack at the top, and the entire wedge was literally hang-

ing at the remainder of what may be called "shearing surface." It is obvious that the stress condition at this surface was not the same as determined by theory, exactly as Professor Cross states.

Two questions arise in the light of these statements: (*a*) Why are "failure lines," as observed in model experiments, more or less close to the trajectories of maximum shearing stress? and (*b*) do small scale models truly represent "failure lines" in an actual case? These questions will be answered simultaneously.

It should be noticed that "failure lines," "slip lines," or "Lüders' lines," as observed in the case of metals also "have a shape similar to the shearing stress trajectories in the most highly stressed portion under a concentrated load.* * * How nearly exact this is, however, has not yet been investigated."[61] The situation being thus rather obscure, only hypotheses can be advanced. In the writer's opinion a quick application of an excessive load to a small model causes instantaneous failure in the "highly stressed portion," both in metals and in soils, which may be compared to an explosion; and shear occurs along the trajectories of maximum shearing stress, which offer least resistance. Apparently, this model action has no analogy in building practice since cases of quick loading (without impact) are exceedingly rare. Another type of model action as described by Mr. Buisson is slow changes in loading. The earth mass consolidates and flows plastically; and, finally, under a "critical load," fails. In this case particles move plastically along certain trajectories close to those of maximum shearing stress.

What happens in reality is that a building of a constant (and not variable) weight begins to tip and after a certain interval of time (in some cases 24 hr), stops. There is no doubt that this is a shear action consisting of both a separation of a part of the mass from its remainder, and plastic flow. Small models as described in the discussion do not explain such failures; and, obviously, "failure lines" in these cases are not trajectories of plastic flow but rather a succession of shearing surfaces as described by Professor Cross.

Experiments conducted by Mr. Buisson are of great value for students of physical properties of soils. For instance, they show clearly the influence of consolidation on the shearing value, or the fact that a soil at the liquid limit still may possess a shearing value. The method of recording the path of particles by taking successive pictures and superposing them is very interesting. W. S. Housel, Assoc. M. Am. Soc. C. E., applied a similar method recording the movement of particles by a time-exposure photograph.[62]

Other Remarks.—Some of the discussions of this paper are veritable contributions to the field of soil mechanics, and only a few items will be mentioned in addition to those the writer has already discussed. Professor Tschebotareff gives a comprehensive discussion of the influence of some geological factors on the design of structures, and makes valuable additions to the list of research projects cited by the writer. Professor Hennes discusses the necessity of "statistical" approach to questions of settlement. The writer believes that similar methods may be adequate in spreading popular knowledge of soil mechanics after corresponding items have been studied formally.

[61] "Plasticity," by A. Nádia, p. 251 (1931).

[62] *Engineering News-Record*, November 24, 1932, p. 626.

In conclusion, the writer wishes to thank all discussers for advancing so many progressive and fruitful ideas.

Correction for *Transactions:* On page 1968, Line 22 from the bottom, insert the following footnote after "friction angle": [41] "Etude de la plasticité dans le cas d'une bande de largeur indéfinie chargée uniformement," by M. M. Buisson, Travaux, Mars, 1937; on page 1968, Line 11 from the bottom, insert the following footnote: [42] "Conditions de stabilité du sol," by M. M. Buisson, Travaux, Decembre, 1936; on page 1969, change the sentence beginning on Line 18, to read "First, the basic hypotheses must be checked. Only by actual testing, etc."; and, on page 1969, add to Footnote 44 (corrected to Footnote 41) "; also, 'Ein Grundgesetz der Tonmechanik und sein experimenteller Beweis,' by Leo Rendulic, *Der Bauingenieur*, Heft 31, 32, 1937." Mr. Buisson is incorrect in stating that the writer has made an error in computing horizontal pressure, P_h, on page 677. By correspondence Mr. Buisson states that he misinterpreted the writer's nomenclature and consequently wishes to delete from his discussion the last ten lines of text (not footnotes) on page 1967, beginning "An error was made" * * * etc. Furthermore, delete the first two paragraphs on page 1968, ending "principal stress trajectories." The first paragraph under "Part I.—The Method." should then end with the sentence "This is an application of a graphic integration method which has been well known for many years, and which makes it easy to solve for the principal stress trajectories."

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
DISCUSSIONS

MEASUREMENT OF DÉBRIS-LADEN STREAM FLOW WITH CRITICAL-DEPTH FLUMES

Discussion

BY MESSRS. HAROLD K. PALMER AND FRED D. BOWLUS, AND HARRY F. BLANEY

HAROLD K. PALMER[14] AND FRED D. BOWLUS,[15] MEMBERS, AM. SOC. C. E. (by letter).[15a]—Débris-laden streams may be divided into three classes: (1) Muddy water in which the débris consists of fine silt, ordinarily considered as that passing a 200-mesh sieve, distributed uniformly throughout the entire cross-section; (2) water carrying a suspended load of coarser sandy material, the bulk of which is carried along in the lower part of the stream; and (3) water transporting rocks which roll and slip along the bed.

Any clear-water measuring device will give satisfactory measurement of the Class (1) stream. The authors state that with a level throat the addition of débris caused the critical point to move down stream, but that with a throat built on a 3% grade the critical point did not move when débris was added. In a throat with a level floor the acceleration of the water is caused by the difference in pressure due to the fall in the water surface. If there is a suspended bed-load which increases the specific gravity of the lower strata, but not of the surface, the force acting on the lower strata will be the same whether or not there is a bed-load. However, the weight of the débris-laden water being greater, this force must act for a greater distance to produce the same velocity. When the throat floor is on a grade, gravity accelerates the heavy water as much as the light water, but since the acceleration is caused by the slope of the bottom and the slope of the surface, the critical point may still be displaced somewhat by the addition of the débris.

All attempts in the past to make use of critical-depth measurements have failed because of the difficulty of locating the point of critical depth. Near

NOTE.—The paper by Messrs. H. G. Wilm, John S. Cotton, and H. C. Storey, was published in September, 1937, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: February, 1938, by Messrs. R. L. Parshall, and Martin A. Mason; and March, 1938, by Edwin S. Fuller, Esq.

[14] Chf. Draftsman, Los Angeles County Sanitation Dists., Los Angeles, Calif.

[15] Res. Engr., Los Angeles County Sanitation Dists., Los Angeles, Calif.

[15a] Received by the Secretary February 28, 1938.

critical velocity there is no appreciable change in energy for small changes in depth, therefore, the water surface is very unstable and the resulting waves interfere with the measurements. Placing the flume on a 3% slope, as the authors have done, causes it to avoid the resulting waves, since the water quickly attains a greater velocity, resulting in a concave water surface with a single point of critical depth. Since this is not at the same position in the throat for all flows, the stilling-well will show critical depth for only a small range of flow. Other flows will require an empirical formula which will apply only to one ratio of channel and throat width.

It is unfortunate that the authors had no way of measuring the quantity of débris added to the stream while making the experiments at San Dimas, because, neglecting other effects, it should at least add its volume to the flow. If the débris consisted of silt uniformly distributed throughout the stream, its only effect on measurements would be to indicate the combined volume of water and silt. If it carried a bed-load in the lower strata, its apparent roughness coefficient in a smooth channel might be slightly greater than for clear water, and it would accelerate more slowly in the throat. If, in addition to the débris carried in suspension, rocks were rolled on the bottom, there would be a further retarding of the flow, with increased uncertainty.

If the change in flow in the throat, due to the addition of débris, is materially different from the change in flow in a smooth channel, a correction for bed-load might be found from a second measurement in a smooth channel about 50 or 100 ft below, after the water surface indicated a stable flow. If the bed-load does not increase the apparent roughness coefficient this second measurement would be useless.

Whatever form the throat and transition may take, the method requires a point of critical velocity which, in turn, is dependent upon a change from mild to steep flow. The mild flow may be produced by means of a jump above the transition if the channel of approach is on a steep grade to prevent deposits of débris, but proximity to a jump is not conducive to accurate measurements. However, considering the past failure to make a successful meter which depended on critical depth measurements, it seems likely that even near a jump a measuring point above the throat would give better results than one near the point of critical depth. This might require a longer transition than the authors suggest, but the writers believe it would be worth trying.

The paper reveals two interesting and important results which have already been established: First, the exact shape of the transition to a Venturi flume is unimportant as long as it does not waste energy; and second, the Parshall flume can be used to greater depths than those covered by previous experiments. This will permit its use in many new places and increase the range for any one flume by using a deep and narrow throat instead of a wide shallow one.

The need for high velocities to prevent débris deposits would require a narrower approach channel than three times the width of the throat. If no measurements are made above the throat it would seem that the channel should be only wide enough to insure critical velocity in the throat at all flows. Probably 25% greater than the throat width would suffice.

As the authors state, they have made only a start and much experimenting will be needed to reach a final solution, and when that solution is found it is probable that the results will be only approximate. Even so, approximate results will be better than those now available, and the problem is well worth solving.

HARRY F. BLANEY,[16] M. AM. SOC. C. E. (by letter).[16a]—Results of experiments on the San Dimas flume are presented in an excellent manner in this paper. The complex problem of measuring intermittent débris-laden streams having wide variations in flow has interposed difficulties in mountain water-shed studies in Southern California for many years.

Since the Parshall flume was included in the studies made by the authors, perhaps it would be well to call attention to the fact that this flume was not originally designed to measure débris-laden streams, such as exist in the mountain water-sheds, and that heretofore it has been used principally in the valley areas. For many years Parshall measuring flumes of various sizes have been used successfully in California under less extreme conditions than those in the San Dimas Experimental Forest.

The application of flumes to measurements in mountain canyons involves problems not ordinarily encountered in valley areas. The stream gradient is steep (often 10%, or more) and the water tends to cut a narrow channel, travel at a relatively high velocity, and carry a large bed-load as well as a considerable volume of suspended material.

A large flume, for example, placed directly in a stream channel where the grade is as steep as 10% may have a rating curve quite different from the standard calibration. It may pass much less water than that given for the lowest gage heights in the standard tables. The reason for this is that a small stream of water entering the center of a wide flume at high velocity tends to proceed through the center without changing its cross-section greatly, leaving dead water along each side of the wide flume. At high stages, the flume may pass more water than is indicated by the gage height in the standard tables, because of the high velocity of approach across the entire width of the flume. Large flumes, when used under such extreme conditions, should be rated in place.

In connection with some experimental studies in 1931, the writer was confronted with the necessity of measuring stream flows varying from 0.1 cu ft per sec to more than 20 cu ft per sec in mountain areas.[17] In order to measure large winter flows and also to obtain an accurate record of small summer flows, a combination flume such as that shown in Fig. 15 was found desirable. This combination consists of two Parshall measuring flumes, one 2 ft and one 3 in. wide, arranged so that both large and small flows pass through the converging section of the 2-ft flume. The small flows are by-passed from the dip in the large flume into a basin above the 3-in. flume and thence through this latter

[16] Irrig. Engr., Bureau of Agricultural Eng., U. S. Dept. of Agriculture, Los Angeles, Calif.

[16a] Received by the Secretary March 5, 1938.

[17] "Water Losses Under Natural Conditions from Wet Areas in Southern California," California State *Bulletin No. 44*, Div. of Water Resources, 1933, p. 88.

flume, while the greater part of the large flows continues on through the 2-ft flume. A record of the larger discharges is obtained by a recorder operated by a float in a stilling-well connected to the larger flume, and a record of the smaller discharges is obtained by a flow recorder operated by a float in a stilling-well connected to the smaller flume. Some overlap is provided, so there is a small range during which a record may be obtained from both flumes. Two separate recorders may be used, or one duplex recorder is sufficient if it is desired to record gage heights only. Several combination flumes of this type were

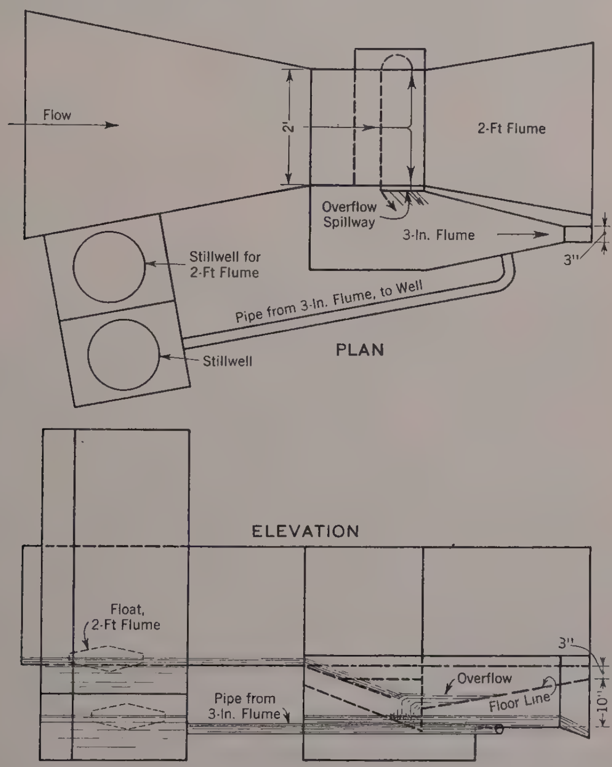

FIG. 15.—COMBINATION PARSHALL FLUME FOR MEASUREMENT OF WATER AT BOTH HIGH AND LOW STAGES

installed in the mountain sections of Coldwater Canyon and Devil Canyon, near San Bernardino, Calif. These flumes gave satisfactory results for the few years they were operated.

There are denuded water-sheds in Southern California where no type of flume will measure the débris-laden stream flow accurately. This was demonstrated by the run-off that occurred in the La Crescenta-Montrose area on January 1, 1934.[18]

[18] "Flood in La Cañada Valley, California, January 1, 1934," by Harold C. Troxell, Assoc. M. Am. Soc. C. E., and John Q. Peterson, *Water-Supply Paper 796–C*, U. S. Dept. of the Interior, 1937.

In 1928, the writer was one of several who suggested replacing a 30-in. rectangular weir with a 12-in. Parshall measuring flume, after the weir had proved to be very unsatisfactory in measuring the run-off from a 44-acre experimental denuded water-shed near San Bernardino. The flume operated satisfactorily under normal conditions but filled with débris when excessive run-off occurred. Some time later it was replaced with a 12-in. San Dimas flume. In February, 1936, this flume was also filled with débris after a heavy rain storm. A larger San Dimas flume has now (1938) been installed, and it is apparently operating satisfactorily under normal conditions. Whether the present installation will or will not solve the problem of measuring flows that carry excessive quantities of bed-load is a question that may be answered when it is put to the same test as its predecessors.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

PRE-STRESSED REINFORCED CONCRETE AND ITS POSSIBILITIES FOR BRIDGE CONSTRUCTON

Discussion

By Messrs. A. Floris, A. A. Eremin, and Ivan A. Rosov

A. Floris,[29] Esq. (by letter).[29a]—The problems of pre-stressing steel in reinforced concrete structures are treated in this interesting paper. In the methods described therein the pre-stressing is done by mechanical devices or by heat. However, in certain cases, the same results can be achieved quite naturally, without external influences.

In reinforced concrete trusses, for instance, the steel of the compression members can be embedded in concrete during construction, but the steel of the tension members is left exposed. After the forms are removed and the structure is under full load, the tension steel is covered with concrete.

The advantage of this method, due to Dr. Ulrich Finsterwalder,[30] is that cracks, otherwise unavoidable in the tension members, are almost eliminated and the appreciable secondary stresses are reduced considerably, because of the flexibility of the exposed reinforcement in the tension members. In this manner the angular changes at the joints, caused by the weight of the structure, take place almost without restraint.

A. A. Eremin,[31] Assoc. M. Am. Soc. C. E. (by letter).[31a]—The possibility of pre-stressing concrete is important in the economic design of reinforced concrete structures. However, the methods of pre-stressing are still very little known.

Note.—The paper by Ivan A. Rosov, M. Am. Soc. C. E., was published in September, 1937, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: November, 1937, by Messrs. Charles S. Whitney and Karl W. Lemcke; and December, 1937, by Messrs. Dean F. Peterson, Jr., and M. Hirschthal.

[29] Dipl.-Ing., Los Angeles, Calif.

[29a] Received by the Secretary January 24, 1938.

[30] "Untersuchungen über Knicksicherheit, die elastische Verformung und das Kriechen des Betons bei Bogenbrücken," von Dr.-Ing. Fr. Dischinger, *Der Bauingenieur*, September 3, 1937, p. 551.

[31] Assoc. Bridge Designing Engr., Div. of Highways, State Dept. of Public Works, Sacramento, Calif.

[31a] Received by the Secretary January 25, 1938.

An interesting method of pre-stressing a reinforced concrete tunnel lining was used in 1930 in the outlet tunnels of the dam across Dordogne River, in France.[32] At their lower ends these tunnels were bored through the ground formation with rock of inferior strength which was incapable of resisting the hydraulic pressure produced by a water head of 335 ft. Designed according to common practice, without pre-stressing, the tunnels would have required a concrete lining 3 ft 3 in. thick, which would have made the cost prohibitive. M. Mary developed a method of pre-stressing which reduced the lining to a thickness of only about 12 in. The arrangement is shown in Fig. 10. The concrete lining,

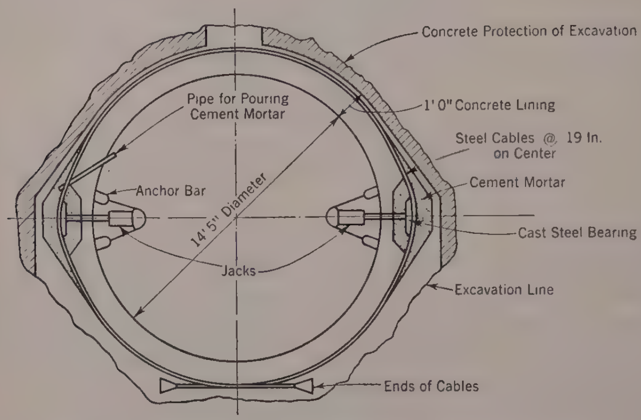

FIG. 10.—METHOD OF PRE-STRESSING A TUNNEL LINING

with an inside diameter of 14 ft 5 in., is reinforced with steel cables placed 19 in. between centers. The ultimate strength of the cables was 242 tons. The computed maximum stress, produced by the hydraulic static pressure, was 30 tons per cable, and the maximum stress developed by hammer action was about 10 tons per cable. The cables were pre-stressed to 152 tons, which was considered sufficient to resist internal hydraulic pressure in the tunnel and to reduce shrinkage cracks in concrete. A small coefficient of safety was used because design assumptions were successfully verified by testing a full-sized model concrete cylinder 20 ft long. The cables were pre-stressed by hydraulic jacks placed at the horizontal diameter of the tunnel, in such a manner that the original circular form of the cables was changed to an elliptical one. The forces from the jacks were transmitted to the concrete lining through steel bar anchors. After pre-stressing, a cement mortar was poured around the cables and bearing plates. The jacks were removed when the cement mortar had set sufficiently to reach the prescribed strength.

In developing the method of pre-stressing considerable attention was given to reducing the bond stresses between the steel cables and the concrete. Bond stresses generally increase in proportion to the increased stresses in the cables during pre-stressing, and are affected by the change in length of the cables as

[32] *Annales des Ponts et Chaussees,* April, 1936, p. 423.

shown by Equation (23). Special provisions for reducing the bond stresses in a concrete lining were found necessary. Heavy asphalt coating of the cables was not considered sufficient; and, furthermore, asphalt reduces the resisting strength of concrete. Finally, it was decided to place the cables in steel pipes permitting them to slide freely. The design and construction of the outlet tunnels at Dordogne Dam provide a valuable demonstration of the possibilities of pre-stressing reinforced concrete and serve to support the author's conclusions. .

IVAN A. ROSOV,[33] M. AM. SOC. C. E. (by letter).[33a]—The pre-stressing of reinforced concrete is a new idea which marks great progress in this branch of engineering. It is only natural that a new idea should meet objection on the grounds that it is purely academic, impractical, difficult to embody in practice, and not suitable to the material.

The writer should acknowledge that his contribution was almost purely theoretical, when it was written early in 1934. Since that time, however, practice has completely justified his suggestions. Several pre-stressed bridges and roof trusses have been built and have developed exceptionally good properties. The writer has already mentioned the girder constructed in Germany in 1936,[16] which fully confirmed all expectations. This girder (which was actually tested) was followed by a design of a girder 198 ft long with a depth of only one-sixteenth of the span.

The Saale Bridge, at Alsleben,[34] Germany, with a span of 223 ft, is an example of a pre-stressed bowstring bridge. The success of this structure encouraged the construction of many other pre-stressed arched structures—especially hangars with spans longer than 320 ft.[35] Trusses are represented by hangars of the Caquot type constructed in France,[36] in which pre-stressing is done in a manner similar to the method described by Mr. Floris. For the most advanced design one should mention the Gerber (girder) Bridge, "Drei Rose," at Basle, Switzerland. Pre-stressing permitted increasing the span of this bridge to 410 ft, which is extremely unusual for a simply supported concrete structure. It is seen that pre-stressed bridges and roof trusses are already well established, at least in Europe. They can no longer be considered "academic."

As to the practical difficulties of pre-stressed construction, stated by Mr. Hirschthal as being "almost insuperable," the writer has already outlined the methods involved. Some of them are very simple, as is the lengthening of the reinforcement by means of electrical heating. The method described by Mr. Floris is even simpler and requires only that the compression members be poured first and the tension members later. In M. Freyssinet's method, which embodies the "treatment"[37] of the concrete, the pre-stress is obtained as a by-product of the latter procedure. This method was used under the difficult

[33] Asst. Engr., New York City Tunnel Authority, New York, N. Y.

[33a] Received by the Secretary March 2, 1938.

[16] *Concrete and Constructional Engineering*, April, 1936.

[34] *Beton und Eisen*, 1932.

[35] International Assoc. for Bridge and Structural Eng., 1936.

[36] *Travaux (Science et Industrie)*, 1936.

[37] *Memoires et Compte Rendu des Travaux* de la Société des Ingenieurs civils de France, September–October, 1935.

conditions presented by a narrow foundation at the Marine Station, at Havre, France. The work of fabricating and sinking the pre-stressed piles attained a speed of 25 ft per working day. Plant equipment was almost automatic and, as is stated by M. Freyssinet, "expenditure in material and labor was much less than for the ordinary process."[38]

Possibly the most complicated was the case of the aforementioned girder in which the degree of pre-stressing varied along the span; the stirrups were also pre-stressed and the concrete was "treated."[16] Even in this case the only additional equipment required, beyond the customary compressors, jacks, and mixers, was a stretching beam and a 5-ft section of a special steel form. To complete the operation for one 5-ft section (which included pouring the concrete, vibrating, hydraulic pressing, and heating) required only 2 hr. The entire girder, consisting of twelve sections, required 60 hr.

This case, and the foregoing list of structures already built, indicate that "insuperable" difficulties do not exist and that progress in methods of pre-stressing is rapid. The new "sursulfaté" cement developed in France is very promising.[39] This cement expands after hardening, the expansion reaching 0.003 in. per in., and the reinforcement, if embedded, would resist it creating an initial compression in the concrete.

Mr. Hirschthal states that the writer was too optimistic regarding the cost of special steels. The extra cost is actually small if proper methods are used to produce such steels. For bars $\frac{5}{8}$ in., or less, in diameter, M. Freyssinet used a mechanism, that he developed, which unrolled bunches of steel, welded the ends, and stretched the steel in one operation. At the cost of a "few centimes per kilogram," straight endless rods (which were tested at every point, including welds) with yield points of 114 000 lb per sq in. were obtained. The general cost for pre-stressed structures, as a rule, is considerably less. Mr. Eremin should be commended for his striking example of a tunnel lining that would be prohibitive in cost unless pre-stressing were used.

The writer cannot agree with those who state that pre-stressing is not adaptable to concrete because that material is not homogeneous and elastic. This general defect handicaps any concrete structure, those that are subjected to tension being affected much more severely than others. In the latter case the bond between particles is decreased (often entirely destroyed), and will not be restored if the tension is removed. This makes concrete unsuitable for repeated stresses which involve a wide range of tensile stress. On the other hand, if concrete is compressed, the particles approach one another, the bond is increased, and releasing it merely restores the concrete to its initial condition.

Concrete can even heal minute cracks by chemical conversion of calcium hydroxide to calcium carbonate,[40] and permanent compression aids this process. Freyssinet describes a case in which a pre-stressed pipe subjected to an internal pressure, ten times greater than normal, at last "failed," resulting in seepage and the appearance of fissures. However, after the head had been reduced,

[38] Rept. to Joint Meeting of Inst. of Structural Engrs. and Société des Ingenieurs civils de France, March, 1936.

[39] *Bulletin Technique* de la Suisse Romande, January, 1937.

[40] *Concrete,* April, 1937.

the fissures closed at once, and a few weeks later, the concrete having sealed the fissures, the pipe was as good as before.[16]

Pre-stressing eliminates tension entirely. Professor Dischinger states that a "reinforced concrete body subjected to compression only, has, like natural stone, almost unlimited life."[35] Mr. Hirschthal's reference to the lack of homogeneity and elasticity in the concrete should be of concern to designers of customary structures much more than to designers of pre-stressed structures.

The writer's method of analysis inspired Mr. Peterson's very interesting contribution containing a more detailed calculation of the loss in pre-stressing. However, when Mr. Peterson refers to the writer's example, and subtracts from the designing pre-stress the loss due to the plastic flow, he did not notice that this had already been done. In the text of the paper preceding the heading, "Theory of Pre-stressed Rectangular Beams," the writer considered possible losses, including the plastic flow, estimated them as 35 000 lb per sq in., and noted that this loss should be subtracted from the "preliminary pre-stress" to give the "designing pre-stress." The preliminary pre-stress in the writer's example is 36 500 + 35 000 = 71 500 lb per sq in., not 60 000 lb per sq in., as Mr. Peterson assumes. The example then remains intact. It is gratifying that Mr. Peterson's calculation for the loss gives 60 000 − 26 300 = 33 700 lb per sq in., which is almost identical with the writer's estimate of 35 000 lb per sq in. Plastic flow, however, has not been studied sufficiently, particularly for "treated" concrete. Such a study would be valuable in connection with pre-stressing. For this kind of concrete Mr. K. Lenk accepts the entire loss as only 21 000 lb per sq in.[41]

For diagonal tension, Equation (31) should be rewritten as $f_d = -\frac{v^2}{f}$. Tensile stresses exist although they are much smaller than for conventional design. In order to cancel them entirely the stirrups are also pre-stressed. In this manner, compression is introduced along two perpendicular axes and the problem of diagonal stresses is eliminated.

Replying to Mr. Hirschthal, the pre-stressed section is compressed at any point. This is the basic principle of the design. Bending moments acting over the section produce compression on one side of the section and tension on the other side. However, the tensile stresses are absorbed by counteracting compression and do not develop cracks. The area of the section remains constant, unlike the conventional case in which, after cracking, the section is decreased and the stresses are redistributed. The stability of the section permits the use of the law of superimposition and the separate consideration of the direct pressure and the bending moments produced either by loads, or by pre-stressing. This fact was ignored by Mr. Hirschthal. However, it is common practice to consider combined direct stresses and bending in the case when the entire section is under compression. If the writer assumed the concrete section as taking tension at one time, and then assuming the entire section to be in compression, it was because he considers superimposed bending moments separately in the former case and all stresses together in the latter case.

[41] *Beton und Eisen*, May 20, 1937.

The writer also "assumes the entire concrete section to be in compression; and yet he indicates the neutral axis in the proximity of the center, implying it to be the gravity axis and so computing his moments of inertia." This was because the neutral axis is considered for the bending moments only and, if the section is not cracked, it is the gravity axis and, of course, the moment of inertia should be computed about this axis.

Mr. Hirschthal considers "confusing" the fact that the writer takes the reinforcement into consideration and then drops it. The writer "drops" the reinforcement in two cases: When he considers the action of outside load, he takes the full section, including the reinforcement; but when he considers the action of the pre-stressing, he subtracts the area of the reinforcement, because the pre-stress is the force produced by the steel and is acting on the concrete in the form of bond between the two. The second case is that in which the writer passes from the general design with both top and bottom reinforcement to a design with bottom reinforcement only. He "drops" the top reinforcement, making $p = 0$. In either case, it could not be done otherwise.

Mr. Hirschthal wants Equations (20) and (21) derived directly. The writer selected the most logical method. He considered the general case and then passed to particular cases, equating some quantities to zero. This is the common method of solving problems. Equations (20) and (21) represent only a special case when $p = 0$ and the reinforcement is small. Certainly, every special case can be derived directly but there seems to be no need of complicating the problem with each separate solution.

Mr. Hirschthal's last remark is that the writer does not apply Test (*D*) to his examples. The writer applied this test (see text following Fig. 5 and Table 1) and found that Test (*D*) does not control, because higher pre-stressing is required by Test (*C*). Therefore, only the latter was considered.

Mr. Whitney prefers to compare the pre-stressed design with his method. The writer made his comparison with a commonly used method because it was the only way to reveal clearly the advantages of pre-stressing. Recently, many new methods of designing reinforced concrete have been offered by Professor Emperger,[42] Professor Steuerman,[43] Dr. Gebauer,[44] Professor Saliger,[45] Mr. Whitney, and others. New methods differ widely from conventional design and are objected to by many concrete authorities.[46] This is not the proper place for discussing them. Personally, the writer is inclined to accept Mr. Whitney's method as logical and well adjusted to tests. However, his formulas cannot be used for pre-stress design without revision. They are based on the assumption that $\frac{a}{d} = 0.59$, which is taken from the tests of Messrs. Slater and Lyse,[47] Mr. Glanoille,[48] U. S. Geological Survey,[49] and Messrs.

[42] *Beton und Eisen*, 1933, Hefte 3 und 4.
[43] *Loc. cit.*, July 20, 1935.
[44] *Loc. cit.*, 1936, Heft 2.
[45] Mitteilungen über Versuche d. Oesterr. Eisen Beton Ausschlusses.
[46] *Concrete and Constructional Engineering*, 1937 (Discussions).
[47] *Journal*, Am. Concrete Inst., June, 1930.
[48] *Concrete and Constructional Engineering*, March, 1937.
[49] *Journal*, Am. Concrete Inst., 1920.

Humphrey and Losse.[50] In all these tests the steel had a yield point of 38 000 to 58 000 lb per sq in. Only the tests conducted by Messrs. Slater and Lyse considered steel with a yield point of 64 800 lb per sq in., but in this case also the steel was not stressed beyond 40 000 lb per sq in. No empirical assumption may be extended beyond the limits of tests. Mr. Whitney's formulas cannot be applied to stresses in steel as high as 90 000 lb per sq in. The writer believes that such stresses in the steel would induce cracks much wider and longer than those in conventional structures; therefore, the ratio, $\frac{a}{d}$, will be smaller, the "compression block" will decrease, and the breaking point for a conventional design will be much lower than that calculated by Mr. Whitney's method. On the other hand, if the reinforcement is pre-stressed, the outside moment should overcome: (1) The resisting moment due to pre-stressing; and (2) compression at the bottom due to the direct pressure from pre-stressing, before cracks would appear—a stage that occurs at the beginning in conventional design. This will delay the breaking of the concrete considerably. Since the stress in the steel increases only 90 000 − 45 000 = 45 000 lb per sq in. from the first tension in the concrete to the breaking point, the cracking will not be greater than for a design with ordinary steel, and the compression block will be about the same. It would be very interesting to see Mr. Whitney's method developed to include a design involving steel with high yield point and based upon pre-stressing. However, until this is done, it will be impossible to use this method in appraising pre-stressed structures.

Some discussers would replace pre-stressing by the customary methods of reducing slab depths, such as by continuity, end restraint, the use of stronger concrete, etc.; but pre-stressing is entirely compatible with these methods; it is based on an entirely different principle and can be combined with each of these methods to great advantage in design. As Mr. Lemcke suggests, the writer purposely used low concrete stresses and simple structures as examples so as not to obscure the advantages of pre-stressing by other elements. More complicated examples were eliminated from the paper as an unnecessary contribution to its length. Among examples that might have been cited was an alternate design of a continuous bridge with a span of 30 ft. Contrary to the opinion expressed by Mr. Hirschthal the comparison was as favorable as for simple structures. The volume of concrete was reduced 20%, the decrease in the depth at the center was 40%, and the reinforcement was straight and simple. Another example included a pre-stressed I-girder as an alternate to a T-beam design for the Delaware, Lackawanna and Western Railroad. The construction proved to be much more economical and expedient, saving 47% in concrete and 42% in steel.

The writer takes the opportunity, presented by Mr. Hirschthal's design of a non-pre-stressed slab with high-strength concrete and high yield-point steel, to demonstrate once more the effect of the pre-stressing.

In general, the use of steel stresses as high as 45 000 lb per sq in. would create important defects in the conventional slab. The concrete cracks when it cannot follow the uniform elongation of the reinforcement. Any increase in

[50] *Technological Paper No. 2,* National Bureau of Standards, 1912.

the elongation of the steel involves a much greater increase in cracking. At present, working stresses in the reinforcement are not exceeding 27 000 lb per sq in.[51] Raising them 70% would produce a badly cracked structure. The deflection also depends on the elongation of the bottom fibers. It is doubtful whether the construction of such a shaky and cracked slab would be allowed.

Moreover, the saving is not large enough in proportion to the quality of the concrete, because the ratio, $n = \frac{E_s}{E_c}$, decreases considerably. Mr. Hirschthal assumed $n = 12$, but this value does not correspond to that for high-strength concretes. Usually n is taken as $\frac{30\,000}{f_c'}$, in which f_c' is the crushing strength of concrete.[52] In England, n is assumed as $\frac{40\,000}{f_{cs}'}$ in which f_{cs}' is the "cube strength" of concrete.[53] Since f_{cs}' is $1\frac{1}{3}$ to $1\frac{1}{2}$ times f_c', the European method gives almost the same value of n as is accepted in the United States. Hence, in Mr. Hirschthal's first example, n should be taken as $\frac{30\,000}{3\,500} = 8.6$ instead of 12. With this correction, and taking into account the additional dead load, the depth of the slab should be as follows: $k = \frac{8.6 \times 1\,400}{45\,000 + 8.6 \times 1\,400} = 0.211$; $j = 0.930$; $K = \frac{0.211 \times 0.930 \times 1\,400}{2} = 137.5$; $d = \sqrt{\frac{50\,600}{137.5}} = 19$; and, $D = 19 + 2.5 = 21\frac{1}{2}$ in. compared to 27.5 in. in the design with plain concrete and steel. The saving is not so great as to warrant sacrificing the stability of the structure.

Now, consider the pre-stressed slab. The depth of the slab is assumed as 10.5 in.; $p = 0.0155$; $n = 8.6$; and $q = \frac{8}{10.5} = 0.761$. From Equation (19), $K = 1 + 7.6 \times 0.0155 = 1.116$; from Equation (18), $k = \frac{1}{1.116} \times (\frac{1}{2} + 7.6 \times 0.0155 \times 0.761) = 0.527$; and, from Equation (17), $C = 0.083 + \overline{0.027}^2 + 7.6 \times 0.0155 \times \overline{0.234}^2 = 0.090$. The dead load is $0.875 \times 150 + 65 = 196$ lb per sq ft; $M_D = 22\,900 \times \frac{196}{265} = 16\,950$ ft-lb; $M_L = 21\,700$ ft-lb; and the total, $M = 38\,650$ ft-lb.

Stresses due to loading, from Equations (5), (6), and (7), are:

$$f_t' = \frac{38\,650 \times 0.527}{0.090 \times 12 \times \overline{10.5}^2} = 1\,702 \text{ lb per sq in.;}$$

$$f_b = -1\,702 \times \frac{0.473}{0.527} = -1\,535 \text{ lb per sq in.;}$$

and,

$$f_s = -\frac{38\,650 \times 8.6 \times 0.234}{0.090 \times 12 \times \overline{10.5}^2} = -6\,620 \text{ lb per sq in.}$$

[51] *Concrete and Constructional Engineering,* September, 1933.

[52] New York City Building Code.

[53] *Concrete and Constructional Engineering,* January, 1937.

Stresses due to pre-stressing, from Equations (20) and (21), are: $f_{pt} = 2 \times 0.0155 \times (3 \times 0.761 - 2) \times f_{ps} = 0.00877\ f_{ps}$; and, $f_{pb} = -2 \times 0.0155 \times (3 \times 0.761 - 1) \times f_{ps} = -0.0398\ f_{ps}$.

Conditions to determine the pre-stress required, are:

(*C*) $F_t = +0.00877\ f_{ps} + 1\ 702 \leq 1\ 400$; hence, $f_{ps} \leq -34\ 400$ lb per sq in.

(*D*) $F_b = -0.0398\ f_{ps} - 1\ 535 \geq 0$; hence, $f_{ps} \leq -38\ 400$ lb per sq in.

and,

(*E*) $F_s = -(f_{ps} - 6\ 620) \leq +45\ 000$; hence $f_{ps} \geq -38\ 400$ lb per sq. in.

Conditions (*D*) and (*E*) are controlling; therefore: $-38\ 400 \leq f_{ps} \leq -38\ 400$ and the final stresses are as follows: $F_t = 1\ 702 - 336 = +1\ 366$ lb per sq in. $\leq +1\ 400$; $F_b = -1\ 535 + 1\ 535 = 0$; and, $F_s = -38\ 400 - 6\ 620 = -45\ 000$ lb per sq in.

Compared to Mr. Hirschthal's design the pre-stressed slab: (*a*) Saves about 51% of concrete; (*b*) eliminates tension and cracking completely; and (*c*) decreases the deflection to a small amount because the steel stresses vary only within the narrow limit of 6 620 lb per sq in.

It is clear that pre-stressing is justified by results achieved by practice; it is not costly; and it results in exceptionally economical structures. Mr. Lenk who observed the German tests with the pre-stressed girder adds to this, the statement that, "the complete lack of cracks and a surface as hard as glass makes the pre-stressed concrete resistant to air, gases, and humidity. Its water-tightness is unrivaled. The expense of maintaining pre-stressed concrete structures is considerably less than other types."[54]

[54] *Beton und Eisen,* May 20, 1937. (Quotation cited is a translation by the writer.)

AMERICAN SOCIETY OF CIVIL ENGINEERS
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DISCUSSIONS

PRACTICAL APPLICATION OF SOIL MECHANICS
A SYMPOSIUM

Discussion

BY MESSRS. LEE H. JOHNSON, JR., AND GREGORY P. TSCHEBOTAREFF

LEE H. JOHNSON, JR.,[70] JUN. AM. SOC. C. E. (by letter).[70a]—In a clear and straightforward manner, Mr. Buchanan has presented a general method of applying soil mechanics to the design of earth levees. He is to be commended for this important contribution to the small group of publications of scientific methods for the design of earth structures. He closes his paper with the remark: "For new units involving materials with which there has been no experience, savings in time and cost are possible through the application of soil mechanics." The writer wishes to cite an instance from his personal experience in which there has been such a saving in cost.

One of the problems with which the U. S. Engineer Office, at Mobile, Ala., has been faced is that of protecting the City of Rome, Ga., from recurrent floods on the Oostanaula and Etowah Rivers which unite in the heart of the city to form the Coosa River. These floods are of much shorter duration than the floods on the Mississippi River and on other large streams. A study of gage heights for floods that occurred during a period of fifty years shows that high water during a flood on these rivers lasts only several days.

A preliminary design of the protection works and incidental structures was prepared and submitted for approval in the form of a preliminary report. The protection works provided for in this report consisted mainly of levees with a land side slope of 1 on 4 and a river side slope of 1 on 3. These slopes were chosen on the basis of a general knowledge of the soil in the vicinity of the river banks at Rome, and the short duration of the high water.

NOTE.—This Symposium was presented at the meeting of the Soils Mechanics and Foundations Division, at Boston, Mass., October 7, 1937, and published in September, 1937, *Proceedings*. Discussion on this Symposium has appeared in *Proceedings*, as follows: September, 1937, by the members of the Committee of the Society on Earths and Foundations; November, 1937, by Messrs. S. C. Hollister, T. T. Knappen, and L. F. Harza; December, 1937, by Edward Adams Richardson, Esq.; January, 1938, by Messrs. Richards M. Strohl, William P. Creager, Jacob Feld, and Y. L. Chang; and February, 1938, by Messrs. Charles Senour, Donald M. Burmister, and Donald W. Taylor.

[70] Acting Dean; Prof. of Civ. Eng., School of Eng., Univ. of Mississippi, University, Miss.

[70a] Received by the Secretary February 3, 1938.

The preliminary report was approved and the final design authorized. It became feasible to analyze the soil to be used in the construction of the levees. Borings were taken along those sections of the banks of the Oostanaula and Coosa Rivers, which are to be used as borrow-pits for the levee material. Representative samples were sent to the Soil Laboratory of the U. S. Waterways Experiment Station, at Vicksburg, Miss., for analysis. There, the samples were classified by means of mechanical analyses and such properties of the soil as angle of internal friction, cohesive strength, and coefficient of permeability were determined. A study of the stability of earth slopes composed of these materials, using the results of the laboratory tests, established the fact that the materials possessed sufficient shearing resistance to stand on a 1 on 3 slope with a satisfactory factor of safety. Furthermore, these materials were sufficiently impervious, by a very large margin of safety, to prevent flood waters from seeping through in the short time they were against the levees and threatening the stability of the land side slope.

In view of these facts, the recommendation was made, and accepted, that the levees at Rome be built with a slope of 1 on 3 for both land side and river side slopes. This produced a saving of about 80 000 cu yd of earth and about $30 000, or more, in the cost of the levees. This example furnishes a striking illustration of the value of soil mechanics when applied to the design of levee units in new and unknown material.

The writer acknowledges his indebtedness to V. K. Wagner, Engineer in Charge of Design and Specifications, U. S. Engineer Office, Mobile, Ala., for making available and authorizing the use of this information.

GREGORY P. TSCHEBOTAREFF,[71] M. AM. SOC. C. E. (by letter).[71a]—It is interesting to note that three of the four papers of the Symposium treat the related subjects of levees, dikes, and embankments, and only one paper deals with foundations of bridges and buildings.

It would appear that this relationship reflects the actual situation. The efforts of most research scientists in the field of practical application of Soil Mechanics are directed toward solving problems of water-retaining works for the construction of which soil is used as an engineering material. The comparatively high total expenditure on such works, as well as the risk of particularly extensive damage to life and property in case of their failure, has led to greater research facilities being provided for this branch of the new science. The use of disturbed and recompacted soil material on actual constructions of the foregoing type facilitates the duplication of these field conditions in the laboratory. Furthermore, from the point of view of the organization of the necessary studies, this branch is not handicapped by the wealth of minor technical details and by obstacles created by professional and by misguided competing commercial interests, all of which are generally present when studies of foundations of structures are concerned.

A few years ago most scientists working in the field of Soil Mechanics published periodical appeals stressing the importance of full-scale settlement

[71] Asst. Prof. of Civ. Eng., Princeton Univ., Princeton, N. J.

[71a] Received by the Secretary March 3, 1938.

studies on buildings and bridges and deploring the difficulty of obtaining reliable data on the subject. Gradually, these appeals have diminished, apparently through sheer exhaustion of their authors, who have turned their attention to more well received subjects of study.

In view of this situation the paper by Professor Terzaghi is of more than outstanding interest. It outlines the knowledge recently acquired in Europe through the study of actual full-scale structures. It should be remembered in this connection that the proposal to make such systematic full-scale observations was first advanced in 1925 by Professor Terzaghi himself[72] for the purpose of the control, correction, and development of soil-testing methods; the necessity of such observations was stressed further in later publications of the Committee

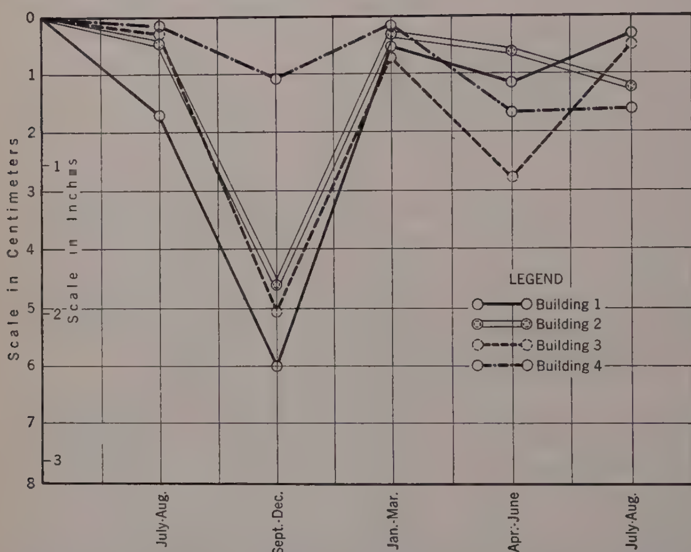

FIG. 68.—GREATEST CONTINUED SETTLEMENT OF FOUR DIFFERENT STRUCTURES OF THE KOUZNETZKSTROI WORKS, FOUNDED ON LOESS, OCCURRED DURING RAINY MONTHS

of the Society on Earths and Foundations in which he participated.[73] Professor Terzaghi not only made this proposal, but actually conducted such systematic observations and studies in Vienna and elsewhere. These studies and others which they have inspired (for instance, in Germany and in Egypt) form the only available source of information concerning the inter-related behavior of superstructure, foundation, and soil, as it actually occurs in Nature. Until the results of these observations became known, facts which are now generally recognized as such were contested vigorously by many engineers. The problems still awaiting solution in this field are likely to be clarified only by a continuation of similar systematic settlement studies in numerous other localities in different parts of the world, and by their correlation. Previously

[72] "Erdbaumechanik," 1925, pp. 5, 386.

[73] Progress Rept. of the Special Committee on Earths and Foundations, *Proceedings*, Am. Soc. C. E., May, 1933, p. 778.

unsuspected facts are likely to emerge from the development of such observations in places where they had not been extensively made before.

Some such interesting examples are to be found in recent Russian publications. For instance, observations of structures founded on loess soils have revealed that the rate of settlement increased during the wet seasons—that is, in the autumn and in the spring.[74] Fig. 68 illustrates such observations on four different structures of the Kouznetzkstroi Works, in Siberia. It should be noted that this diagram does not show time-settlement curves in the usual manner of their presentation, but that the ordinates give the additional total settlement which occurred during periods of three months each. The large settlement during the rainy months was attributed to a breaking down of the porous structure of the loess in the upper moistened layers. Load tests (see Fig. 69), and other investigations in different parts of Russia showed that this was actually the case (to a varying degree for different types of loess) when the soil was loaded, but that preliminary wetting alone, not accompanied by the application of load to the soil surface, did not produce this break-down. Sand piles were used for structures on such loess soils with satisfactory results, as proved by full-scale settlement observations.

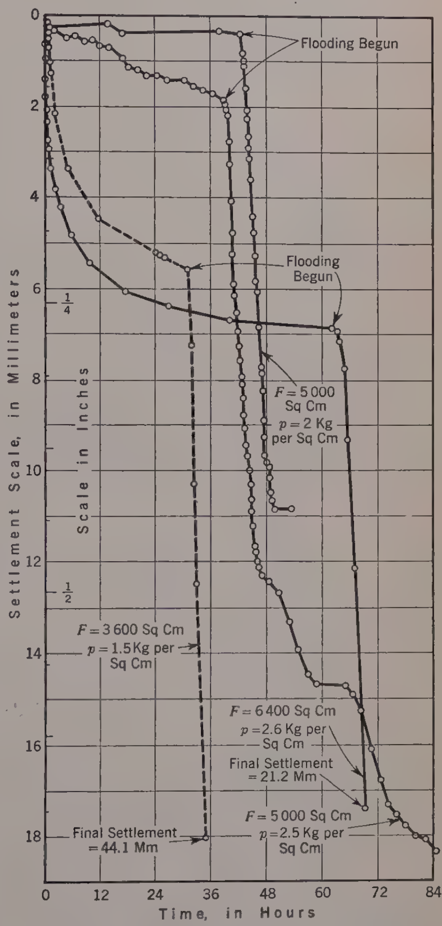

FIG. 69.—SETTLEMENTS OF TEST FOOTINGS ON LOESS SOILS FOUND TO INCREASE AFTER FLOODING OF SOIL SURFACE

The suggestion by Professor Terzaghi that the procedure for making settlement observations should be standardized is an important one and should be actively supported. The methods of observation he proposes should be adopted.

Several hundred reference-point leveling plugs of the type designed by Professor Terzaghi and illustrated in Fig. 42 were used for settlement studies in Cairo and in some provincial cities of Egypt, with which the writer was connected. These plugs were found to be very convenient and practicable when levelings were to be begun after the structure had reached the surface of the ground—that is, in most cases. A special method of leveling must be designed

[74] "Praktika Stroitelstva na Lessovidnih Gruntah," by Eng. I. M. Abeleff, *Onti*, Moscow, 1934 (in Russian).

to meet different conditions only when a relatively considerable portion of the weight of the structure is beneath the ground level and when, therefore, a study of the early part of the time-settlement curve appears to be advisable from the very moment the foundation raft or the pile-capping is cast.

In Egypt the writer also had the opportunity of using the precision water-level system illustrated in Fig. 42. It was found to be invaluable for observations inside a building where the use of an optical instrument is impossible in most cases. The rubber hose of this water level could be guided easily around several corners of narrow corridors and small rooms or around machinery (in the case of factory buildings). Some precautions were required when this hose came to rest alternatively on floors exposed to the sun and in the shade. At first, the thermal expansion or contraction of the hose then caused slight fluctuations of the level of the water in the glass cylinder, which could produce errors in the readings unless these readings were carefully timed. It is probable that an increase in the diameter of these glass cylinders to at least three or four times the value of the diameter of the rubber hose would decrease the aforementioned fluctuations of the level of the water in the cylinders, and, therefore, would constitute a desirable minor improvement of this valuable instrument for special use in hot climates.

It is to be hoped that Professor Terzaghi's paper will stimulate interest in the United States for the development of systematic regional settlement studies of new structures erected on compressible deposits. Organized effort is required to obtain tangible results.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

THE DESIGN OF ROCK-FILL DAMS

Discussion

BY MESSRS. PAUL BAUMANN, O. W. PETERSON, AND GEORGE W. HOWSON

PAUL BAUMANN,[18] M. AM. SOC. C. E. (by letter).[18a]—In his interesting and very valuable paper the author mentions, among other dams, San Gabriel Dam No. 2 of the Los Angeles County Flood Control District. This structure was built of relatively large rock of granitic gneiss secured from a quarry approximately two miles from the dam site. The rock was hauled by means of 10-yd trucks and was placed dry, that is, without the application of sluicing water.

During the so-called New Year's storm of 1934 which swept over Southern California, heavy rainfall occurred which, in the vicinity of this dam, yielded as much as 12 in. in 24 hr. Due to the sluicing effect of this rainfall, settlement occurred in the dam, which at that time was nearly completed as far as the rock-fill proper was concerned. These settlements amounted to a maximum of about 4 per cent. Subsequent sluicing (which continued for several months) increased the maximum settlement to about 6 per cent.

During the construction of the dam a truck count was kept and recorded, from which it was possible to determine the shrinkage of the rock-fill during construction. Fig. 16 shows a curve that was plotted from the ratio of actual quantity in the dam and truck count as established at the end of each month during construction. From this diagram it is apparent that, although at the beginning of the job (that is, for the first 25-ft lift), each truck load of rock yielded 11.3 cu yd of rock-fill, it yielded only 9.4 yd at the end of the job. Hence the shrinkage due to self-compaction that occurred during construction was roughly 18 per cent. Data are from estimates and inspectors' reports.

NOTE.—The paper by J. D. Galloway, M. Am. Soc. C. E., was published in October, 1937, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: December, 1937, by Messrs. Cecil E. Pearce, and H. B. Muckleston; January, 1938, by Harold K. Fox, M. Am. Soc. C. E.; February, 1938, by Messrs. Charles H. Paul, and A. Floris; and March, 1938, by Messrs. Howard F. Peckworth, Oren Reed, Walter L. Huber, Samuel B. Morris, and L. F. Harza.

[18] Jun. Asst. Chf. Engr., Los Angeles County Flood Control Dist., Los Angeles, Calif.

[18a] Received by the Secretary February 25, 1938.

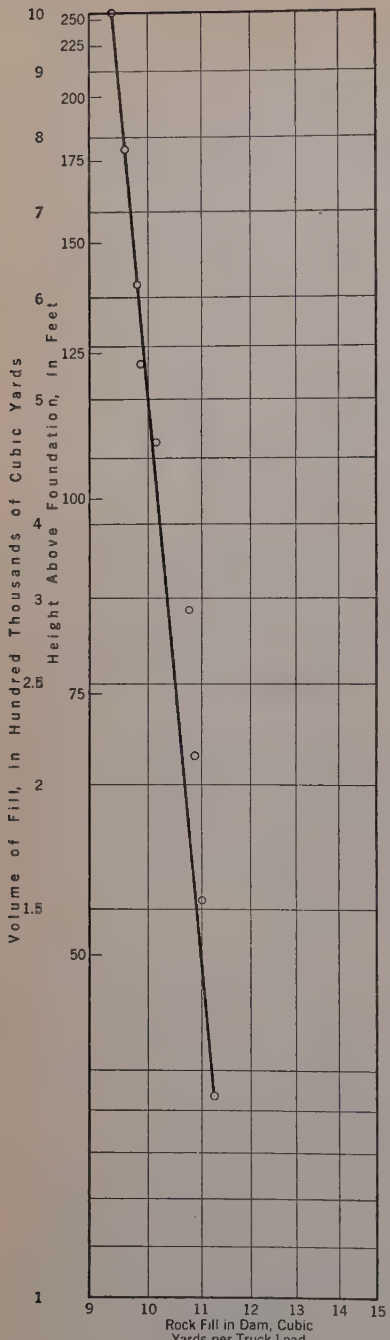

FIG. 16.—SAN GABRIEL DAM NO. 2, SHRINKAGE OF ROCK-FILL AFTER PLACING IN DAM

As mentioned previously, the additional shrinkage due to natural and artificial sluicing amounted to about 6%; hence, the total shrinkage of this dam amounted to approximately 24 per cent.

During the construction of San Gabriel Dam No. 1, which consists of 6 236 000 cu yd of loose rock-fill and 3 206 000 cu yd of rolled rock-fill, the total shrinkage of the loose rock-fill amounted to about 26%, and the total shrinkage in the rolled rock-fill amounted to about 34 per cent. The loose rock-fill was placed wet; that is, with the application of approximately 2 cu yd of water to 1 cu yd of rock. The analysis shows that the application of water caused an initial shrinkage (that is, at the time of placing) of from 4 to 6 per cent. It is interesting to note that this corresponds substantially to the shrinkage in San Gabriel Dam No. 2 when water was applied after the completion of the rock-fill.

From the shrinkage measurements due to self-compaction, it follows that since the density of the rock-fill is directly proportional to the shrinkage, the average weight probably increases from 100 lb per cu ft of fill at the time of placing to approximately 125 lb per cu ft of fill at the time of completion for dams of such magnitude. This, in turn, means that the porosity of rock-fill dams of such height, particularly near the base, is relatively small due to the break-down of the contact points of the rock.

O. W. PETERSON,[19] M. AM. SOC. C. E. (by letter).[19a]—A comprehensive presentation of the design of modern rock-fill dams is presented in this paper. Nevertheless, it contains several statements with which the writer is not in full accord. Mr. Galloway believes that a fill composed of individual rocks of fairly uniform size is best and that "any wide

[19] Engr. of Gen. Constr., Pacific Gas & Elec. Co., San Francisco, Calif.

[19a] Received by the Secretary February 28, 1938.

divergence in size will cause excessive and unequal settlements, something to be avoided wherever possible" (see heading, "Design: Rock-Fill"). Quarry rock ranges from fines to the maximum size, the latter depending on the physical characteristics of the rock mass and the method of quarrying. As Mr. Galloway states, the maximum size placed in Salt Springs Dam was about 25 tons, that being the size to which the larger rock fragments were reduced by secondary drilling and blasting so that they could be handled by the equipment provided. Fully 50% of the rock volume consisted of pieces heavier than 10 tons, which were loaded over the top of the power shovel buckets; but there was no usual size, as Mr. Galloway states. In the writer's opinion an unsegregated quarry-run mass, with excess fines wasted, will give the highest degree of contact, rock to rock, and the greatest resulting fill density. Such a fill will have ample rock-to-rock contact and will still have sufficient voids to provide adequate drainage.

Instead of stressing the importance of the nozzle pressure of the streams of water used for washing the finer rock particles into the voids between the larger rock, the writer believes the volume of water supplied should be emphasized as being of critical importance in reducing the frictional resistance to the movement of individual rocks against each other and in causing the fines to lodge in void spaces. The lubricating effect of water is invaluable in obtaining maximum settlement of a rock-fill mass during the construction period. This effect has been utilized in flattening the slopes of high tunnel dumps of hard rock free from earth, thereby stabilizing the mass. At Salt Springs Dam, as many as six 2-in. to 3-in. streams, each delivering about 600 gal per min, were used. Another advantage of the low-pressure stream is that it can be applied more economically than that of a higher pressure stream, which requires more man-power as well as more power for pumping.

At Salt Springs Dam the fines that accumulated at the top of each lift were loosened by light blasting and then washed into the voids of the underlying larger rock. This blasting was done with small charges of powder without damage to the adjacent larger rock; such vibration as occurred to the fill mass was considered beneficial in assisting settlement.

Moderately high lifts are both economical and beneficial in producing a compact fill. For a fill of 200 to 300 ft, or more, lifts of 60 to 90 ft, or more, in height can be placed successfully without undue segregation of rock. The impact of well-lubricated rock masses sliding from such heights helps to break weak or fractured rock, to remove points and thin edges, and to ram the rock firmly into stable position.

Mr. Galloway states that each stage or lift should be fairly well complete before the next above is placed. The writer can see no reason for delaying a subsequent lift after the previous one has been carried sufficiently forward to assure safety to men and equipment operating at lower levels. If the slopes are kept at a moderate angle through the use of ample sluicing water, this condition will also provide an ample margin of stability for the rock masses. Under this plan the up-stream and down-stream faces of each new lift should approximate the required final positions of the loose fill. The face that is advancing across the stream from either abutment should, in the interest of safety to the workmen, be kept back from the forward edge or crest of the

underlying lift a distance about equal to twice the height of the upper fill or lift. To minimize unequal settlement, lifts advanced from the two sides of the canyon should be kept reasonably uniform in height and at the same general level.

The writer sees no reason why gates, if of conservative design and of generous proportions, should not be installed on the spillway lip of a rock-fill dam. This conclusion presupposes the necessary vigilance and intelligent operation of the gates.

George W. Howson,[20] Assoc. M. Am. Soc. C. E. (by letter).[20a]—A very thorough and complete paper on the rock-fill dam has been presented by the author. The writer was connected with the construction and operation of three of the dams listed in Table 1 and offers the following comments as the result of this experience. Three sub-headings are discussed: the "Rock-Fill," "Settlement," and the "Facing."

Rock-Fill.—At Relief and Strawberry Dams the rock-fill was spoken of as the "drop fill section," the specifications calling for the rock to be dropped into place from a height of not less than 10 ft. The idea was to hammer the rock into place. At Strawberry Dam most of the rock was transported by cableway and automatically dumped. The dropping was distributed over the entire area of the fill. It was found that drops in excess of 50 ft damaged the rock. There was a limited amount of sluicing, but several hundred cubic feet of water per second passed through the fill during a flood at the time of construction.

At Dix Dam each 16-yd car was classified as to the quality of the rock, the larger and better rock going to the up-stream half of the dam. All dirt and decomposed rock was discarded.

During the construction of a rock-fill dam all three phases of the work are generally done at the same time. As the loose fill is being placed, the rubble section is laid on the up-stream slope, and below this the concrete ribs and apron are being poured. The rubble section is built simultaneously with the loose fill and follows directly below. Progress and workmanship are best with rocks of suitable size and with one or more good faces. The selected rocks are easily delivered to the masons on the rubble wall by dumping down the up-stream slope. The rubble wall should always be extended back far enough to have a good bearing against the rock-fill section. The fill should be built to the proper lines and not carried out to meet the back of the rubble wall already built. The rubble must be supported everywhere by the compacted rock-fill.

Settlement.—All settlement is the result of crushing of the rock-to-rock bearing points, as described by Mr. Galloway. The larger the rock the fewer will be the bearing points for any given depth of fill resulting in less settlement, but construction equipment and methods generally limit the size of rock.

A fill of good granite or limestone, made as described by Mr. Galloway and properly sluiced, should not exceed 1% or 2% at the most. A reasonable settlement is to be expected and can be designed for, provided it is uniform. A settlement of 2% of the height would be reasonable. However, a rock-fill

[20] Berkeley, Calif.

[20a] Received by the Secretary February 28, 1938.

dam having a semi-rigid facing may be a partial failure with moderate settlements if the settlement of adjacent areas is not fairly regular. It is difficult to design a concrete facing to accommodate itself to large, sharp, irregular movements.

Large rocks are generally least damaged in quarrying, and have the soundest surfaces and points upon which the rock-to-rock load, as described by Mr. Galloway, is carried; consequently they offer the greatest resistance to crushing, thereby reducing settlement. Very good results may be expected if the bearing rocks weigh 2 tons and more. Smaller rock subsequently sluiced into the interstices of the bearing rocks adds weight to the fill and supporting power as the bearing points of the large rock crush.

The beneficial effect of sluicing cannot be over-stressed. Even in a fill of clean, large rock, sluicing wets all bearing surfaces, which gives a certain degree of lubrication, and assists the slipping and readjustment of the rocks to a better and more permanent position. Settlement is aided.

In considering the settlement of a rock-fill dam one generally means that settlement which occurs after the dam is completed. However, a considerable settlement is taking place in the lower parts of a dam during the construction period as the latter assumes the increased weight of the fill above. These movements are of interest to the engineer.

The rubble section settles as it follows the movement of the supporting loose fill. In most large dams with a concrete face there are vertical supporting ribs under each vertical expansion joint. The ribs are poured in advance of the concrete face. If the ribs are poured too close to the top of the rubble wall as it is being built up, the initial settlement may be sufficient to crush the green concrete ribs. At Dix Dam, it was found advisable to keep the concrete ribs about 50 ft below the top of the rubble wall.

The settlement, to a reasonable depth, is not so much a criterion of the quality of the fill as is the relative settlement of adjacent areas. In a dam in which the total settlement may be only 2%, trouble may be expected if such settlement is uneven; and uneven settlement may be expected in a fill with poor distribution of material, insufficient sluicing, or where a considerable time element exists between placing adjacent areas.

During the construction period at Dix Dam three temporary points were established on the 690-ft level (where the fill was 185 ft high) and maintained 41 days when they were covered by rock for the next lift above. During this period a flood piled up about 60 ft in front of the partly completed dam. Much water flowed freely through the rubble wall and loose fill, emerging at the base of the down-stream slope. It was impossible to measure the quantity of water passing through the dam, but the fill received a perfect sluicing below the hydraulic gradient. Fig. 17 shows a record of the settlement of these three points and is of especial interest as it shows the extent of the early settlement, the settlement caused by the flood, and the increase in settlement toward the down-stream slope of the dam. The increase in settlement toward the down-stream slope is believed to be due entirely to the fact that the poorer material was placed in the down-stream half of the dam. This section of the dam may have had some small rock separating the rock-to-rock bearing points of the larger rocks,

and the small rocks were displaced by the flood. Settlement varied from 0.32% at the up-stream slope to 1.02% of the height of the fill at the down-stream slope. All these points were on the loose fill section of the dam.

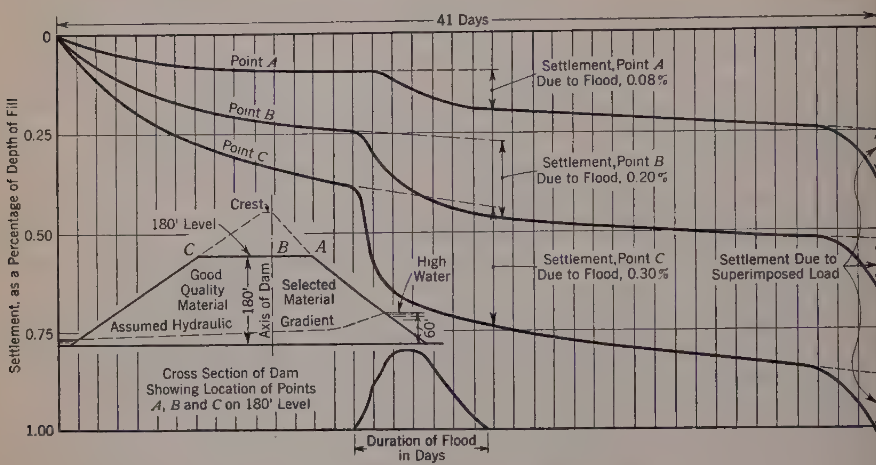

FIG. 17

Another temporary point set on the face of the rubble wall showed the following movement during the 70-day period of record before it was covered by the concrete facing. There was a movement toward the center of the canyon of 0.28 ft, an up-stream movement of 0.07 ft, and a vertical settlement of 0.28 ft. The dam at this point was 100 ft high and on the side of the canyon. Consideration should be given these movements when establishing lines for the work.

At Strawberry Dam a similar flood occurred during construction. It is believed these floods were of material benefit to both dams.

Mr. Galloway gives settlement data for Dix Dam during the first three years after its completion. Later measurements, made eleven years after completion, show the maximum settlement at the crest to be: Vertical, 2.48 ft; horizontal, 1.93 ft; and down on the face of the dam, 55 ft below the crest, where the depth of fill is 215 ft, there is a vertical settlement of 2.28 ft and a horizontal settlement of 1.44 ft. The maximum vertical settlement at the crest is 0.90% of the height of fill.

Facing.—No one standard design of facing can be claimed to be best for all conditions. Each dam is its own problem.

Strawberry Dam has an apron of reinforced concrete sliding leaves over the upper two-thirds of the dam, each approximately 12 in. thick, 60 ft wide, and 135 ft long, fastened at the bottom but free to slide upon a smooth plastered surface covered with roofing paper. Over the lower one-third of the structure the concrete facing was poured directly against the dam, but not until all cracks between the rocks had been plastered to prevent the concrete from running in between the rocks. This was thought to be a measure of economy. There is

no evidence that the leaves over the upper two-thirds of the dam have ever moved.

It is interesting to note that the expansion joints at Strawberry Dam have no seal. The original design called for the joints to be packed with asphalt or oakum. High water overtook the water before the seal was installed but the joints proved tight and were never packed. These joints were constructed about 2 in. wide and terminated at the back against the underlying concrete rib.

At Dix Dam concrete was poured directly against the face of the rubble section of the dam and worked in between the rocks. Vertical and horizontal expansion joints were provided. At the east abutment large differential settlements were anticipated and the spacing of expansion joints was decreased. The bottom 180 ft of water in this dam has never been emptied, but inspection at the most critical point, which is above the draw-down, after 10 yr of service indicates a relative movement of about 2 in. at the expansion joint, with some twisting. There are no open cracks in the slabs that are visible. Joints have taken all movements. There were no construction joints as each panel was completed in one continuous pour.

The writer believes that if a concrete facing is adopted it should be poured directly against the dam and between the face rock, with adequate expansion joints. Spacing of these joints should be such as to accommodate the larger settlement occurring over sections of the dam where there is an abrupt change in the depth of fill. Vertical joints should be constructed open or closed according to their position in the dam.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

DESIGN OF REINFORCED CONCRETE IN TORSION

Discussion

BY MESSRS. E. NEIL W. LANE, AND A. A. EREMIN

E. NEIL W. LANE,[21] JUN. AM. SOC. C. E. (by letter).[21a]—Due mainly to the desire for simplicity and a lack of adequate information, torsional stresses have been largely ignored in structural design. Generally, they are small in comparison with the flexural and shearing stresses, which makes a two-dimensional solution satisfactory for most structural problems.

Professor Andersen shows the application of the moment distribution method to the analysis of three-dimensional frames, but dismisses the solution by the slope deflection method with a general statement of procedure. Since certain methods are more adaptable to one type of problem than others, it is deemed advisable to show the application of the slope deflection method to a frame similar to that analyzed in Table 1.

The writer has assumed a length of 15 ft for the side members, and a modulus of elasticity of 3 000 000 lb per sq in. The position of the assumed 10-in. width of the members, relative to the assumed 24-in. depth, was such that the members, $A\,B$, $A\,C$, $A'\,B'$, and $A'\,C'$, were to take the greater proportion of the external torsion; and the frame, $DA\,A'D'$, was to take the greater proportion of the flexure.

Using stiffnesses, as in Equations (4) and (6), the following joint equations are obtained:

$$M_{f(AA')} + M_{f(AD)} + M_{t(AB)} + M_{t(AC)} = 25.0\,\theta_A + 2.4\,\theta_{A'} - 1.0 = 0 \quad \text{(24a)}$$

$$M_{f(A'A)} + M_{f(A'D')} + M_{t(A'B')} + M_{t(A'C')} = 50.0\,\theta_{A'} + 4.8\,\theta_A + 1.0 = 0 \quad \text{(24b)}$$

$$M_{f(AB)} + M_{f(AC)} + M_{f(AD)} + M_{t(AM)} = 145.9\,\phi_A - 1.0 = 0 \quad \text{(24c)}$$

$$M_{f(A'B')} + M_{f(A'C')} + M_{f(A'D')} + M_{t(A'M)} = 265.4\,\phi_{A'} - 1.0 = 0 \quad \text{(24d)}$$

NOTE.—The paper by Paul Andersen, Assoc. M. Am. Soc. C. E., was published in October, 1937, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: December, 1937, by Messrs. C. W. Deans, and L. E. Grinter; January, 1938, by Messrs. Bruce G. Johnston, and Dean Peabody, Jr.; and March, 1938, by Messrs. Walter H. Wheeler, and A. Floris.

[21] Bridge Detailer, State Dept. of Roads and Irrig., Lincoln, Nebr.

[21a] Received by the Secretary February 26, 1938.

In solving, the rotations become $\theta_A = +0.0423$ radian, and $\theta_{A'} = -0.0242$ radian in the plane, $DA\ A'D'$; and $\phi_A = -0.00685$ radian, and $\phi_{A'} = -0.00367$ radian in the planes, $AB\ CD$, and $A'B'\ C'D'$, respectively. Thus, a torsional moment of $+ 1.96 \times 10^6$ in.-lb and a flexural moment of $- 6.85 \times 10^6$ in.-lb are obtained at End A of the member, $A\ M\ A'$. Side-sway, although present, has been neglected to make this solution more comparable to the moment distribution method, as shown by the author.

It is hoped that Professor Andersen's worthy paper will interest others in the study of torsional analysis in connection with three-dimensional structural frameworks.

A. A. EREMIN,[22] ASSOC. M. AM. SOC. C. E. (by letter).[22a]—An interesting method of distributing torsion moments in rigid frames is developed in this paper. Professor Andersen assumed that the centers of joints in rigid frames remain at their original position at all variation of loading sustained by the frame. This assumption is generally used in computing torsion moments by the method of least work. It simplifies computation without serious error for practical purposes.

In the distribution of the bending moments in rigid frames a convenient rule for checking the computations was derived: The sum of bending moments at any joint in a rigid frame, at any loading, is equal to zero. There is no similar simple rule for checking computations of the torsion moments in rigid frames. However, a simplified approximate computation of torsion moments may be used for checking of the more exact computations.

The relation of the torsion moments at the ends of Member $A\ A'$ in the rigid frame shown in Table 1 may be expressed by the following equations:

$$M_A + M_A' = M \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (25)$$

and,

$$K_{TA}\ M_A\ (1 - \Sigma\ K_A)\ m + K_{TA}'\ M_A'\ (1 - \Sigma\ K_A')\ n = 0 \ldots\ldots (26)$$

in which, M_A and M_A' are the torsion moments in the member, $A\ A'$, at the ends, A and A', respectively; K_{TA}, K_{TA}' are torsion stiffnesses in the member, $A\ A'$, at the ends, A and A', respectively; and $\Sigma\ K_A$, $\Sigma\ K_A'$ are the sum of the stiffness factors of the adjacent members at Ends A and A', respectively.

Solving Equations (25) and (26) simultaneously, the torsion moments, M_A and M_A', may be determined. The torsion moments in Member $A\ A'$, with the loading considered in Table 1 as computed by Equations (25) and (26), are $M_A = 200$ and $M_A' = 100$. These moments are the same as those computed for the fixed ends in Member $A\ A'$. Evidently, a slight degree of fixation at the ends of Member $A\ A'$, caused by the action of adjacent members on the distributed torsion moments, is due to their low flexural resistance and the symmetry of the frame. Evidently, the maximum error of computed torsion moments is only 8 per cent. This error is not excessive considering

[22] Associate Bridge Designing Engr., Div. of Highways, State Dept. of Public Works, Sacramento, Calif.

[22a] Received by the Secretary March 7, 1938.

various approximate assumptions used in the more exact method. The distribution of torsion moments in a multiple-story, rigid frame may likewise be computed by Equations (25) and (26) by considering the frame to be composed of elementary frames similar to that shown in Table 1. Therefore Equations (25) and (26) may be used for approximate computations of torsion moments and for checking the distribution of torsion moments determined by the more exact method.

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DISCUSSIONS

GRIT CHAMBER MODEL TESTS FOR DETROIT, MICHIGAN, SEWAGE TREATMENT PROJECT

Discussion

BY THOMAS R. CAMP, M. AM. SOC. C. E.

THOMAS R. CAMP,[7] M. AM. SOC. C. E. (by letter).[7a]—This paper describes an excellent experimental work on grit chambers. The author states in the "Synopsis" that his paper "is not a theoretical approach to the problem of sedimentation in flowing water, but rather a presentation of data in a practical and usable form heretofore not available to the sanitary engineer." These data have assisted the engineers of the Detroit project in preparing efficient designs for the grit chambers. The data are of even greater importance, however, in furnishing, for the first time, full-scale experimental checks of the validity of the rational theory of sedimentation of discrete particles in flowing water.

The Detroit engineers have previously used models successfully for studies of the flow distribution in the settling tanks of the Springwells water purification plant.[8] For this purpose model studies appear to produce reliable results. The conclusions reached by Mr. Hubbell, regarding the passage of oil and scum, however, seem questionable. In this case surface tension and viscosity probably come into play and neither is accounted for by the Froude model law. It is probably justifiable to conclude that if a draw-off velocity of 1.29 ft per sec in the model will remove scum, a somewhat higher velocity in the prototype will do likewise; but there seems to be little reason for concluding that the corresponding prototype velocity for scum removal is 5 ft per sec. In fact, it seems likely that grease will be removed at about the same velocity in both model and prototype since there is no method of reducing the scale of the grease.

Studies of the settling characteristics of suspended matter can be made better and more easily at full scale than with models. The particular difficulty

NOTE.—The paper by George E. Hubbell, Assoc. M. Am. Soc. C. E., was published in December, 1937, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: March, 1938, by G. M. Ridenour, Assoc. M. Am. Soc. C. E.

[7] Assoc. Prof. of San. Eng., Mass. Inst. Tech., Cambridge, Mass.

[7a] Received by the Secretary February 11, 1938.

[8] *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 344.

in the use of models for this purpose arises in selecting and obtaining the proper size of material to settle in the model. The difficulty may be overcome in the case of discrete particles, but it is insurmountable with flocculent suspensions. In Equations (1), (2), and (3), the author attempts to show how the proper size of particle may be found for the model; but since Equation (1) is based upon the assumption that the settling velocity varies with some power of the particle size (an assumption which the author admits is incorrect) the method is rejected. This method of attack is much simplified and may be made correctly if the settling velocities are considered without regard to particle size. Settling velocities are readily measured by quiescent settling tests, but it is difficult to estimate them accurately from sieve analyses.

For both model and prototype the path of a settling particle will be the direction of the vector sum of its own settling velocity and the velocity of the liquid in which it is settling. If the velocity of the liquid is taken as the nominal or mean tank velocity, it follows from Equation (2) that the particle settling velocities should bear the same ratio as the tank velocities for equal removal. That is, the settling velocity for the model grit particles should be $\frac{1}{\sqrt{l}}$ times the settling velocity of the prototype grit. Since no quiescent settling tests were made for the Detroit grit, it is necessary to estimate the settling velocities from the particle size.

Fig. 12 shows the settling velocities of spheres with a specific gravity of 2.65 in water at 10° and 20° C. The values on the two curves are for the region beyond the range of validity of Stokes' law, and were computed from the curve of experimental values presented by Schiller[9] in terms of the Reynolds number. Fig. 12 also shows a curve for the settling velocity of sand at 18° C in terms of the grain size. It was estimated by the writer from experiments[10] of Richards, Hazen, and others. The position of this curve is only approximate, of course, and will vary with the shape of the particles. Very angular grains will settle at slower velocities, the retardation being greatest for the larger particles. For the critical grit size of 0.2 mm selected for Detroit, the corresponding settling velocity from Fig. 12 is about 2.3 cm per sec. The proper settling velocity for the model, therefore, would be $\frac{2.3}{\sqrt{15}}$, or 0.6 cm per sec, which corresponds with a grain size of about 0.085 mm.

The author is incorrect in the statement that Stokes' law does not hold for particles settling in moving water. The settling velocity of a particle is a function only of the particle size, shape, and density, and of the viscosity and density of the fluid. When these are constant the settling velocity is constant both within the range of validity of Stokes' law and beyond. The settling velocity of a particle must be considered with relation to the surrounding fluid, however, and the direction and velocity of motion of a particle with relation to the tank correspond with the vector sum of the settling velocity and the velocity of the fluid with respect to the tank.

[9] "Handbuch der Experimental Physik," von Schiller, Band 4, 2 Teil, 1932, p. 369; see, also, *Sewage Works Journal*, September, 1936, pp. 743–744.

[10] *Transactions*, Am. Inst. Mining Engrs., 1907, p. 283; also, *Transactions*, Am. Soc. C. E., Vol. LIII (December, 1904), p. 45.

Theory of Sedimentation of Discrete Particles.—A simple theory[11] of the removal of discrete particles in continuous-flow, rectangular settling tanks has been presented by the writer which involves the following assumptions: (1) The direction of flow is horizontal, and both direction and velocity are the same in all parts of the tank; (2) the concentration of suspended particles of each size is the same at all points in the vertical plane perpendicular to the

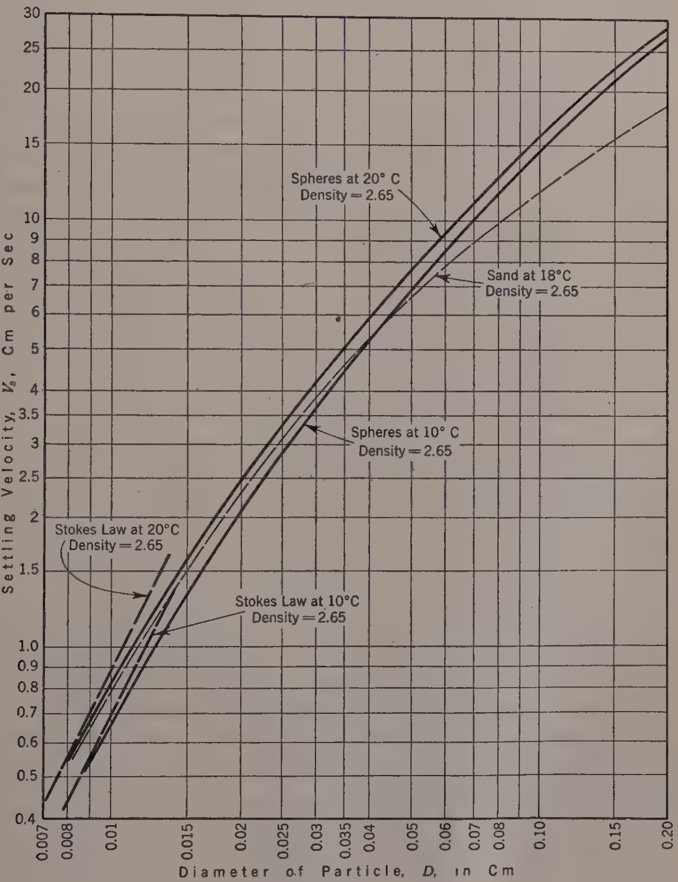

FIG. 12

direction of flow at the inlet end of the tank; (3) all suspended particles maintain their size, shape, and individuality during settling, and the temperature of the fluid remains constant throughout the tank; hence, each particle is assumed to settle at constant velocity; and, (4) a particle is removed when it strikes the bottom.

In view of these assumptions, all particles settling at velocities such that their paths will be parallel to, or steeper than, the path from the liquid surface

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[11] *Transactions*, Am. Soc. C. E., Vol. 102 (1937), p. 306; and *Sewage Works Journal*, September, 1936, p. 742.

at the tank inlet to the bottom at the outlet end will be removed. The settling velocity corresponding to this path is known as the tank "overflow rate," and it is equal to the discharge per unit of surface area. The removal of particles which settle at any velocity less than the overflow rate is,

$$r = \frac{V}{V_0} = \frac{A\,V}{Q} \quad \text{.......................(4)}$$

in which r = the removal as a ratio; V_0 = the overflow rate $= \frac{Q}{A}$; Q = the discharge; and, A = the surface area of the tank.

Equation (4) was first presented by the late Allen Hazen,[12] M. Am. Soc. C. E., and represents sedimentation in a basin in which, according to Hazen, the water is absolutely quiet. For a continuous flow basin it represents the condition of no "vertical mixing" of suspended matter. According to Hazen, "in an actual settling basin the water is mixed more or less from top to bottom in the process, with the result that the sediment does not go down in the manner indicated by the assumption." The nearest approach to this "theoretical maximum" removal which may be obtained in an actual basin, according to Hazen's theory, is represented approximately by Curve C of Fig. 1 in Hazen's paper. For conditions which produce a removal of 100% as given by Equation (4), Hazen's Curve C gives only 66% removal. It has been previously shown by the writer[11] that, except for the proposition corresponding to Equation (4), Hazen's theory is fundamentally unsound. The effect of vertical currents upon the retardation of settling of grit is very much less than is indicated by Hazen's theory, as will be shown subsequently in comparing the results of the Detroit experiments with removals computed by means of Equation (4).

In order to apply Equation (4) to a suspension of particles of varying settling velocity, it is necessary to have a "settling velocity analysis curve" analogous to a sieve analysis curve of sand. Such an analysis may be obtained by quiescent settling tests of the suspension. Lacking settling tests, the curve may be estimated for grit from the sieve analysis by means of Fig. 12. Curve 1 of Fig. 13 is prepared in this manner from the sieve analysis of the sand used in the experiment at Grand Rapids, Mich. As has been shown[11] previously, the total removel for a given overflow rate is,

$$r = 1 - P_0 + \frac{1}{V_0}\int_0^{P_0} V\,dP \quad \text{...................(5)}$$

in which P_0 = the value of P from the settling velocity analysis curve corresponding to V_0.

Curve 1, Fig. 13, indicates how this removal may be evaluated graphically from the settling-velocity analysis curve. The last term of Equation (5) is equal to the average vertical distance between the curve and the horizontal line for $P = P_0$.

Correlation of Theory with Results of Hubbell Experiments.—The "count and weigh" sizes of sand grains are normally 10 to 15% greater than the sizes of

[12] "On Sedimentation," by Allen Hazen, *Transactions*, Am. Soc. C. E., Vol. LIII (December, 1904), p. 45 (see Curves C and A, Fig. 1).

sieve openings. This is due to the grain shape, and the more the shape departs from the spherical the greater this discrepancy. On the other hand, the more the shape departs from the spherical the less the settling velocity will be. As the "count and weigh" size, and, hence, the grain shape, was not given for the

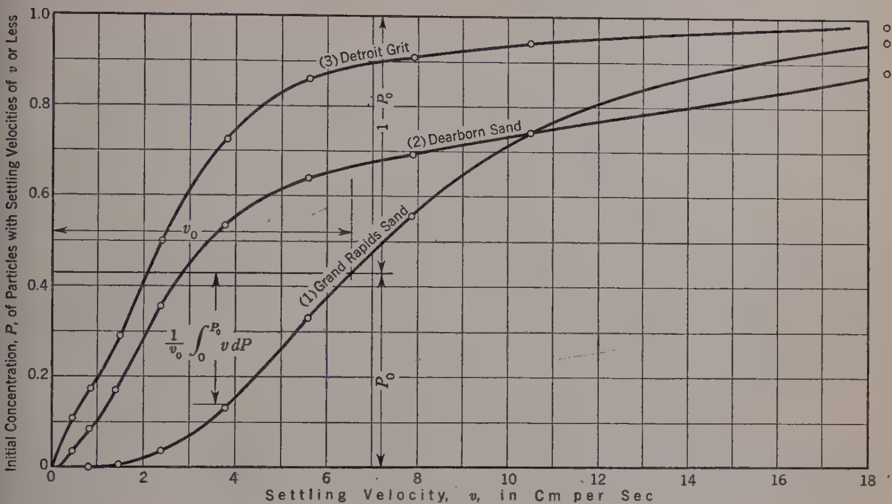

FIG. 13.—SETTLING VELOCITY ANALYSES

Hubbell tests, the writer has assumed for an approximation that these two effects are balanced; and the grain size has been taken as the size of sieve opening.

TABLE 7.—REMOVAL IN GRIT CHAMBERS

Section	1	2	3	4	5	6	7	8
(a) GRAND RAPIDS, MICHIGAN								
Cumulative surface area, in square feet	42	122	202	282	362	442	522	....
V_0, in feet per second	1.035	0.357	0.215	0.154	0.12	0.098	0.083	....
V_0, in centimeters per second	31.6	10.9	6.56	4.7	3.66	2.99	2.53	....
P_0	....	0.765	0.43	0.23	0.12	0.07	0.045	....
Computed removal, r	....	0.67	0.86	0.934	0.969	0.982	0.99	....
Actual removal	....	0.475	0.687	0.806	0.863	0.90	0.917	....
Efficiency	....	0.71	0.80	0.86	0.89	0.92	0.93	....
(b) DEARBORN, MICHIGAN								
Cumulative surface area, in square feet	25	50	75	100	125	150	175	203
V_0, in feet per second	0.26	0.13	0.087	0.065	0.052	0.043	0.037	0.032
V_0, in centimeters per second	7.94	3.97	2.65	1.98	1.59	1.31	1.13	0.98
P_0	0.695	0.55	0.40	0.28	0.196	0.15	0.12	0.10
Computed removal, r	0.542	0.725	0.827	0.881	0.92	0.942	0.954	0.962
Actual removal	0.272	0.509	0.645	0.74	0.776	0.82	0.845	0.868
Efficiency	0.50	0.70	0.78	0.84	0.85	0.87	0.89	0.90

Table 7(*a*) shows the results of computations for removal in the Grand Rapids grit chamber using Equation (5) and Curve 1, Fig. 13. The results

show cumulative removals up to, and including, each section of the grit chamber in which the deposit was weighed in the Hubbell experiments. For comparison, the actual removals as measured by Hubbell (the ratios of the weights of deposited sand to the total weight of sand added) are also shown. The ratio of the actual removal to that computed by means of Equation (5) is shown under the caption, "Efficiency." Table 7(*b*) shows the comparison of the computed values for removal with the actual removal obtained in the Dearborn grit chamber. Curve 2, Fig. 13, gives the settling velocity analysis for this sand.

The sand used in the model of the Detroit grit chambers was quite uniform in size, being retained between 140-mesh and 150-mesh sieves. The writer has assumed a constant settling velocity for this sand which he has estimated from Fig. 12 at 0.85 cm per sec. The theoretical removal in the model is given by Equation (4) and is equal to $\frac{0.85}{V_0}$. The predicted removals in the Detroit grit chambers of sand 0.2 to 0.3 mm in size, based upon removals obtained in the model, are given in Table 3. Presumably these removals are the same as the actual values obtained in the model. Table 8 shows the removals computed by means of Equation (4) for the model compared with those shown in Table 3.

TABLE 8.—REMOVAL IN MODEL GRIT CHAMBER

Length, in feet	$\frac{60}{15}=4$	$\frac{80}{15}=5.33$	$\frac{100}{15}=6.67$	$\frac{120}{15}=8$	$\frac{140}{15}=9.33$	$\frac{150}{15}=10$
Cumulative surface area, in square feet	4.26	5.68	7.12	8.53	9.95	10.67
V_0, in feet per second	0.068	0.051	0.041	0.034	0.029	0.027
V_0, in centimeters per second	2.07	1.56	1.25	1.04	0.89	0.82
Computed removal, r	0.41	0.545	0.68	0.818	0.955	1.0
Actual removal	0.72	0.79	0.85	0.90	0.94	0.96
Efficiency	1.75	1.45	1.25	1.10	0.985	0.96

Table 9 shows the removals computed by means of the theory for the Detroit grit chambers compared with the removals estimated by Mr. Hubbell. Curve 3, Fig. 13, gives the settling velocity analysis for the assumed Detroit grit.

TABLE 9.—ESTIMATED REMOVAL IN PROPOSED DETROIT, MICHIGAN, GRIT CHAMBERS WITH NOMINAL VELOCITY OF 0.9 FOOT PER SECOND

Length, in feet	50	100	150	50	100	150
Cumulative surface area, in square feet	800	1 600	2 400	800	1 600	2 400
	(*a*) CHAMBERS 15 FEET DEEP			(*b*) CHAMBERS 6 FEET DEEP		
V_0, in feet per second	0.27	0.135	0.09	0.108	0.054	0.036
V_0, in centimeters per second	8.24	4.12	2.74	3.3	1.65	1.1
P_0	0.91	0.765	0.56	0.65	0.335	0.215
Computed removal, r	0.364	0.577	0.715	0.655	0.827	0.878
Hubbell's estimated removal	0.368	0.542	0.633	0.594	0.73	0.78
Efficiency	1.00	0.94	0.89	0.91	0.88	0.89

The correlation of the results obtained by the theory with those obtained in all three sets of experiments is very good for the entire length of the chamber, the discrepancy being less than 10 per cent. The errors are greatest for the

shortest lengths considered, which may well be expected because of the difficulty of distributing both sand and water uniformly over the cross-section of the chamber at the inlet end and for other reasons which will be explained subsequently. The errors for the chambers complete, although small, are explainable in terms of factors not accounted for by the theory.

Factors Unaccounted for in Theory.—Reference to the assumptions made in the development of the theory will indicate that only one of the assumptions is fully met in the settling of sand; namely, that the particles do not coalesce in settling. Actual sewage grit contains a variety of substances other than sand, and some of the particles are mixed, or coated, with grease or organic matter. Some flocculation or coalescence in settling, therefore, probably does occur with sewage grit. Nevertheless, sewage grit is the only suspension commonly dealt with by sanitary engineers which approximates the assumption of no flocculation. It is the only suspension, therefore, which can be considered in terms of the foregoing theory. It is not possible to make a settling-velocity analysis of a flocculent suspension, because the particles are changing their settling velocities continuously.

The errors introduced by the assumption of uniform horizontal velocity in the chamber arise from two sources, short-circuiting and turbulence.

The effect of short-circuiting is to send some of the liquid through the chamber in less than the detention period and to retain some of it for longer periods. The overflow rate, therefore, is different for different portions of the contents of the chamber. A correction for this effect may be made if dye studies are available for the chamber provided, of course, that the velocity is great enough to maintain stable flow. The author presents results of dye studies for only the model grit chamber. These results are shown approximately by the writer in the form of a curve (see Fig. 14). Since the effect of short-circuiting is to reduce the removal given by the theory, and since the actual removal in the model was greater than that given by the theory, this curve cannot be used to explain the discrepancies for the model chamber.

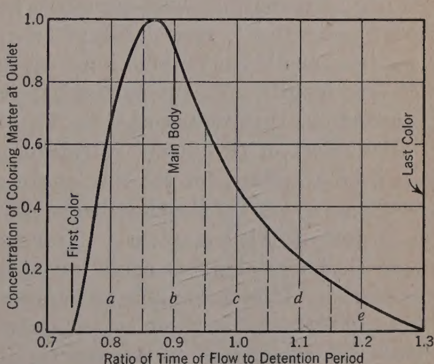

FIG. 14.—DYE CURVE TO SHOW SHORT CIRCUITY IN MODEL GRIT CHAMBER

The writer is at a loss to state precisely why the actual removal in the model exceeded the theoretical, whereas in both the Grand Rapids and Dearborn tests it was less than the theoretical. He suggests, however, that the manner of adding the sand at the inlet of the model might have been such as to carry much of it directly to the bottom. The second of the assumptions upon which the theory is premised may thus have been wide of the truth for the model and approximately true for both the Grand Rapids and Dearborn experiments. Since no measure of the accuracy of this assumption was made, this explanation must be regarded as hypothetical.

In order to explain the procedure to be used in correcting for short-circuiting, let it be assumed that the short-circuiting in the Dearborn chamber is also represented by Fig. 14 for the chamber as a whole. The time of settling of the suspension may be assumed equal to the time of passage of the dye through the chamber. The portion represented by a, Fig. 14, is approximately 26.6% of the whole, and it passed through the chamber in 81.5% of the detention period. The overflow rate for this portion is, therefore, $\frac{1}{0.815}$ times the average value of V_0 for the chamber, or 1.20 cm per sec. The corresponding removal from Curve 2, Fig. 13, is 94.8 per cent. These values are given in Table 10 with similar values for the other parts of the dye curve of Fig. 14. The weighted average

TABLE 10.—EFFECT OF SHORT-CIRCUITING IN DEARBORN, MICHIGAN, CHAMBER.

Area under dye curve	a	b	c	d	e	Total
Ratio to total	0.266	0.376	0.198	0.097	0.063	1.0
Time ratio	0.815	0.895	0.995	1.095	1.2	0.9
V_0, in centimeters per second	1.20	1.095	0.985	0.895	0.817	....
P_0	0.13	0.115	0.10	0.09	0.08	....
Removal ratio	0.948	0.956	0.962	0.967	0.973	....
Fractional removal	0.252	0.360	0.191	0.094	0.061	0.958
Removal neglecting short-circuiting	....	....	....	....	....	0.962

removal corrected for short-circuiting is 95.8%, as compared with 96.2% without the correction. Hence, it may be concluded that the effect of short-circuiting is very small in a chamber in which the increase in removal is very small with increased length. This is particularly true if the short-circuiting is of as little magnitude as that represented by Fig. 14.

The effect of turbulence in retarding settling in the author's experiments should be examined for the full length of the chambers only, inasmuch as the deposit in any part of the chamber is influenced by transportation of sand along the bottom after it has settled. Transportation out of either the Grand Rapids or the Dearborn chamber could have occurred only if the material were held in suspension. It is reasonable to assume that most of the discrepancy between the actual removal in these two chambers and the removal given by the simple theory can be charged to transportation in suspension.

The phenomena of transportation of matter in suspension due to turbulence is just beginning to be understood. The recent developments in the theory of fluid turbulence in pipes make available much of value in the study of sedimentation. In turbulent flow, the shearing forces due to velocity gradients are accompanied by eddy currents. The magnitude of these eddies may be stated in terms of the "mixing length" and "mixing velocity." The effect of these eddies in retarding settlement is nil if the concentration of suspended matter is uniform throughout. If, however, there exists a concentration gradient, the effect of an eddy is to carry material from regions of higher concentration to regions of lower concentration in a manner precisely analogous to the process of diffusion. If, then, the concentration of grit in suspension is greater near the bottom of the chamber than above it, the effect of turbulence is to retard

settling. If the concentration gradient with depth is just such that the downward motion due to settling is balanced by the upward motion due to turbulence the particles will be held in suspension indefinitely.

von Kármán, Ippen, and Rouse[13] have shown the application of the mechanics of fluid turbulence to the transportation of sediment in suspension, and have presented an equation for the concentration gradient at equilibrium in terms of the settling velocity of the particle and the so-called "friction velocity." Since the "friction velocity" may be stated in terms of the mean chamber velocity, this equation may be given in terms of the ratio of the particle settling velocity to the chamber velocity. Mr. Rouse also presents a diagram, first developed by Ippen, for the rapid solution of the equation.

The writer has examined this diagram for the grit-chamber experiments presented by Mr. Hubbell, in order to estimate what portion of the sand might be subject to permanent transportation in suspension if the grit chambers were long enough for equilibrium to be reached. The examination reveals that about 1% of the Grand Rapids sand, 32% of the Dearborn sand, and 35% of the Detroit grit (in chambers, 15 ft deep) are small enough to be subject to such suspension. This statement does not mean that the actual transportation in suspension will be anything like these values; because, for transportation to take place, the proper concentration gradient must exist before the particles settle out. The extension of the turbulent flow theory to cover the effect of turbulence in retarding settling before equilibrium has been reached has not yet been made.

The fourth assumption made in the development of the simple theory of sedimentation of discrete particles (namely, that a particle is removed when it strikes the bottom) is correct for a tank as a whole, provided the outlet is above the bottom. For a correlation of the Hubbell experiments with the theory, however, the assumption is not correct except for the whole chambers. Movement of particles along the bottom by traction, which was observed by the author to take place in at least one case, results in less deposit near the inlet and greater deposit near the outlet. The removal "efficiency" of the sections near the inlet for both the Grand Rapids and the Dearborn chambers, it will be noted, was less than that of the entire chamber. A part of this effect is doubtless due to "bed movement." Many studies have been made in recent years of the factors influencing stream-bed transportation, but the results thus far obtained are not sufficiently consistent to be used in predicting bed movement.

Conclusion.—A thorough understanding of the mechanism of settling of non-flocculent particles is essential before a rational approach can be made to the theory of settling of flocculent suspensions. The forces at work in the settling of discrete particles are also at work in the clarification of flocculent suspensions, but, in addition thereto, is the phenomenon of coalescence.

Data such as those presented by Mr. Hubbell are of great value in the development of the sedimentation theory. Many data have been collected upon the clarification of flocculent suspensions in full-scale settling tanks and in small pilot plants, but practically all these data are valueless in the development of the sedimentation theory because of the failure of the experimenters to measure all the variables. Most of the experiments that have been made, have

[13] *Transactions*, Am. Soc. C. E., Vol. 102 (1937), pp. 534–537.

had as their purpose the solution of particular sedimentation problems. The writer submits that if all the variables are measured in such experiments the particular problem may be solved with greater facility, and what is more important, the work will be of value to others. The ultimate goal is a theory which will enable engineers to design tanks intelligently from small-scale quiescent settling tests made upon the suspension to be clarified. In practically all experiments thus far made, settling characteristics, as distinct qualities of the suspensions, have not been measured. These qualities have been obscured by the characteristics of the basins in which the experiments have been run.